

The 1051 s period of the interacting binary white dwarf AM CVn

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Summary. The close binary system AM CVn consists of two white dwarfs; the lower mass white dwarf, a helium white dwarf, is transferring mass to its higher mass companion. The light curve of AM CVn has a double humped variation with a period of 1051 s that previously was identified with the orbital period of the system. An earlier measurement of the 1051 s period seemed to show that it was increasing rapidly, too rapidly to be easily understood. We have, therefore, remeasured the rate of change of the 1051 s period. We find that the period is changing two orders of magnitude more slowly than previously thought: $dP/dt = (-3.2 \pm 0.6) 10^{-12} \text{ s s}^{-1}$. Since the mass losing star in AM CVn is a white dwarf, the orbital period of the system must be increasing. We have found that the 1051 s period is decreasing, and, therefore, it cannot be caused by orbital motion. We reinterpret the 1051 s period as the rotation period of the accreting white dwarf, which must then be magnetized. We have also found a 1011.4 s period in the light curve that may be related to the orbital period, but cannot itself be the orbital period. Finally, we show that the double humped light curve may really be a single humped light curve with a period of 525.5 s.

Key words: close binaries – white dwarfs – cataclysmic variables – AM CVn

I. Introduction

Although AM CVn can be found in both the Malmquist (1936) list of faint stars and the Humason and Zwicky (1947) list of faint blue objects, its truly peculiar nature was first noted by Greenstein and Matthews (1957). They found that its spectrum consisted of extremely broad ($\sim 100 \text{ \AA}$), shallow ($\sim 5\%$ deep) absorption lines of neutral helium. The widths of the lines were similar to the widths of the lines of DB white dwarfs, but the strengths and profiles of the lines were unlike those of any white dwarf. Shortly thereafter Smak (1967) discovered that AM CVn has a double humped brightness variation with an amplitude of $\sim 2\%$ and a period of only 1051 s. A flood of models for AM CVn appeared in the literature following Smak's discovery (see Robinson and Faulkner, 1975 for references to these models), but the rapid flickering found in its light curve by Warner and Robinson (1972) and the failure of AM CVn to show strong X-ray emission (Coe et al., 1978) eliminated all but one class of models: AM CVn must be a close binary system in which a white dwarf is accreting mass transferred to it from its companion star. The structure of AM CVn must, in fact, be similar to the structure of the cataclysmic variables (Robinson, 1976).

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Faulkner et al. (1972) were the first to recognize that both stars in AM CVn are white dwarfs. They argued that the mass losing star must be a helium star because there are no hydrogen lines visible in the spectrum of AM CVn, but it must not be burning helium because the luminosity of AM CVn is too low. The mass losing star must, therefore, be a white dwarf with a helium atmosphere. In addition, they assumed that the orbital period of the binary was equal to the 1051 s photometric period, and that the only mechanism driving the evolution of the system was the removal of orbital angular momentum by gravitational radiation. From these assumptions they could predict that the rate of change of the orbital period should lie between $3.6 10^{-13}$, and $7.3 10^{-13} \text{ s s}^{-1}$.

In the Faulkner et al. model, the rate of change of the orbital period is set by the rate of emission of gravitational radiation from the binary, so a measurement of the rate of change of the period would test the theory of gravitational radiation. As a result, considerable effort has been expended in measuring the period and the rate of change of the period of AM CVn. The early attempts to measure the period produced conflicting results for reasons we will discuss in Sect. II, but the conflicts seemed to have been resolved by the unusually extensive observations of Patterson et al. (1979). They found that the period of AM CVn was increasing at a rate of $3.8 10^{-10} \text{ s s}^{-1}$. This rate is 1000 times faster than the rate predicted by gravitational radiation; indeed, it is so rapid that it cannot easily be produced by any known mechanism (J. Faulkner, private communication).

The difficulty in accounting for the rapid period change found by Patterson et al. (1979) suggested to us that there may have been some problem in their period determination. Therefore, we made an extensive series of photometric observations of AM CVn to determine precise value of the 1051 s period and its rate of change. This paper presents the results of the observations. The period change derived by Patterson et al. is incorrect; the true rate of change of the period is more than 100 times smaller, and, furthermore, the period is *decreasing*. We also show that the orbital period of AM CVn cannot be decreasing, and therefore that the 1051 s period cannot be the orbital period. We reinterpret the 1051 s period as the rotation period of the accreting white dwarf.

II. Observations

a) The new data

The new observations of AM CVn consist of 22 nights of high speed photometry, mostly in unfiltered light, obtained in 1982 and

Table 1. Observations of AM CVn

RUN NUMBER	START TIME (UT)	RUN LENGTH	INT TIME	TELESCOPE	FILTER	OBSERVERS*
2632	1982 Jan 24, 9 ^h 5 ^m	3 ^h 20 ^m	5 ^s	McD 2.1 m	none	EN, KO
2638	1982 Jan 27, 9 ^h 25 ^m	2 ^h 48 ^m	6 ^s	McD 2.1 m	none	EN, KO
2641	1982 Jan 28, 9 ^h 8 ^m	3 ^h 26 ^m	6 ^s	McD 2.1 m	none	EN, KO
35	1982 Feb 17, 8 ^h 36 ^m	4 ^h 0 ^m	5 ^s	McD 0.9 m	none	JES
42	1982 Feb 21, 8 ^h 42 ^m	3 ^h 58 ^m	5 ^s	McD 0.9 m	none	JES
44	1982 Feb 22, 7 ^h 7 ^m	0 ^h 52 ^m	5 ^s	McD 0.9 m	none	JES
58	1982 Apr 17, 2 ^h 59 ^m	3 ^h 2 ^m	5 ^s	McD 2.1 m	none	RB, DM
60	1982 Apr 25, 3 ^h 10 ^m	4 ^h 5 ^m	5 ^s	McD 2.1 m	none	JES
78	1982 Apr 28, 7 ^h 16 ^m	1 ^h 40 ^m	5 ^s	McD 2.1 m	none	JES
2666	1982 May 22, 3 ^h 16 ^m	3 ^h 31 ^m	5 ^s	McD 0.9 m	none	ER, DW
82	1982 May 29, 3 ^h 57 ^m	3 ^h 38 ^m	5 ^s	McD 0.9 m	none	JES
88	1982 May 31, 3 ^h 52 ^m	3 ^h 9 ^m	5 ^s	McD 0.9 m	none	JES
95	1982 Jun 2, 4 ^h 24 ^m	2 ^h 40 ^m	5 ^s	McD 2.1 m	none	JES
100	1982 Jun 4, 3 ^h 27 ^m	1 ^h 57 ^m	5 ^s	McD 2.1 m	none	JES
103	1982 Jun 5, 3 ^h 36 ^m	2 ^h 51 ^m	5 ^s	McD 2.1 m	none	JES
2677	1982 Jun 17, 3 ^h 33 ^m	1 ^h 24 ^m	5 ^s	McD 2.1 m	none	ER, SB
2760	1982 Dec 14, 11 ^h 2 ^m	1 ^h 8 ^m	6 ^s	McD 2.7 m	B	ER, EN
2765	1982 Dec 15, 10 ^h 45 ^m	1 ^h 35 ^m	5 ^s	McD 2.7 m	B	ER, EN
135	1983 Jan 19, 3 ^h 12 ^m	3 ^h 25 ^m	20 ^s	LP 0.6 m	none	JES, RBK
136	1983 Jan 21, 1 ^h 27 ^m	3 ^h 25 ^m	20 ^s	LP 0.6 m	none	JES
2782	1983 Mar 19, 4 ^h 51 ^m	1 ^h 58 ^m	5 ^s	McD 0.9 m	U	DW, SB
2799	1983 Apr 16, 5 ^h 19 ^m	1 ^h 57 ^m	5 ^s	McD 0.9 m	U	SB, DW

*OBSERVERS: EN = R. E. Nather, KO = S. O. Kepler, JES = J. -E. Solheim,
 RB = R. P. Binzel, DM = J. D. Mulholland, ER = E. L. Robinson,
 RBK = R.B. Kalleberg, DW = D. E. Winget, SB = S. Balachandran

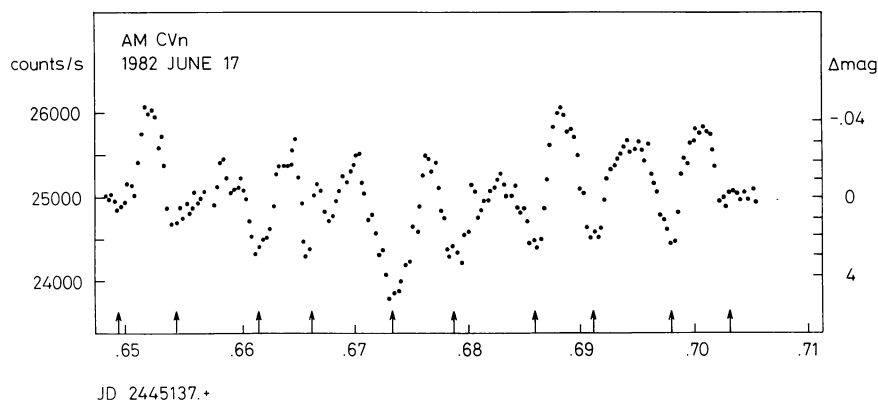


Fig. 1. A portion of the light curve of AM CVn on the night of 1982 June 17. The light curve was measured with a blue sensitive photomultiplier tube used without any filters, and approximates a *B* light curve. The integration time in the original data was 5 s, but the data have been averaged to 25 s per point for the figure. The arrows point to minima in the light curve. The obviously uneven spacing of the arrows emphasizes that the displacements in the times of minima of AM CVn are a real property of the light curve, not artifacts of the measurement process

1983 on the 0.9 m, 2.1 m and 2.7 m telescopes at McDonald Observatory and on the Swedish 0.6 m telescope at La Palma. The data obtained at McDonald Observatory are similar in most respects to the data on AM CVn published by Warner and Robinson (1972) and by Patterson et al. (1979); a complete discussion of our procedure for taking and reducing the data can be found in those papers. The La Palma data were obtained with a two aperture, single channel photometer. These data have a 26 s cycle time (20 s on AM CVn, 2 s dead time, 2 s on sky, 2 s dead

time); otherwise they are similar to the McDonald data. Table 1 gives more detailed information on the individual observing runs. A portion of the light curve of AM CVn on the night of 1982 June 17 is shown in Fig. 1.

b) The times of minima and the alias problem

Without exception, the published descriptions of the 1051 s variation agree that it has two humps per period. The two minima

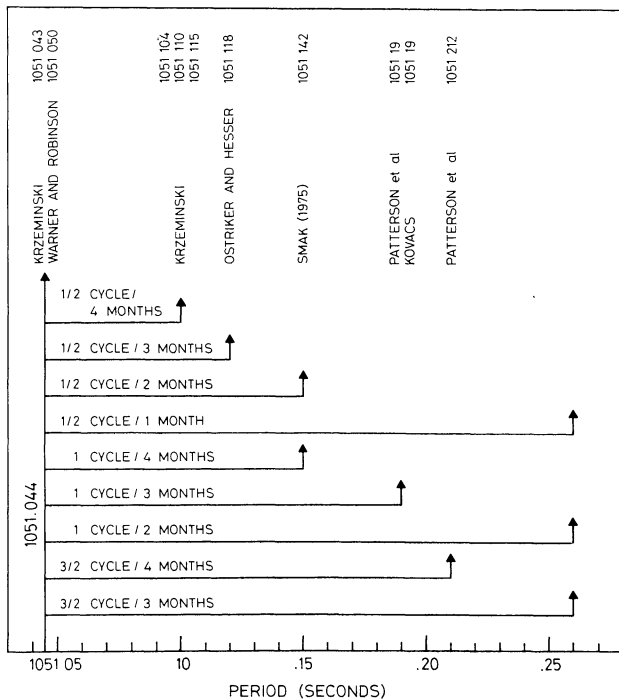


Fig. 2. The half-cycle aliases of the 1051 s period. The best period for AM CVn in 1982–1983 was 1051.044 s; the arrows show the periods that would be found if the counts of the number of elapsed cycles in the light curve were in error by 1/2 cycle in an integer number of months. The month is the lunar month, the interval between successive new moons and the typical interval between successive observing runs. All published periods of AM CVn are given at the top of the figure (we have used the the individual periods published by Krzemiński rather than the discredited period corresponding to a cyclic *O–C* diagram). The correspondence between the alias periods and the published periods demonstrates that aliasing is a severe problem in AM CVn, and that claims that the two maxima in the light curve of AM CVn can be reliably distinguished are suspect

between the humps have unequal depths, the two maxima have unequal heights, and the lengths of the two humps are nearly but not exactly equal. In consequence, it has been thought possible to distinguish between the two maxima or minima, at least if the light curve is long enough to average out random variations. For example, Smak (1967) identified the deeper minimum as the primary minimum, and Krzemiński (1972) identified the depression preceding the broader maximum as the primary minimum.

In the next section we will demonstrate that this description of the 1051 s variation is not correct, and that the minima and maxima are so similar they cannot be uniquely identified, even in principle. Even if they could be identified, there is rarely enough data available to identify them unambiguously because the light curve of AM CVn is extremely irregular. It has random flickering with an amplitude that often exceeds the amplitude of the 1051 s variation and can totally mask it (Warner and Robinson, 1972; Patterson et al., 1979). There are also systematic distortions in the amplitude and shape of the 1051 s variation. For example, the 1051 s variation in the light curves obtained by Warner and Robinson (1972) was sometimes double humped, sometimes single humped, and sometimes disappeared altogether. There can even be systematic displacements in the *phases* of the minima – an unusually clear example of this is shown in Fig. 1. The

irregularities in the light curve of AM CVn, particularly the systematic distortions, can only be averaged out with long light curves. Thus, the mean light curves constructed by Warner and Robinson (1972) from data taken on three different nights are different from each other even though more than two hours of data were used for each mean light curve.

Observing runs devoted to AM CVn typically are scheduled near new moon and tend to be separated by an integer multiple of one lunar month. If the minima of the 1051 s variation are misidentified, a cycle count error of 1/2 cycle in an integer number of months will be introduced. The incorrect cycle count will cause an incorrect period, which we call a half-cycle alias. Figure 2 shows that these half-cycle aliases have been causing problems in finding the correct period of AM CVn. The best period for AM CVn in 1982–1983 was 1051.044 s; the arrows in the figure point to the 1051.044 s period and all its important half-cycle aliases. At the top of the figure we have plotted all the published periods of AM CVn. The correspondence between the alias periods and the published periods is perfect, demonstrating first, that half-cycle aliasing is a severe problem, and second, that claims that the primary and secondary minima can be distinguished reliably are suspect. We believe that most of the confusion over the period of AM CVn has been caused by a failure to recognize the importance of these half-cycle aliases.

Because of the problem with the half-cycle aliases, we have extracted the times of minima from our data as follows. For each night of data we measured the time of every minimum that was clearly present in the original light curve, and calculated the phases of the minima with respect to a 1051 s period. We then divided the minima into two groups separated by about 0.5 in phase and calculated a mean phase for each group. Finally, we calculated the heliocentric Julian date for both mean phases; both mean phases are recorded in Table 2 without prejudice. This procedure is, of course, identical to the usual way of finding the times of minima for AM CVn except that we never attach the terms primary and secondary to the times of minima.

c) The old data

We have collected all published times of minima of AM CVn and have listed them in Table 2 along with our new timings. If two timings were reported for any night, one each for putative primary and secondary minima, we have included both timings, but without the designations “primary” and “secondary”. Most of the timings made at McDonald Observatory before 1982 have already been published by Warner and Robinson (1972) or by Patterson et al. (1979), but we have remeasured the times of minima from the original data and have used the remeasured timings in the table, not the published timings. All the remeasured timings agree with the published timings except the minimum at HJD 3640.70334 in Table 1 of Patterson et al. (1979) which we have replaced by HJD 3640.65877. We have also included timings from several archival light curves that have not been published previously.

The data obtained by Ostriker and Hesser (1968) was published only as a figure showing the mean light curve of AM CVn. We extracted times of minima from the figure and, using the same ephemeris used by Ostriker and Hesser, we updated the times of minima to a day on which they actually observed. It was necessary to refer the timings to a day on which AM CVn was observed because the period used by Ostriker and Hesser was an alias of the true period and gives incorrect timings on all days except the days of actual observations.

Table 2. Heliocentric times of minima of AM CVn

	JD ₀ 2,400,000+	
37700.00975	43578.73808	44993.94601
37700.01603	43578.93338	44993.95252
37732.00511	43579.72390	44996.97517
37732.01148	43579.82090	44996.98090
37815.77202	43580.70923	44997.94881
39527.98427	43582.89948	44997.95498
39527.99084	43584.90598	45017.94564
39528.98239	43599.85792	45017.95185
39528.98884	43601.68275	45021.93721
39589.86658	43601.86546	45021.94351
39589.87298	43609.67704	45022.82566
39650.40110	43609.87100	45022.83126
39650.40767	43610.72328	45076.68639
39673.79429	43611.85444	45076.69353
39673.80078	43612.86319	45084.71563
39909.77985	43613.83726	45084.72166
39909.78630	43615.78360	45087.82909
40680.82312	43635.71979	45087.83573
40976.94640	43637.76976	45111.69592
41032.79876	43640.65877	45111.70276
41032.80520	43660.88739	45118.72722
41034.76930	43664.53795	45118.73343
41836.73571	43667.57959	45120.73508
41837.72139	43714.65222	45120.74140
42070.94140	43722.64774	45122.74114
42070.94790	43846.94515	45122.74750
42071.87830	43865.98794	45124.68791
42071.88460	43866.96728	45124.69412
42072.94890	43867.99541	45125.72210
42099.83550	43899.92083	45137.67896
42099.84180	44291.59724	45137.68618
42189.70799	44291.60362	45317.96860
42849.77990	44344.44859	45317.97518
42900.74560	44344.45464	45318.97864
42904.70070	44345.33732	45318.98561
42925.66380	44345.34296	45353.74661
42926.64890	44345.42219	45353.75245
42927.68250	44345.42829	45355.64494
42928.66820	44347.34490	45355.65101
43575.72125	44347.35088	45412.74484
43576.87696	44347.43583	45412.75074
43577.98386	44347.44325	45440.77196

Times of minima in the light curves obtained by Smak in 1962 (Smak, 1967) were extracted and reported by Krzemiński (1972). Unfortunately, Krzemiński did not refer the times to any of the days on which Smak observed, choosing instead to refer the time to a day near the middle of the observing season. Therefore, using the same ephemeris used by Krzemiński, we have updated the times of minima to a day on which Smak did observe.¹

The remaining timings were culled from Krzemiński (1972), Smak (1975), and Kovacs (1981). The timings given by Krzemiński (1972) for his own data were times of a “midpoint” in the light curve; we have modified these timings to correspond to times of minima. We have not included the timing derived by Smak (1975) from his multicolor data because he assigned a low reliability to the timing, but except for this one timing, Table 2 contains every time of minimum for AM CVn of which we are aware.

III. Determination of the period

a) Power spectrum analysis

We calculated the power spectrum of the light curve of AM CVn to search for additional periods in the light curve, to eliminate any possibility that the 1051 s period was a 24 h alias of the true period, and to provide a reliable initial period for refinement by further analysis. For these purposes it was necessary to take the power spectrum of a light curve stretching over several successive nights (with gaps during the days). There were two subsets of data suitable for the power spectrum, both subsets from old data. One subset consisted of the four nights of data from 1976 May 27 to May 30; the other consisted of the five nights of data from 1978 March 8 to March 12 [the light curves from 1978 March 11 and March 12 are shown in Patterson et al. (1979)]. We calculated the power spectra using the method described in Kepler et al. (1982). The two subsets yielded nearly the same results, but the power spectrum of the 1978 March data was of much higher quality because there were more nights in the subset and because the light curves on each night were much longer. The two significant portions of the power spectrum of these data are shown in Fig. 3.

Despite the many peaks in the power spectrum, there are only two significant periods, one at 525.5 s and one at 1011.4 s. All the other peaks are aliases of these two periods; the strongest peaks are one day aliases. The 1023.6 s alias of the 1011.4 s period was slightly stronger than the 1011.4 s period in the power spectrum of the 1976 May data, but since the power spectrum of the 1978 data had a higher quality, the 1011.4 s period is preferred. The strong peak at 525.5 s confirms that the correct period of AM CVn is, indeed, either 525.5 s or 1051 s rather than a one day alias of these periods. Surprisingly, however, there is no significant power at 1051 s in either power spectrum. The lack of power at 1051 s has three important implications.

1. Since there is no power at 1051 s, there is no permanent difference between two successive minima in the 1051 s period. The perception that there is a difference has been caused by the 1011.4 s period. When the 1011.4 s period – and its harmonics – are added to the 525.5 s period, successive minima will look different and will be displaced in phase by different amounts, giving an appearance

¹ Krzemiński’s measurements of the times of minima in the Ostriker and Hesser (1968) data suffered from the same problem. A comparison of Krzemiński’s timings for the Ostriker and Hesser data with our independent timings for the Ostriker and Hesser data confirmed the validity of our procedure for correcting his timings

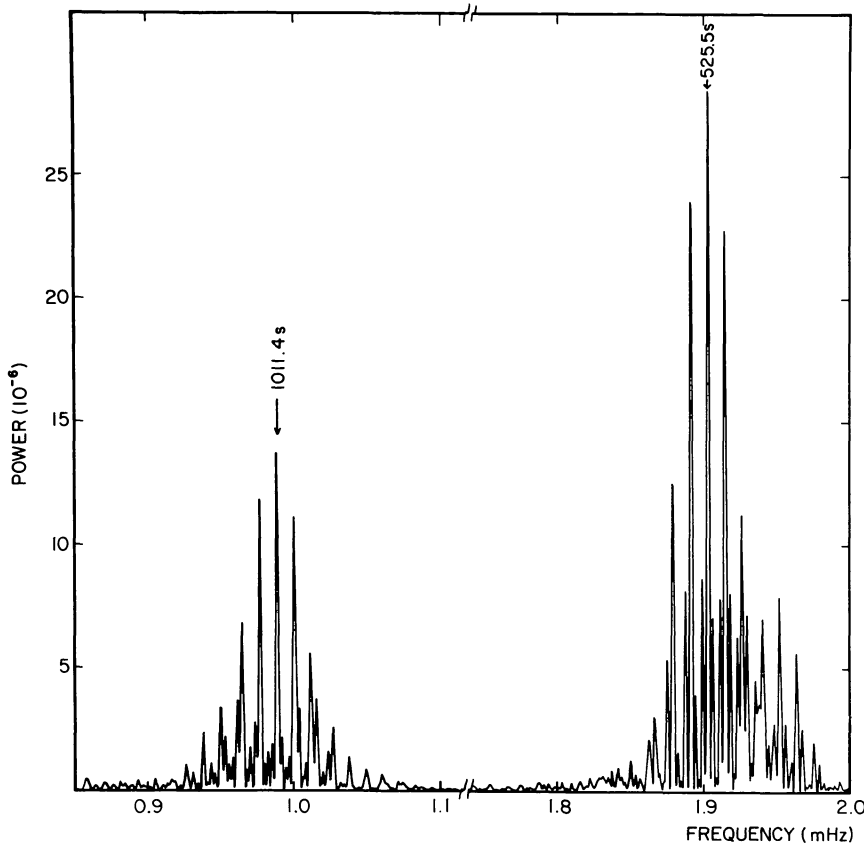


Fig. 3. The power spectrum of the light curve of AM CVn. The total light curve was made up from the individual light curves obtained on five successive nights in 1978 March; the lengths of the individual light curves were between 2.3 and 6.7 h. Only the two portions of the power spectrum showing the two significant periods are shown. One period is at 525.5 s, and the other is at 1011.4 s (or possibly 1023.6 s). All of the other peaks in the power spectrum are aliases of these two periods. There is no significant power at 1051 s

that there is a primary and a secondary minimum. However, the phase of the 1011.4 s period will slip with respect to the 525.5 s period over the course of several hours, and when it has slipped by 525.5 s the minima that are called primary and secondary will switch. The switch occurs every 3.7 h and the return to the original configuration takes 7.4 hr (or 10.9 hr if the secondary period is 1023.6 s).

2. There is no evidence whatsoever that the period of AM CVn is 1051 s rather than 525.5 s. Unfortunately, we cannot eliminate the 1051 s period because of a peculiar accident: the 1051 s period falls exactly on a side lobe of the 1011.4 s period in our power spectra (i.e. it is a 24 h alias of 1011.4 s). A 1051 s component could be present in the light curve but hidden by the side lobe, and if it were present, the true period of AM CVn would be 1051 s. We admit to a prejudice that the true period of AM CVn is 525.5 s, but to emphasize that the period is uncertain, we will continue to call the period a 1051 s period. We will refer to the 525.5 s period explicitly when our results change if the period is 525.5 s.

3. The 1011.4 s period is incommensurate with the 1051 s period, and, therefore, it is not possible for both periods to be caused by orbital motion. At least one other physical mechanism must be invoked. Furthermore, it now becomes necessary to ask whether we have properly identified which, if either, of these two periods is the orbital period.

b) The 1051 s Period

The method we used to refine the 1051 s period of AM CVn was to examine the (*O-C*) diagrams for ever larger subsets of the times of minima in Table 2, fitting linear ephemerides or, in the last step, a quadratic ephemeris to the (*O-C*) diagrams of possible periods. All

the (*O-C*) diagrams showed a large scatter in the times of minima. The obvious displacements of the minima in the light curve shown in Fig. 1 demonstrate that the scatter is an intrinsic property of the light curves caused by systematic distortions from the 1011.4 s period and from other unknown sources. The intrinsic scatter has a large standard deviation, about ± 0.09 cycle, and therefore even good periods produce (*O-C*) diagrams that look poor: a scatter of ± 2 standard deviations is equivalent to a peak-to-peak scatter of 0.36 cycle.

The large scatter in the times of minima also exacerbates the problems with aliasing. The problems are illustrated by Fig. 4 which shows the (*O-C*) diagram of the 1982–1983 minima of AM CVn for a period of 1051.12 s. Any point in an (*O-C*) diagram can be displaced vertically by an integer number of cycles. We have shown these possible displacements explicitly in Fig. 4 by repeating each point five times, separated by one cycle each time. Any straight line in the (*O-C*) diagram that passes through one repeat of every point is a good ephemeris for the times of minima. Because of the intrinsic scatter no straight line in Fig. 4 satisfies this requirement well. Conversely, a combination of $1/2$ cycle and 1 cycle ambiguities in the cycle count allows four straight lines, and thus four periods to fit the times of minima moderately well: 1050.99 s, 1051.04 s, 1051.21 s, and 1051.26 s.

Figure 5 shows the (*O-C*) diagrams of the 1982–1983 minima for these four periods in the more usual format. For the sake of clarity, filled circles have been used for minima clustered near an (*O-C*) value of zero, and open circles for minima clustered near 0.5. The 1051.044 s period yields an (*O-C*) diagram with the least scatter and is the preferred period, but the 1051.206 s and 1050.990 s periods also give acceptable (*O-C*) diagrams. The 1051.26 s period gives an (*O-C*) diagram with a larger scatter near

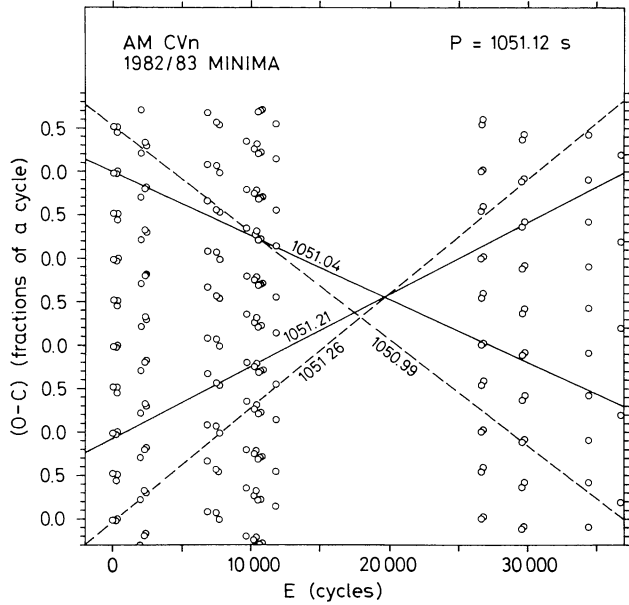


Fig. 4. The $(O-C)$ diagram of the 1982–1983 minima of AM CVn for a period of 1051.12 s. The diagram has been repeated five times, each repeat displaced vertically by one cycle with respect to the previous repeat. The diagram shows that a combination of 1/2 cycle and 1 cycle ambiguities in the cycle count allow several periods to fit the times of minima moderately well: 1050.99 s, 1051.04 s, 1051.21 s, and 1051.26 s. The 1051.04 s and 1051.21 s periods, shown by solid lines, are the best fits to the times of minima

$E=10,000$ cycles and is not a preferred period, but it has been included in Fig. 5 because the period would have been near this value if the ephemeris of Patterson et al. (1979) were correct. Thus, despite 22 nights of observations in the 1982 and 1983 observing seasons, three acceptable periods still remain.

There are 29 nights of data from 1978 and 1979, enough to eliminate all but two periods for those years. The two periods are 1051.043 s and 1051.205 s; the $(O-C)$ diagrams for the two periods are shown in Fig. 6. The reason we find a period of 1051.205 s instead of 1051.212 s, as found by Patterson et al. (1979) for these same years is, that we modified one time of minimum used by those authors and added times of minima on two additional nights. The difference is within the measurement error and is not significant. The 1051.043 s period is a halfcycle alias of the 1051.205 s period and was missed by Patterson et al. It should be noted that the times of minima show large systematic residuals for both periods. These systematic residuals are present no matter what period is chosen, and are not a symptom that the periods are incorrect.

Although we were unable to identify the correct period of AM CVn unambiguously up to this point, the possible periods in 1982–1983 were the same as the periods in 1978–1979 to within the measurement errors. We could conclude safely that the period of AM CVn is either 1051.04 s or 1051.21 s and that regardless of which period is correct, the period of AM CVn did not change substantially in the four years that elapsed between the two sets of observations. Linear least squares fits to the $(O-C)$ diagrams yielded periods of 1051.0437 ± 0.0012 s and 1051.2064 ± 0.0015 s for the 1982–1983 data, and 1051.0430 ± 0.0024 s and 1051.2047 ± 0.0019 s for the 1978–1979 data. Comparing the periods we find that $|dP/dt| < 3 \cdot 10^{-11} \text{ s s}^{-1}$.

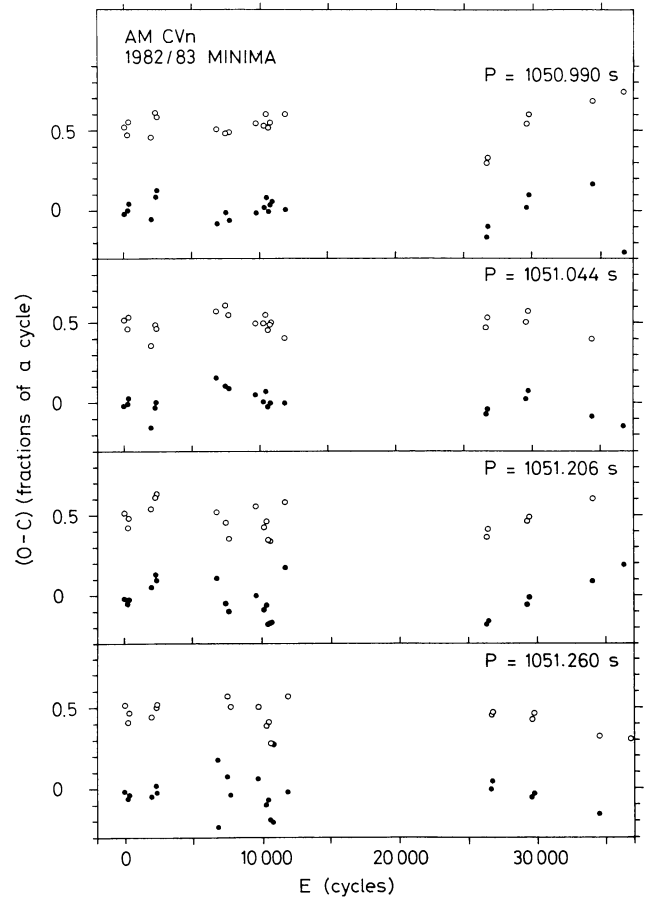


Fig. 5. The $(O-C)$ diagrams of the 1982–1983 minima of AM CVn for four periods. The periods are all half-cycle aliases of each other. For the sake of clarity, filled circles are used for minima clustered near an $(O-C)$ value of zero and open circles for minima clustered near 0.5. The 1051.044 s period produces an $(O-C)$ diagram with less scatter than the other periods and is the preferred period, but the 1051.206 s period also gives an acceptable $(O-C)$ diagram

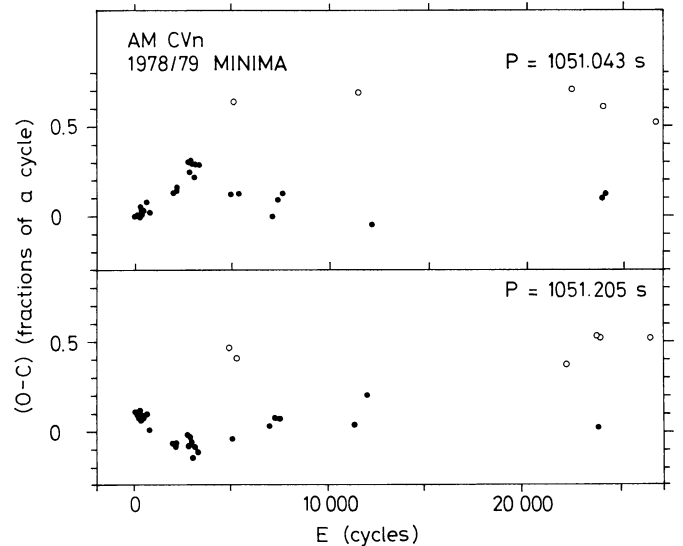


Fig. 6. The $(O-C)$ diagrams of the 1978–1979 minima of AM CVn for two possible periods. The two periods are half-cycle aliases of each other. For the sake of clarity, filled circles are used for minima clustered near an $(O-C)$ value of zero and open circles for minima clustered near 0.5. The scatter in the two $(O-C)$ diagrams is similar, and neither period is preferred

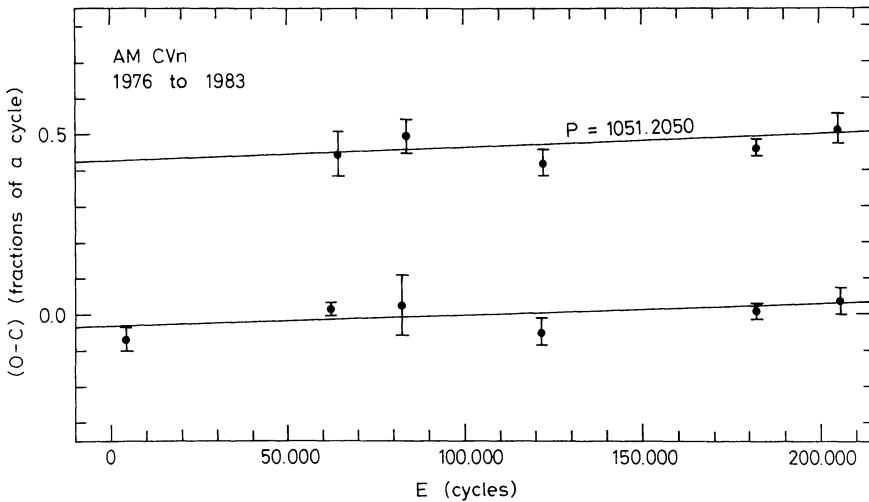


Fig. 7. The (*O*–*C*) diagram for the 1976 to 1983 minima of AM CVn with respect to a period of 1051.2047 s. All the times of minima measured in individual seasons have been averaged together to form seasonal normal points, one normal point for minima falling near phase zero and one for minima falling near phase 0.5. The best fitting linear ephemeris has a period of 1051.2050

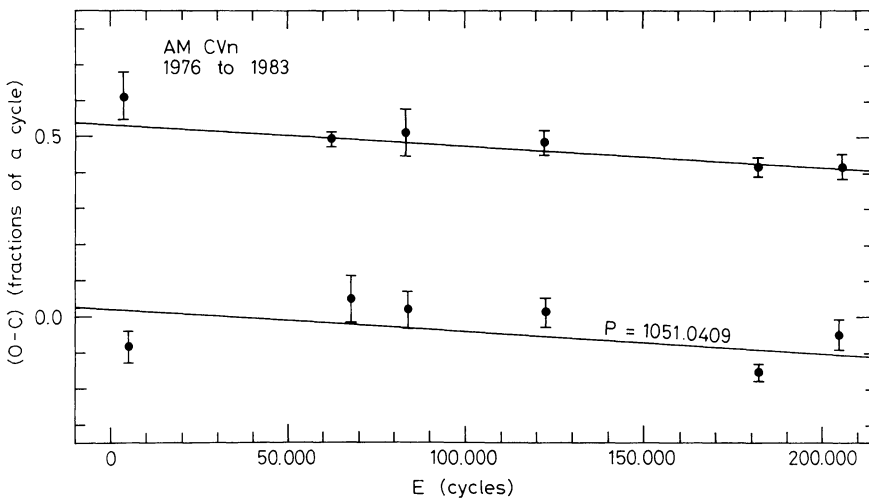


Fig. 8. The (*O*–*C*) diagram for the 1976 to 1983 minima of AM CVn with respect to a period of 1051.0415 s. All the times of minima measured in individual seasons have been averaged together to form seasonal normal points, one normal point for minima falling near phase zero and one for minima falling near phase 0.5. The best fitting linear ephemeris has a period of 1051.0409 s

These limits on the rate of change of the period were small enough that we could join together data taken several years apart. Therefore, our next step was to combine all the times of minima from 1976 to 1983. We constructed two (*O*–*C*) diagrams for these times of minima, one with respect to a period of 1051.0415 s and one with respect to a period of 1051.2047 s. Instead of plotting individual times of minima, we averaged together all the times of minima measured in individual seasons to form seasonal normal points, one normal point for minima falling near phase zero and one for minima falling near phase 0.5. Least squares fits of linear ephemerides to the (*O*–*C*) diagrams yielded periods of 1051.0409 ± 0.0002 s and 1051.2051 ± 0.0002 s. Both periods fit the times of minima reasonably well; up to this point it was again impossible to choose between the two periods. There was no evidence for any secular variation in either period. The upper limit to the rate of change of both periods during these years is $|dP/dt| < 5 \cdot 10^{-12} \text{ s s}^{-1}$, or $|dP/dt| < 1.6 \cdot 10^{-4} \text{ s yr}^{-1}$, both limits corresponding to the 70% confidence level.

The final step was to combine all the times of minima from 1962 to 1983. Once again we constructed two (*O*–*C*) diagrams for these times of minima, one for a period of 1051.0409 s and one for a period of 1051.2051 s. The 1051.2051 s period produced an unacceptable (*O*–*C*) diagram; even allowing for period changes, we

were unable to reduce the scatter in the diagram to a tolerable level. The 1051.0409 s period gave a satisfactory (*O*–*C*) diagram, but the (*O*–*C*) diagram required a decreasing period. The (*O*–*C*) diagram for a period of 1051.04086 s is shown in Fig. 9. For the sake of clarity, we have used two different symbols for the two groups of minima separated by 0.5 in phase. The solid lines in the figure are second order polynomials that have been fit to the times of minima by least squares, and the dashed lines correspond to a scatter of one standard deviation (0.09 in phase) about the solid lines. The two solid lines are parallel to within the measurement error, permitting us to combine the two lines into one equivalent ephemeris with two different zero points, one zero point for each set of minima. The resulting ephemeris is

$$T_{\min} = T_0 + 0.0121648423 E - 1.90 \cdot 10^{-14} E^2, \\ \pm 25 \pm 0.37$$

where E is the cycle number, and T_0 equals $\text{HJD } 2437699.9992 \pm 0.0005$ for one minimum and $\text{HJD } 2437700.0054 \pm 0.0006$ for the other minimum. The two minima are separated by 1/2 cycle to within the measurement error.

If the true period is 1051 s, the rate of change of the period is $(-3.2 \pm 0.6) \cdot 10^{-12} \text{ s s}^{-1}$; if the period is 525.5 s, the rate of change

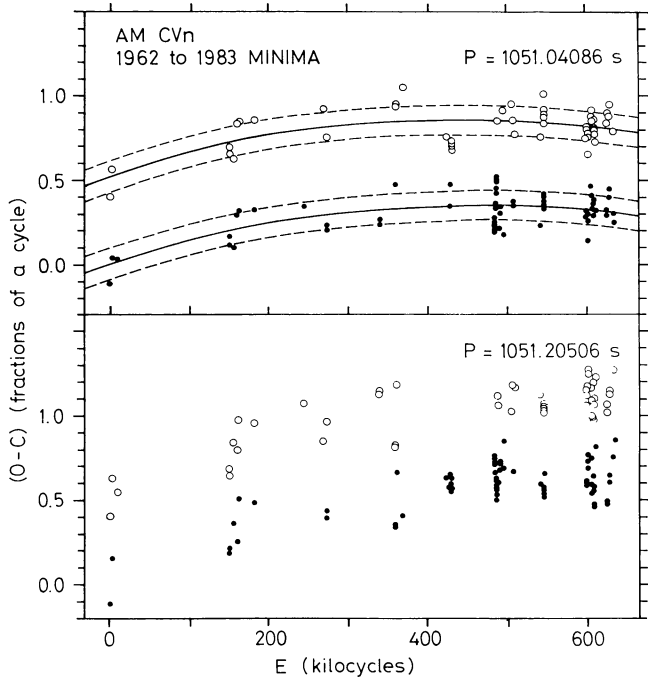


Fig. 9. The $(O-C)$ diagram with respect to a period of 1051.04086 s for all the minima of AM CVn. For the sake of clarity two different symbols have been used to show the two groups of minima separated by 0.5 in phase. The solid lines correspond to a 1051 s period changing at a rate of $-3.2 \cdot 10^{-12} \text{ s s}^{-1}$. The dashed lines correspond to a scatter of ± 0.09 cycle about the solid lines. The lower panel shows the $(O-C)$ diagram for a period of 1051.20506 s which gave an unacceptable fit

is $(-1.6 \pm 0.3) \cdot 10^{-12} \text{ s s}^{-1}$. Formally, the period change is significant to five standard deviations, but the distribution of points in the $(O-C)$ diagram is highly uneven, raising the possibility that the period change depends only on a small subgroup of the times of minima. To test for this possibility, we removed the 1962 timings from the $(O-C)$ diagram and recalculated the ephemeris. The resulting rate of change of the period was the same as for the whole data set to within the measurement errors, but the significance of the result decreased – as expected – by about the square of the ratio of the lengths of time spanned by the two data sets. We believe, therefore, that the period change we have found does not depend unduly on any one subset of the data.

The residuals in the $(O-C)$ diagrams, particularly the residuals in the $(O-C)$ diagrams shown in Figs. 5 and 6, increase and decrease systematically on time scales of a few months. To test whether these variations might be periodic, we calculated power spectra of the residuals. The residuals varied on a time scale of about 80 d in 1978–1979 and on a time scale of about 140 d in 1982–1983, in both cases with an amplitude of about 40 s, but we were unable to find any long lasting periodic components to the residuals.

IV. Discussion

Our observations show that the 1051 s period in the light curve of AM CVn is decreasing at a rate of $(-3.2 \pm 0.6) \cdot 10^{-12} \text{ s s}^{-1}$ [or at a rate of $(-1.6 \pm 0.3) \cdot 10^{-12} \text{ s s}^{-1}$ if the period is 525.5 s]. This rate is more than 100 times smaller than the rate found by Patterson et al.

(1979), and thus, although the new rate is still somewhat larger than the rate predicted by Faulkner et al., $\sim 5 \cdot 10^{-13} \text{ s s}^{-1}$, we have removed the discrepancy that motivated our observations. We have, however, replaced the original problem by a new one: the predicted rate of change is positive, but the observed rate of change is negative. The negative sign is a serious problem since, as we shall now show, the orbital period of AM CVn must be increasing if the mass losing star is a white dwarf.

a) The necessity of an increasing orbital period in AM CVn

The absolute magnitude of AM CVn has not changed appreciably in more than 50 yr (Malmquist, 1936; Humason and Zwicky, 1947; Greenstein, 1958), and the mean light curve of the 1051 s variation has not changed in more than 20 yr (Smak, 1967). We are justified, therefore, in assuming that the properties of the mass transfer, including the mass transfer rate, are constant on a time scale of years. To maintain a constant rate of transfer, the mass losing star must be in contact with its Roche lobe, $R_2 = R_L$, and the rate of change of its radius with mass must equal the rate of change of the Roche lobe radius with mass,

$$\frac{dR_2}{dM_2} = \frac{dR_L}{dM_2}. \quad (1)$$

Taken together these two requirements yield

$$q = \frac{d \ln(R_2)}{d \ln(M_2)} = \frac{d \ln(R_L)}{d \ln(M_2)}, \quad (2)$$

where q can be recognized as the exponent of the power-law mass-radius relation for the lobe filling star:

$$R_2 = b M_2^q. \quad (3)$$

For the large mass ratios we expect in AM CVn, the radius of the Roche lobe is given by

$$R_L = 0.459 A (M_2/M)^{1/3}, \quad (4)$$

where A is the semimajor axis and M is the total mass of the system (Kopal, 1959). Combining Eq. (4) with Kepler's third law, and then with Eq. (2) we find

$$\frac{d \ln(R_L)}{d \ln(M_2)} = \frac{1}{3} - \frac{2 M_2 \dot{n}}{3 n M_2} = q \quad (5)$$

where n is the orbital angular velocity. Since mass is being transferred in AM CVn, we must have $\dot{M}_2 < 0$, and since the mass losing star is a white dwarf, it must have $q \leq -1/3$ in its mass-radius relation (Hamada and Salpeter, 1961). Equation (5) then yields $\dot{n} < 0$. In summary, the orbital period of AM CVn must be increasing if the mass losing star is a white dwarf; the basic reason for this behavior is that the radius of a white dwarf increases as its mass decreases.

This argument would break down if the the mass of the lobe-filling star were so low that it could no longer be degenerate (i.e. if it were a helium planet), because it could then have a mass-radius relation with $q \sim 1/3$ (Zapolsky and Salpeter, 1969). A helium planet must, however, have a density less than about 10 g cm^{-3} . Faulkner et al. (1972) have shown that the mean density of the lobe filling star in a binary system with a large mass ratio is related to the orbital angular velocity by

$$\langle \rho \rangle = 3.7 \cdot 10^7 n^2, \quad (6)$$

where n is in s^{-1} and ρ is in g cm^{-3} . According to Eq. (6) the orbital period of AM CVn would have to be longer than about $1.2 \cdot 10^4$ s for the lobe filling star to be a non-degenerate helium planet. Since this is more than ten times longer than any period observed in AM CVn, the mass losing star cannot be a planet.

Although the orbital period of AM CVn must be increasing, the 1051 s period is decreasing. The 1051 s period cannot, therefore, be the orbital period of AM CVn. We note in passing that there is no direct evidence that the 1051 s period is the orbital period, and that, in fact, an attempt to find orbital radial velocity variations at this period was unsuccessful (Robinson and Faulkner, 1975).

b) A magnetic rotator model for AM CVn

We propose that the 1051 s period of AM CVn is produced by rotation of a magnetized, accreting white dwarf. The mass accreting star in AM CVn must then have a dipole magnetic field strong enough to dominate the accretion flow near the white dwarf, and it must be rotating with a period of either 1051 s if both poles of the field are visible, or 525.5 s if one pole dominates. The variation in the light curve is caused by changes in the visibility of the accretion columns as the white dwarf rotates. We emphasize that AM CVn is still a binary white dwarf with mass transfer from a low mass helium white dwarf to a higher mass white dwarf, but the orbital period is now indeterminate. In effect, AM CVn is the binary white dwarf equivalent to the intermediate polars (Warner, 1983).²

The model we propose has several advantages. First, the decrease in the 1051 s period becomes a spin-up of the rotating white dwarf caused by mass accretion. From angular momentum considerations, we can show that the mass accretion rate necessary to cause the observed spin-up is small. The rate at which angular momentum is added to the white dwarf by accretion is about

$$\dot{J}_{\text{spin}} = (GM_1 r_c)^{1/2} \dot{M}_1, \quad (7)$$

where r_c is the radius out to which the magnetic field dominates the accretion flow. Equation (7) implicitly assumes that the accreting material forms an accretion disk at radii greater than r_c . The angular momentum added to the white dwarf by accretion spins up the white dwarf at a rate

$$\dot{\omega} = I^{-1} (GM_1 r_c)^{1/2} \dot{M}_1. \quad (8)$$

The moment of inertia of the white dwarf, I , is equal to $r_g^2 M_1 R_1^2$, where r_g is the radius of gyration. Equation (8) and the equivalent of Eq. (3) for the mass accreting white dwarf together yield

$$\begin{aligned} \dot{M}_1 &= r_g^2 b^{3/2} G^{-1/2} (R_1/r_c)^{1/2} \dot{\omega} \\ &= 8.7 \cdot 10^{32} (R_1/r_c)^{1/2} \dot{\omega}, \end{aligned} \quad (9)$$

where, for a carbon-oxygen white dwarf we have adopted $r_g^2 = 0.25$, and $b = 9.3 \cdot 10^{19}$ (Hamada and Salpeter, 1961), and \dot{M}_1 is in g s^{-1} for $\dot{\omega}$ in s^{-2} . We expect R_1/r_c to lie between 0.1 and 1.0, and we have observed $\dot{\omega} = 1.82 \cdot 10^{-17} \text{ s}^{-2}$, so the mass accretion rate

² We are not the first to make this suggestion. Among others, Ostriker and Hesser (1968), Krzemiński (1972), and Voikhanskaya (1982) were aware of the magnetic rotator model for AM CVn, although only Voikhanskaya tried to develop the model. The implementation of the magnetic rotator model discussed by Voikhanskaya is different from ours, however

should be in the range

$$7.6 \cdot 10^{-11} < \dot{M}_1 < 2.4 \cdot 10^{-10} M_{\odot} \text{ yr}^{-1}.$$

The corresponding accretion luminosity is between 0.2 and $0.7 L_{\odot}$. Both \dot{M}_1 and L are a factor of two larger if the rotation period is 525.5 s instead of 1051 s.

This accretion luminosity is low, but it is consistent with other evidence that AM CVn is faint. The parallax of AM CVn, $0''.012 \pm 0''.007$, is marginally significant (Vasilevskis et al., 1975). With this parallax, the absolute visual magnitude of AM CVn is $+9.5_{-2.0}^{+1.0}$, making AM CVn only slightly brighter than single white dwarfs. The proper motion of AM CVn is 0.023 yr^{-1} (Holdenried, 1972). If the tangential velocity of AM CVn is less than 50 km s^{-1} , its distance must be less than 450 pc and its absolute visual magnitude must be fainter than +5.8, again suggesting a low luminosity for the system.

Another advantage to our model for AM CVn is that the morphology of its light curve becomes, if not comprehensible, at least less incomprehensible. Although the light curve of AM CVn has often been compared to the orbital light curves of cataclysmic variables, double humped light curves like that of AM CVn are rare among these variables. WZ Sge, the system most often compared to AM CVn, is one of the few cataclysmic variables that does have a double humped light curve (Robinson et al., 1979), but even WZ Sge does not compare well to AM CVn. Specifically, the light curve of WZ Sge is *always* double humped, but the light curve of AM CVn is sometimes single humped and occasionally has no humps at all (Warner and Robinson, 1972). Also, the average timings of the minima of AM CVn have a large scatter, about ± 0.1 of the 1051 s period, but the scatter in the timings of individual eclipses of WZ Sge is only about ± 0.005 of an orbital period, 20 times smaller than the scatter in AM CVn. Thus, the light curve of AM CVn is much less regular than the light curve of WZ Sge.

If, however, the 1051 s period of AM CVn is the rotation period of its accreting white dwarf, the light curve of AM CVn should be compared to the pulse light curves of the DQ Her stars or the intermediate polars, not to orbital light curves. The similarities between the light curves of these stars and the light curve of AM CVn are striking. The pulse light curves of the DQ Her stars and the intermediate polars usually have amplitudes between 0.5% and 20%, and, as in the case of the 33 s pulse of AE Aqr, can have two unequal maxima per cycle (Patterson, 1979). The scatter in the timings of the pulses of the DQ Her stars and the intermediate polars can be as large as the scatter in the timings of the minima in AM CVn. For example, both the 33 s pulse of AE Aqr and the 4022 s pulse of EX Hya have a scatter in their timings that can be up to ± 0.2 cycle (Sterken et al., 1983). Like the light curve of AM CVn, the pulse light curves of the DQ Her stars and intermediate polars are highly variable in shape and amplitude (Warner, 1983). The 33 s pulse of AE Aqr can even disappear altogether for ~ 100 cycles. Thus, the light curve of AM CVn is much more like the pulse light curves of the DQ Her stars and the intermediate polars than it is like the orbital light curves of the normal cataclysmic variables.

c) The orbital period of AM CVn

The orbital period of AM CVn cannot be determined with certainty from the available data, but we can place a lower limit to the orbital period from angular momentum considerations. The method is similar to the method used by Faulkner (1971) and

Paczynski (1967) except for the inclusion of the spin angular momentum of the accreting white dwarf.

The angular momentum in AM CVn can be stored in orbital motion and in the spin of the mass accreting star. It is likely that angular momentum is being removed from the system by several physical mechanisms, but only one mechanism, gravitational radiation, has been positively identified. Restricting, then, the mechanisms for angular momentum loss to gravitational radiation we have

$$\dot{J}_{\text{orb}} + \dot{J}_{\text{spin}} = \dot{J}_{\text{grav}}. \quad (10)$$

The rate of change of the spin angular momentum is given by

$$\dot{J}_{\text{spin}} = r_g^2 M_1 R_1^2 \dot{\omega}. \quad (11)$$

Using Eq. (3) with $q = -1/3$, $b = 9.3 \cdot 10^{19}$, $r_g^2 = 0.25$, and $\dot{\omega} = 1.82 \cdot 10^{-17} \text{ s}^{-2}$, we find

$$\dot{J}_{\text{spin}} = 3.94 \cdot 10^{22} M_1^{1/3}. \quad (12)$$

To find an expression for the rate of change of the orbital angular momentum, we first combine Eqs. (6) and (3) to find a period-mass relation for a lobe filling white dwarf. We use $q = -1/3$ and $b = 9.8 \cdot 10^{19}$ in Eq. (3) since the lobe filling star is a low mass helium white dwarf (Hamada and Salpeter, 1961). We find

$$M_2 = kn, \quad (13)$$

where $k = 1.21 \cdot 10^{34}$. This equation is implicit in Fig. 2 of Faulkner et al. (1972). If the total mass of the AM CVn system is being conserved during the mass transfer process, Eq. (13) also gives

$$\dot{M}_1 = -k\dot{n}. \quad (14)$$

The orbital angular momentum of a binary system is given by

$$J_{\text{orb}}^3 = \frac{G^2 M_1^3 M_2^3}{M n}, \quad (15)$$

where M is the total mass of the system. Making use of Eqs. (13) and (14), we find

$$\frac{\dot{J}_{\text{orb}}}{J_{\text{orb}}} = \frac{\dot{M}_1}{M_1} - \frac{2\dot{M}_1}{3kn}. \quad (16)$$

The second term on the right is much larger than the first, so

$$\dot{J}_{\text{orb}} = -6.97 \cdot 10^{10} M_1^{2/3} n^{-1/3}, \quad (17)$$

where we have adopted a transfer rate of $10^{-10} M_{\odot} \text{ yr}^{-1}$, and we have assumed that the mass ratio of the binary is large. The angular momentum emitted by gravitational radiation is (Faulkner, 1971)

$$\frac{\dot{J}_{\text{grav}}}{J_{\text{orb}}} = -\frac{32 G^{5/3} M_1 M_2 n^{8/3}}{5 c^5 M^{1/3}}. \quad (18)$$

Again using Eq. (13) and assuming a large mass ratio we find

$$\dot{J}_{\text{grav}} = -0.70 M_1^{4/3} n^{13/3}. \quad (19)$$

Inserting Eqs. (12), (17), and (19) into Eq. (10), we have

$$-6.97 \cdot 10^{10} M_1^{2/3} n^{-1/3} + 3.94 \cdot 10^{22} M_1^{1/3} = -0.70 M_1^{4/3} n^{13/3}. \quad (20)$$

This equation gives the orbital period as a function of M_1 . For $M_1 = 1.0 M_{\odot}$, the orbital period is 1990 s, and for $M_1 = 0.5 M_{\odot}$, the orbital period is 2070 s. The orbital period changes only slightly if the rotation period is 525.5 s: for $M_1 = 1.0 M_{\odot}$, the orbital period is 1750 s, and for $M_1 = 0.5 M_{\odot}$, the orbital period is 1860 s.

It is gratifying that we have found a second period in the light curve of AM CVn, the 1011.4 s period, since our model for AM CVn requires an orbital period in addition to the 1051 s (or 525.5 s) rotation period of the white dwarf. Unfortunately, none of the observed properties of AM CVn are known to vary at periods between 1750 and 2070 s. Twice the 1011.4 s period falls exactly within the range of predicted orbital periods if the rotation period is 1051 s, but whether this is a mere coincidence or it means that the orbital period of AM CVn really is 2022.8 s is unclear at present.

A more serious cause for concern is that Eq. (10) explicitly assumed that the only way angular momentum can be lost from the system is through gravitational radiation. As Eggleton has noted repeatedly (e.g. Eggleton, 1983), a small amount of systemic mass loss – by a stellar wind, perhaps – can totally dominate the angular momentum balance of a close binary system if the mass being lost is coupled to the binary system by a magnetic field. All the ingredients necessary for wind driven angular momentum loss are present in AM CVn, and if this mechanism is acting, gravitational radiation losses would not be necessary to drive mass transfer. The orbital period of AM CVn could, then, be much longer than 2000 s.

V. Conclusion

The original discrepancy that motivated our observations has now been removed, although in an unexpected way: the 1051 s (or 525.5 s) period changes far more slowly than originally reported, but the change is a decrease in the period, not an increase. We have also shown that the orbital period of AM CVn cannot be decreasing, and, therefore, the 1051 s period cannot be the orbital period. We have identified the 1051 s period with the rotation period of the accreting white dwarf in the system instead. Our work seems to have raised more questions about AM CVn than it has answered, however. Is the rotation period of the white dwarf 1051 s or 525.5 s? What is the orbital period of AM CVn? What mechanism is responsible for the 1011.4 s period, and how does that period relate to the orbital period? What role does gravitational radiation play in the angular momentum balance of AM CVn? What role does wind driven angular momentum loss play? We can hardly claim yet to understand AM CVn.

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