1986A&A...156..111C

Astron. Astrophys. 156, 111–120 (1986)

ASTRONOMY
AND
ASTROPHYSICS

H ɪɪ region age indicators

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Received August 31, 1984; accepted June 5, 1985

Summary. The evolution of some properties of H ɪɪ regions such as the relative volume R of the He^+ and H^+ zones, the equivalent width W_{H_β} of the H_β emission line and the ratio [O ɪɪɪ]/H_β of the lines [O ɪɪ] λλ4959, 5007 to H_β, was studied through models which take into account: a single burst for the formation of the ionizing association with different initial mass function IMF ($1 \leqq \chi \leqq 3$) and upper stellar mass limits ($30 \leqq M_u/M_\odot \leqq 120$); different chemical composition models of stellar evolution with and without mass loss. It was found that W_{H_β}, R and [O ɪɪɪ]/H_β decrease monotonically as a function of time and consequently they are good H ɪɪ region age indicators.

Key words: H ɪɪ regions – evolution models – age determination

1. Introduction

The H ɪɪ regions are objects specially important for the study of the stellar formation. Their presence prove the existence of a recent cycle of star formation. The exciting stars are of the types O or early B. These are massive, hot stars with a life shorter than about 10^7 yr. Then after this period the H ɪɪ region disappears. The age of an H ɪɪ region ionized by a group of young stars, intended as the time elapsed since the last burst of star formation, can be evaluated through the direct study of the exciting stars or from the properties that they induce in the ionized region. The observations of the exciting stars are often unfeasible. On the other hand, the strength of the H ɪɪ region spectra allows to study them even in extragalactic systems. Hence age indicators based on the integrated radiation of the regions could be of wider applicability.

Some evolutive models of global properties of H ɪɪ regions were developed recently. Dottori (1981) showed that the equivalent width W_{H_β} of the hydrogen H_β emission line depends strongly on the stage of evolution of the ionizing association. Lequeux et al. (1981) studied the evolution of the ultraviolet monochromatic spectrum at the wavelengths 1254, 1392, 1600 and 1900 Å, the Lyman continuum photon flux, the production of heavy elements and the colour temperature of the ionizing radiation of H ɪɪ regions.

These models were successfully applied to age determinations of extragalactic H ɪɪ regions (Dottori and Bica, 1981; Lequeux et al., 1981).

Following these ideas, we searched for global properties of H ɪɪ regions which have a suitable dependence on the time evolution to be usable as age indicators. From all the parameters analysed, three were selected for this purpose, namely the relative volume R of He^+ to H^+ zones, the equivalent width W_{H_β} of the H_β emission line and the ratio [O ɪɪɪ]/H_β of the lines [O ɪɪɪ] λλ4959, 5007 to H_β.

As an extension of the thermodynamical point of view these are *intensive parameters* (Planck, 1954, p.267; Callen, 1960, Chap. II), since they do not depend on the total mass of stars in a given system, but rather on the relative number of them. Such is not the case of the line intensities, which are *extensive parameters* (Callen, 1960, p.9).

2. The H ɪɪ models hypothesis

The evolutive behavior of the parameters W_{H_β}, R and [O ɪɪɪ]/H_β were studied by H ɪɪ region models which take into account the following hypothesis:

a) For the exciting stellar association

1. a burst of stars formation;
2. an initial mass function (IMF) of the form

$\psi(M)\alpha M^{-(1+\chi)}$ *with* $\chi = 1, 1.5, 2, 2.5$ and 3;

3. an upper stellar mass limit M_u

$30 \leqq M_u/M_\odot \leqq 120$;

4. a lower stellar mass limit of $0.5\,M_\odot$;
5. models of stellar evolution of Maeder (1980, 1981a,b), Hellings and Vanbeveren (1981) and Brunish and Truran (1982), for the more massive stars, with and without mass loss, and of Alcock and Paczyński (1978) and Mengel et al. (1979) for the less massive ones;
6. different chemical compositions, appropriate for different objects, according to the models of point 5:

(i) $X = 0.70$, $Y = 0.27$ and $Z = 0.03$ (Solar vicinities)
(ii) $X = 0.742$, $Y = 0.25$ and $Z = 0.008$ (LMC)
(iii) $X = 0.76$, $Y = 0.237$ and $Z = 0.003$ (SMC)
(iv) $X = 0.70$, $Y = 0.28$ and $Z = 0.02, 0.01, 0.001$ and 0.0002

(for H ɪɪ regions in galaxies with metallicity gradients);

7. Kurucz's (1979) stellar atmosphere models.

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b) For the ionized region

1. The hydrogen line are formed according to case *B* of Baker and Menzel (1938);
2. The influence of dust is neglected.

With regard to the stellar association, observational evidences show that at least the more massive stars are formed in a burst (Lequeux et al., 1981; Larson, 1982). There are also evidences that in a given H II region star formation can occur in series of distinct episodes (Hyland et al., 1978). Observational and theoretical considerations of the IMF (Miller and Scalo, 1979; Lequeux et al., 1979; Garmany et al., 1982; Vereshchagin, 1982; Bisiacchi et al., 1983) indicate that its slope is $1.0 \leqq \chi \leqq 3.0$. Larson (1982) shows that there is a correlation between the mass of the prestellar molecular cloud and the upper mass limit M_u of the formed stars. Hence M_u was taken as a free parameter. The adopted rates of stellar mass loss comprise the values currently quoted in the literature. The solar abundance was adopted for reason of simplicity for stellar atmosphere models, since Kurucz (1979), do not present models of hot stars with other abundances. The influence of the line blanketing on the emergent flux of ionizing photons and consequently in the structure of ionization, was studied by Köppen et al. (1983). These authors point out in their conclusion that: "inspection of the tables of Balick and Sneden (1976) and Borsenberger and Stasinska (1982) shows that similar results are obtained when unblanketed LTE or unblanketed non-LTE instead of blanketed LTE stellar models are used. Consequently "inconsistent" nebular abundance analyses in which the nebular metallicity is varied but, for reason of simplicity, the stellar metallicity is assumed to be solar, will yield reasonable results". With regard to the very important suggestion of Panagia (1980) that higher-than-solar metal abundances in stars may still have direct consequences for the appearence of H II regions, through a metal-dependent $T_{\rm eff} - L$ relation for main sequence stars or through a metal-dependent cut-off mass for groups of stars, the problem was released by considering models with differents upper mass limits. This problem is discussed more extensively later on. The influence of dust was not considered, because many new free parameters appear in dusty H II region models due to the lack of knowledge about the scattering and absorption cross section of the dust for $\lambda < 912$ Å, its abundance, distribution and chemical composition. Hence, the inclusion of dust in H II region models is subject to so many controversies (Herter et al., 1983) as to make it as arbitrary as excluding it.

3. The intensive parameters

3.1. *The equivalent width $W_{\rm H_\beta}$ of the $\rm H_\beta$ emission line*

The definition of the equivalent width of a line centered at λ_0 with a profile L_λ superposed on a continuum L_{λ_c} is

$$W_{\lambda 0} = \int_0^\infty \frac{L_\lambda - L_{\lambda_c}}{L_{\lambda_c}}\, d\lambda .$$

It is necessary to emphasize that the interest is in the integral equivalent width of the $\rm H_\beta$ emission line, i.e., taking into account the whole of the $\lambda = 4861$ Å photons produced by the H II region. In this sense the continuum must include both the photons produced by the stars of the exciting association and those originated in the ionized gas.

If the continuum is assumed to vary slowly with wavelength, $W_{\rm H_\beta}$ may be written as

$$W_{\rm H_\beta} = \frac{I_{\rm H_\beta}}{L_{\lambda 4861}}$$

where $I_{\rm H_\beta}$ is the total intensity of the $\rm H_\beta$ emission line and $L_{\lambda 4861}$ is the continuum emission adjacent to $\rm H_\beta$.

3.1.1. *The intensity of the $\rm H_\beta$ emission line*

The intensity of $\rm H_\beta$ can be expressed as (e.g. Osterbrock, 1974):

$$I_{\rm H_\beta} = h\nu_{\rm H_\beta}\, N^0_{912}\, \frac{\alpha^{\rm eff}_{H_\beta}({\rm H}^0, T_e)}{\alpha_B({\rm H}^0, T_e)}$$

where $\alpha^{\rm eff}_{\rm H_\beta}({\rm H}^0, T_e)$ is the effective recombination coefficient of $\rm H_\beta$ at the electronic temperature T_e; $\alpha_B({\rm H}^0, T_e)$ is the recombination coefficient to all excited levels and N^0_{912} is the number of photons capable to ionize the hydrogen. The dependence of this expression on the electronic temperature is very small (Osterbrock, 1974). Adopting $T_e = 10^4$ K it is obtained

$$I_{\rm H_\beta} = 4.762\ 10^{-13}\ N^0_{912}.$$

3.1.2. *The continuum emission $L_{\lambda 4861}$ around $\rm H_\beta$*

The continuum is generated partially by the stars of the ionizing association and partially by the gas, through the free-free and bound-free processes relative to $\rm H^+$ and $\rm He^+$, and by two photons emission resulting from the forbidden transition $2S^{1/2}$ to $1S^{1/2}$. The luminosity of the continuum can be written as

$$L_\lambda = L^*_\lambda + \frac{\gamma_\lambda({\rm H}^0, T_e) + \gamma_\lambda(2q, T_e) + \gamma_\lambda({\rm He}^0, T_e)\, y^+}{\alpha_B({\rm H}^0, T_e)}$$

where L^*_λ is the stellar continuum, $\gamma_\lambda({\rm H}^0, T_e)$, $\gamma_\lambda({\rm He}^0, T_e)$ and $\gamma_\lambda(2q, T_e)$ are respectively the continuum emission coefficients of H II, He I and two photons process, and y^+ is the abundance by number of $\rm He^+$ (see Sect. 3.3). The continuum radiation was calculated for $\lambda = 4861$ Å taking $T_e = 10^4$ K using the emission coefficients obtained by Brown and Mathews (1970). Since 60% to 90% of the continuum is contributed by the stellar association and about half of the rest is due to the two photons emission, which is temperature-independent, $L_{\lambda 4861}$ is weakly sensitive to the electronic temperature.

The continuum around $\rm H_\beta$ is strongly dependent on the IMF of the ionizing association due to the important contribution of the less massive stars.

3.2. *The relative volume R of $\rm He^+$ and $\rm H^+$ zones*

In the study of the ionization structure of H II regions containing hydrogen and helium the main property of the stellar radiation is the ratio between the number of photons which are able to ionize He and H (Mathis, 1971). The ratio between the total number He-photons ($228 \leqq \lambda \leqq 504$ Å) and H-photons ($504 \leqq \lambda \leqq 912$ Å), i.e.

$$\gamma = \frac{N^{228}_{504}}{N^{504}_{912}}$$

determine the relative volume R of the $\rm He^+$ and $\rm H^+$ zones, when the chemical composition is given and the dust is absent (Mathis, 1971). R is defined as

$$R = \frac{\int_{\mathrm{He}^+} N_p^2 \, dV}{\int_{\mathrm{H}^+} N_p^2 \, dV},$$

where N_p is the density of protons.

The relation R vs y for different chemical compositions as given by Mezger and Smith (1976) is adopted here.

3.3. *The ratio* [O III]/H$_\beta$

The forbidden lines of heavy ions such as O^+, O^{++}, N^+ and S^+ are among the brightest in the H II regions, and often the [O III] lines are stronger than the hydrogen ones. The intensity ratio [O III]/H$_\beta$, defined as the sum of the [O III] $\lambda\lambda$4959, 5007 lines relative to H$_\beta$, is given in the catalog of H II regions models of Stasinska (1982) where the quotient $x(\mathrm{He}^+)/x(\mathrm{H}^+)$ of He$^+$ and H$^+$ ionic fractions is also given. Figure 1 displays the relation between [O III]/H$_\beta$ and $x(\mathrm{He}^+)/x(\mathrm{H}^+)$ – this latter quantity can be expressed as:

$$\frac{x(\mathrm{He}^+)}{x(\mathrm{H}^+)} = \frac{y^+}{y}$$

where y is the helium abundance in number and y^+ is the He$^+$ abundance defined as

$$y^+ = \frac{\int_{\mathrm{H}^+} N_e N_{\mathrm{He}^+} \, dV}{\int_{\mathrm{H}^+} N_e N_p \, dV}$$

being N_{He^+}, N_p and N_e respectively the densities of He$^+$, protons and electrons. In turn R is related to y and y^+ by (Churchwell et al., 1974)

$$\frac{y^+}{y} = \frac{(1+y)R}{1+yR} \cong R.$$

Therefore, for any given value of R the ratio [O III]/H$_\beta$ can easily be estimated by using Fig. 1.

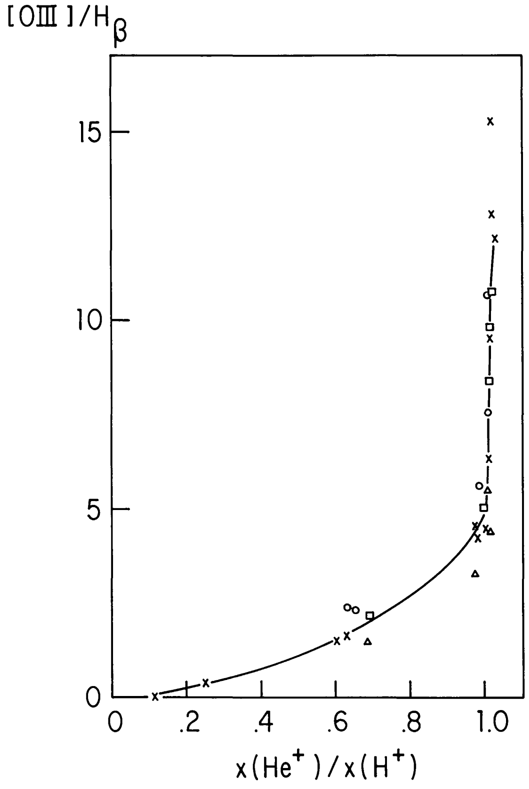


Fig. 1. [O III]/H$_\beta$ vs $x(\mathrm{He}^+)/x(\mathrm{H}^+)$ ($= y^+/y \cong R$) obtained from the Stasinska's (1982) models with hydrogen density $N_{\mathrm{H}} = 100\,\mathrm{cm}^{-3}$ and oxygen abundance O/H = 6.8 10^{-4} (x), 3 10^{-4} (o), 1 10^{-4} (□) and 5 10^{-5} (Δ)

4. The evolution of the intensive parameters W_{H_β}, R and [O III]/H$_\beta$

The evolution of the mentioned parameters was evaluated under the hypothesis of the Sect. 2 for the formation of the exciting stellar association and for the physical conditions of the ionized gas.

From the values of mass $M/M_\odot$, luminosity $L/L_\odot$ and effective temperature T_{eff} given by the adopted models (Sect. 2) for each step of the stars evolution, the values of the radius r and the surface gravity g is obtained through the relations

$$L/L_\odot = (r/r_\odot)^2 (T_{\mathrm{eff}}/T_\odot)^4$$

$$\log g = \log g_\odot + \log M/M_\odot - 2 \log r/r_\odot,$$

using $r_\odot = 6.960\ 10^{10}$ cm (Allen, 1973), $\log g_\odot = 4.44$ (in cm s^{-2}) and $T_\odot = 5770$ K (Kurucz, 1979). The values of T_{eff}, $\log g$ and r allow us to calculate $L_{\lambda 4861}$, N_{504}^{228} and N_{912}^{504} from the Kurucz's tables (1979). The integrations for N_{504}^{228} and N_{912}^{504} were performed elsewhere (Copetti and Bica, 1983). Since N_{228}^{0} is negligible for the stellar temperature of the models, the Lyman continuum photon flux can be written as

$$\mathrm{N}_{912}^{0} = \mathrm{N}_{504}^{228} + \mathrm{N}_{912}^{504}.$$

The intensive parameters W_{H_β}, R and [O III]/H$_\beta$ were obtained following the procedures of Sect. 3, in time intervals of 0.5 10^6 yr from the formation until 15 10^6 yr after.

4.1. *The equivalent width* W_{H_β}

The figures 2, 3, 4 and 5 show that the W_{H_β} decrease almost monotonically with the age of the H II region. The influence of the electronic temperature and density is negligible. The upper stellar mass limit M_u is important in the initial phase of the H II region evolution until about 4 10^6 yr. W_{H_β} is strongly sensitive to the IMF, being reduced to 1/7 by varying χ from 1 to 3. This is due to the considerable contribution to the continuum around H$_\beta$ provided by the dwarf stars which do not ionize the gas. The chemical composition also influences greatly the W_{H_β}, through the effects of the metal content (see for example Fig. 5). The stellar mass loss play a secondary role, at least for the present evaluations of the ejection rates (Maeder, 1981a, b).

However, despite the strong dependences on χ, M_u and Z, W_{H_β} is basically determined by the evolution of the H II region, and, therefore, can be used as a good age indicator.

4.2. *The relative volume* R *of* He$^+$ *and* H$^+$ *zones*

The evolution of R was obtained from the relation R vs y of Mezger and Smith (1976). Figures 6 show the behavior of R for different values of the IMF, upper stellar mass limit M_u, mass loss rates and chemical compositions. Generally in the first stages of

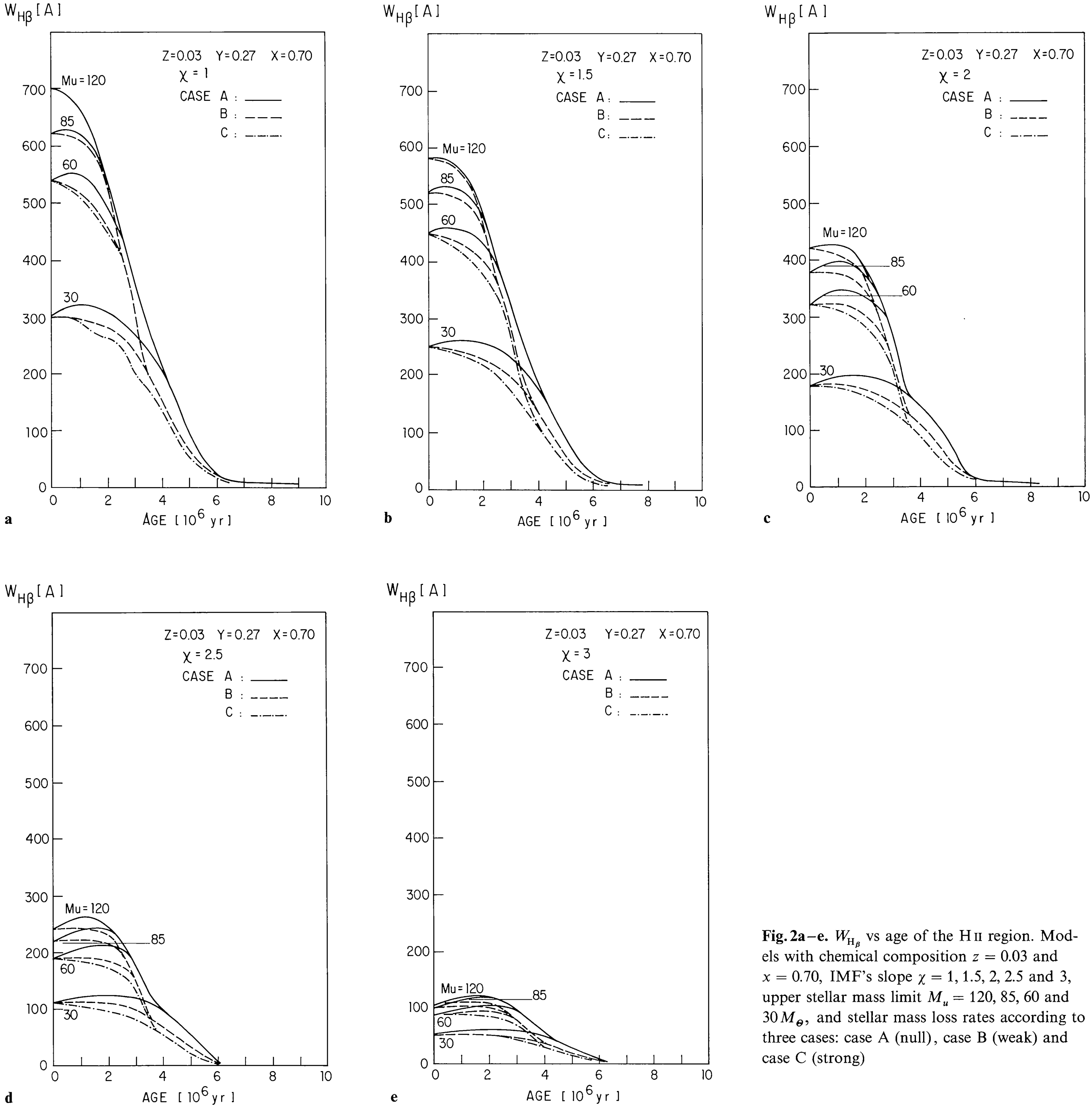


Fig. 2a–e. W_{H_β} vs age of the H II region. Models with chemical composition $z = 0.03$ and $x = 0.70$, IMF's slope $\chi = 1, 1.5, 2, 2.5$ and 3, upper stellar mass limit $M_u = 120, 85, 60$ and $30\,M_\odot$, and stellar mass loss rates according to three cases: case A (null), case B (weak) and case C (strong)

the evolution, when still sufficiently hot stars exist to ionize the H II region (the first 2 or 3 millions years), the He^+ and H^+ zones coincide with each other and $R = 1$. After the evolution of the most massive stars the helium becomes neutral and R falls quickly. R is totally insensitive to mass limits higher than $60\,M_\odot$ and to the IMF slope, being the hottest stars which determines the helium ionization structure (Copetti and Bica, 1983). Then R is also a good H II region age indicator, mainly for the evaluation of upper limits, instead of the influence of the stellar mass loss.

4.3. The ratio [O III]/H_β

The results on the behavior of the ratio [O III]/H_β are shown in Figs. 7. It is strongly determined by the evolution of the H II region in a similar way as the parameter R, then [O III]/H_β is also a good age indicator. However, contrary to what occurs with the other discussed properties, [O III]/H_β is affected by the density since the ionization structure of the O^{++} depends on the distribution of matter within the nebula (Staninska, 1980).

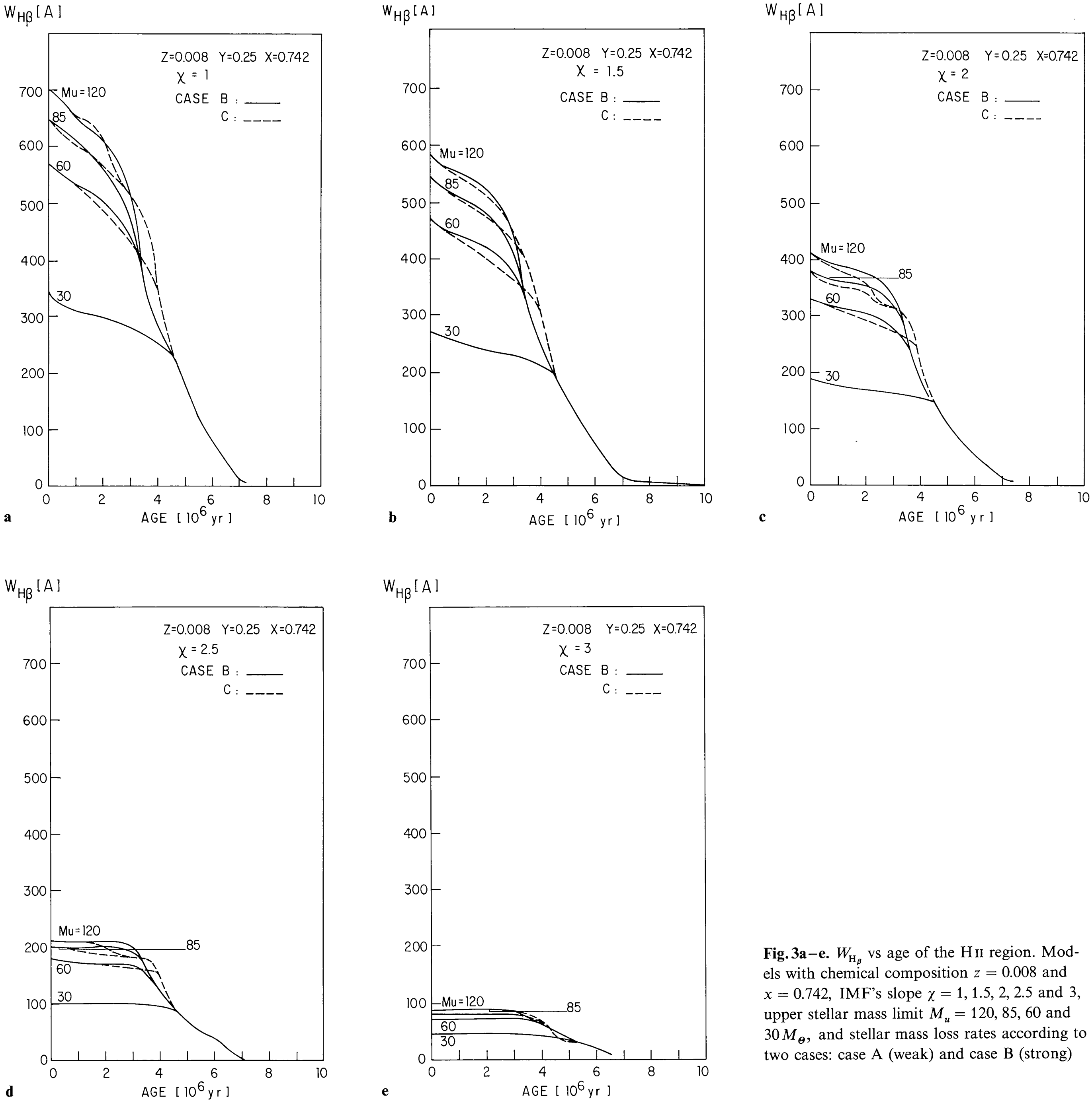


Fig. 3a–e. W_{H_β} vs age of the H II region. Models with chemical composition $z = 0.008$ and $x = 0.742$, IMF's slope $\chi = 1, 1.5, 2, 2.5$ and 3, upper stellar mass limit $M_u = 120, 85, 60$ and $30\,M_\odot$, and stellar mass loss rates according to two cases: case A (weak) and case B (strong)

5. Accuracy and limitations of the method

Among the limitations of the proposed method we could mention the adoption of a dust-free model with no leakage of Lyman continuum photons (case *B*) from the ionized region. The statistical comparison of the models with observations will probably in the future enlight this controversial question. As a matter of fact it is generally accepted that bubbles in the LMC are partially transparent to Lyman – *c* photons. With regard to the ionizing association our models are subject to the uncertainties of the model calculations of stellar evolution and stellar atmospheres. The problem of the Lyman continuum blanketing by metallic lines is an important question. Its influence on W_{H_β} and [O III]/H$_\beta$ can be readily evaluated from the work of Köppen et al. (1983). In fact for a nebular of constant metallicity, ionized by a star of $4\ 10^4$ K, taking into account line blanketing for the Lyman – *c* photons, and varying its metallicity between $-0.5 \leqq \log Z/Z_\odot \leqq 1.0$, the intensity of the H$_\beta$ line remain unaltered (Table 1, row 1, Köppen et al., 1983) and the continuum flux at H$_\beta$ varies by 5% when metallicity goes from $Z/Z_\odot = 0$ to $Z/Z_\odot = 1.0$ (a stellar type FO was considered for the continuum; if $T_{\rm eff} > 10\,000$ K, the continuum is also unaltered) which shows that W_{H_β} is increased by approximately 5% due to the influence of the blanketing in the cold stars, while the ratio of [O III]/H$_\beta$ (Table 1, row 3, Köppen

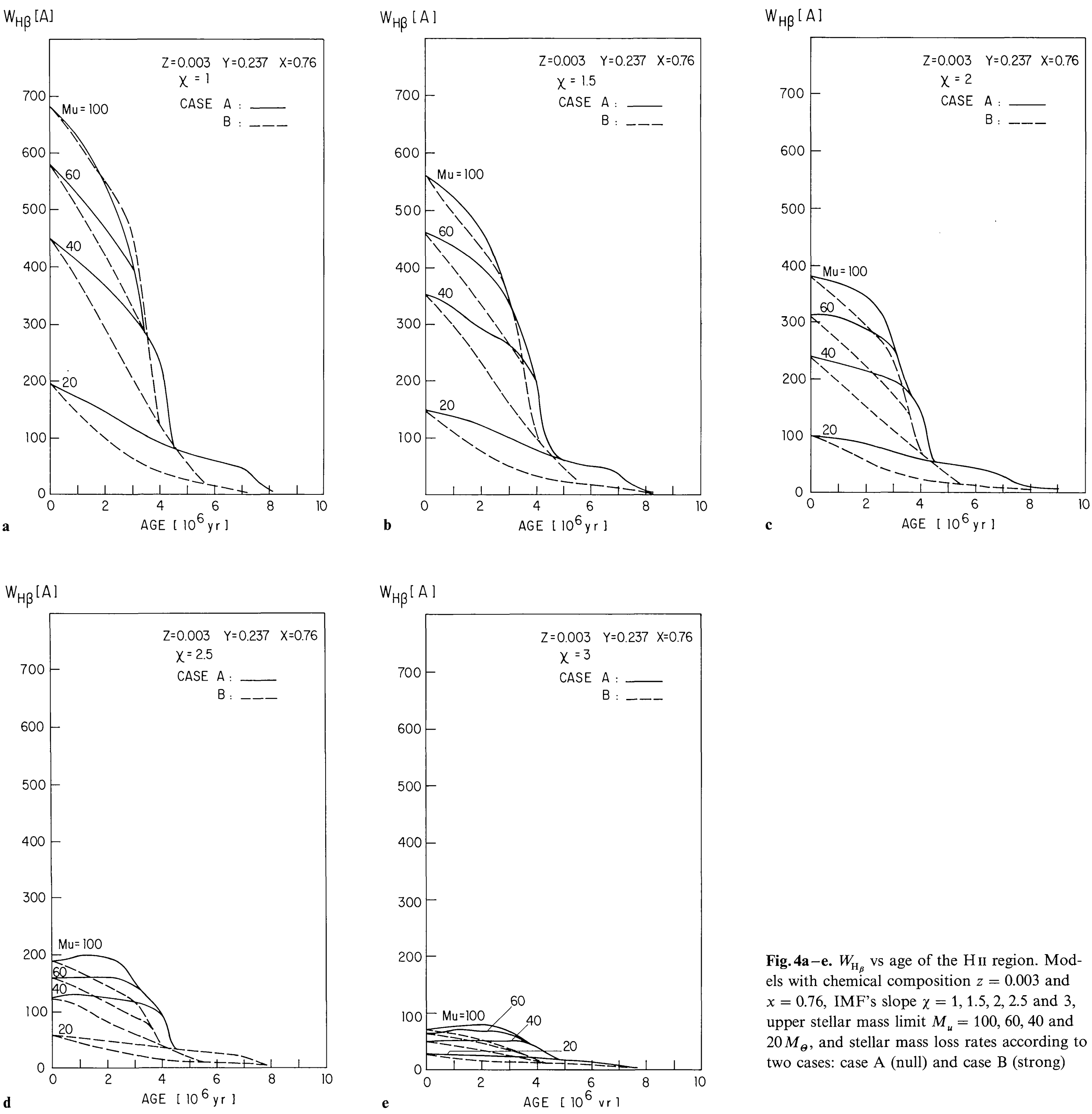
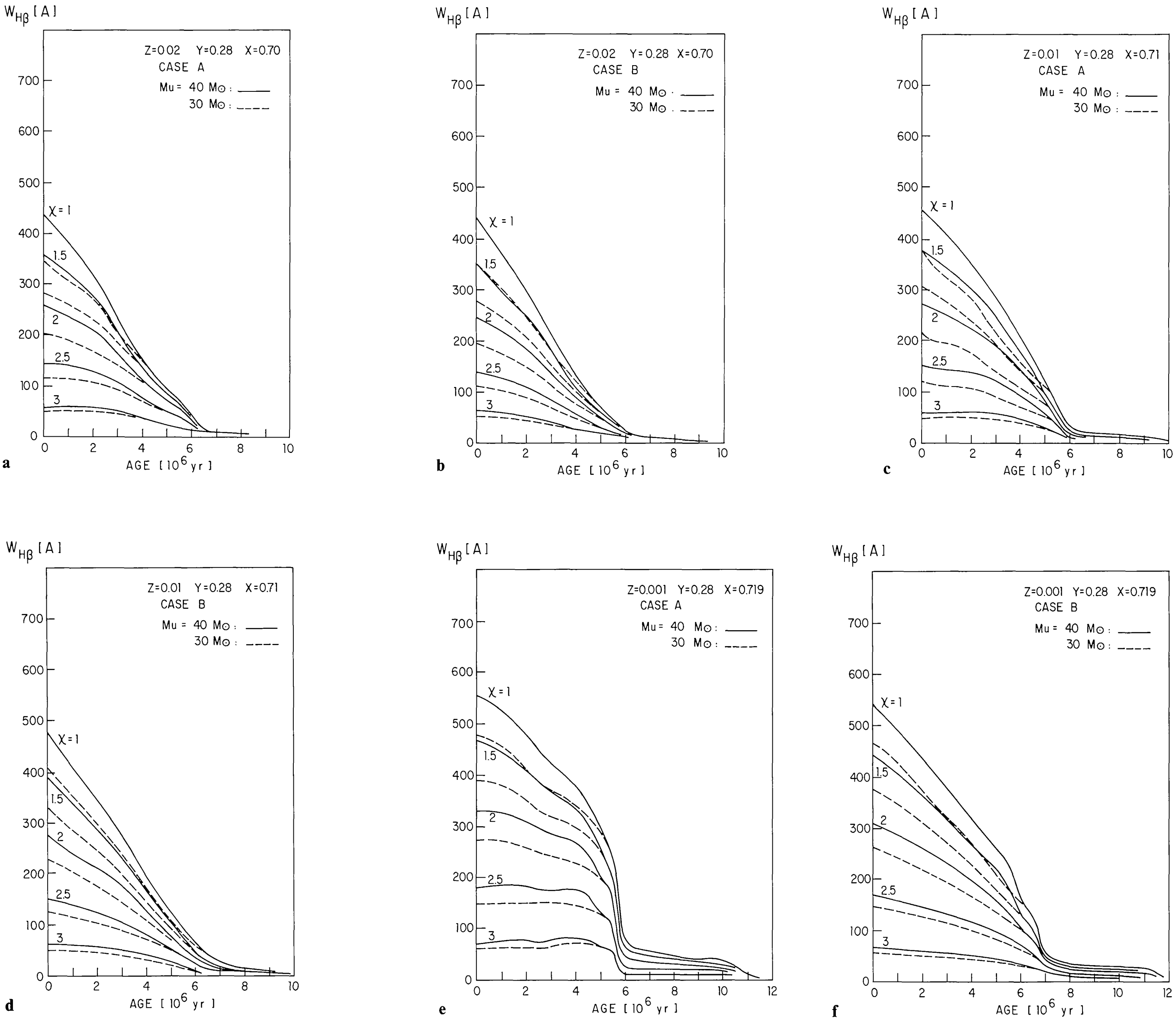


Fig. 4a–e. W_{H_β} vs age of the H II region. Models with chemical composition $z = 0.003$ and $x = 0.76$, IMF's slope $\chi = 1, 1.5, 2, 2.5$ and 3, upper stellar mass limit $M_u = 100$, 60, 40 and $20\,M_\odot$, and stellar mass loss rates according to two cases: case A (null) and case B (strong)

et al., 1983) gives a mean difference of about 20%. For stars of $4\ 10^4$ K or hotter, $R = 1$, i.e. He is totally ionized and, consequently, there is an excess of photons with $\lambda \gtrsim 504$ Å (Fig. 2, Copetti and Bica, 1983), which makes the effects of line blanketing negligible. This can be also seen from the unsensilibity of the R vs age relation (Fig. 6) to changes of M_u in the range $40-120\,M_\odot$. Consequently, during the first 3 million years the age evolution will be not affected by line blanketing.

Another interesting fact to be taken into account is that both diagrams [O III]/H_β and R vs age (Figs. 6 and 7) have a steep slope for ages between 2 and 4 million years. If we take into account that the curves are used to obtain ages from the values of [O III]/H_β and R, we conclude that the line blanketing can introduce an error of the order of 15% for ages between 4 and 5 million years and 20 to 25% for older regions. That is to say uncertainties no larger than those introduced by other factors (such as mass loss rate, evolution model, etc.). As a last comment, for a star of, say $4\ 10^4$ K, the influence of line blanketing will make it mimic one of $3.7\ 10^4$ K. But taking into account the evolution, the considered star evolves so quickly from $4\ 10^4$ K that the blanketing does not influence drastically the age scale, i.e. not more $5\ 10^5$ yr on the average.



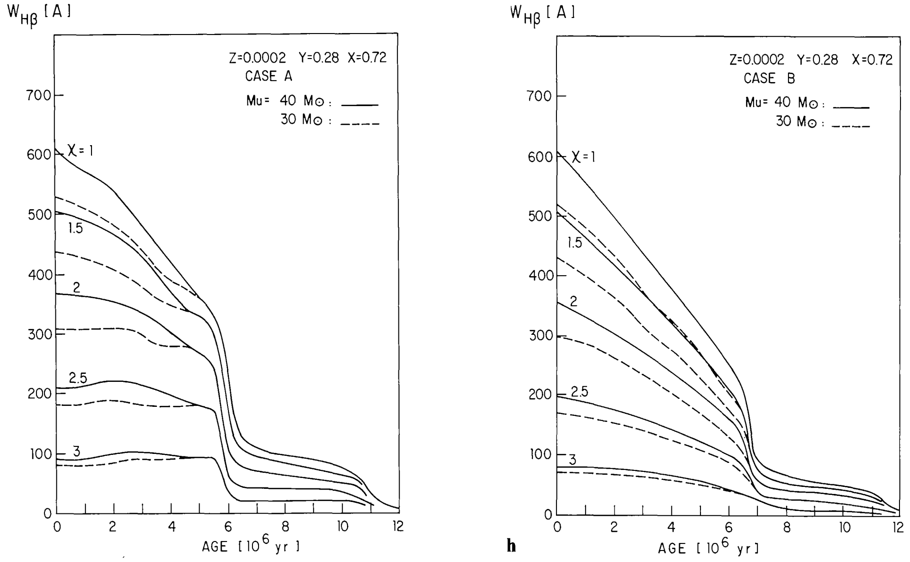


Fig. 5a–h. W_{H_β} vs age of the H II region. Models with chemical composition $y = 0.28$ and $z = 0.02, 0.01, 0.001$ and 0.0002, IMF's slope $\chi = 1, 1.5, 2, 2.5$ and 3, upper stellar mass limit $M_u = 40$ and $30\,M_\odot$, with and without stellar mass loss (B and A cases, respectively)

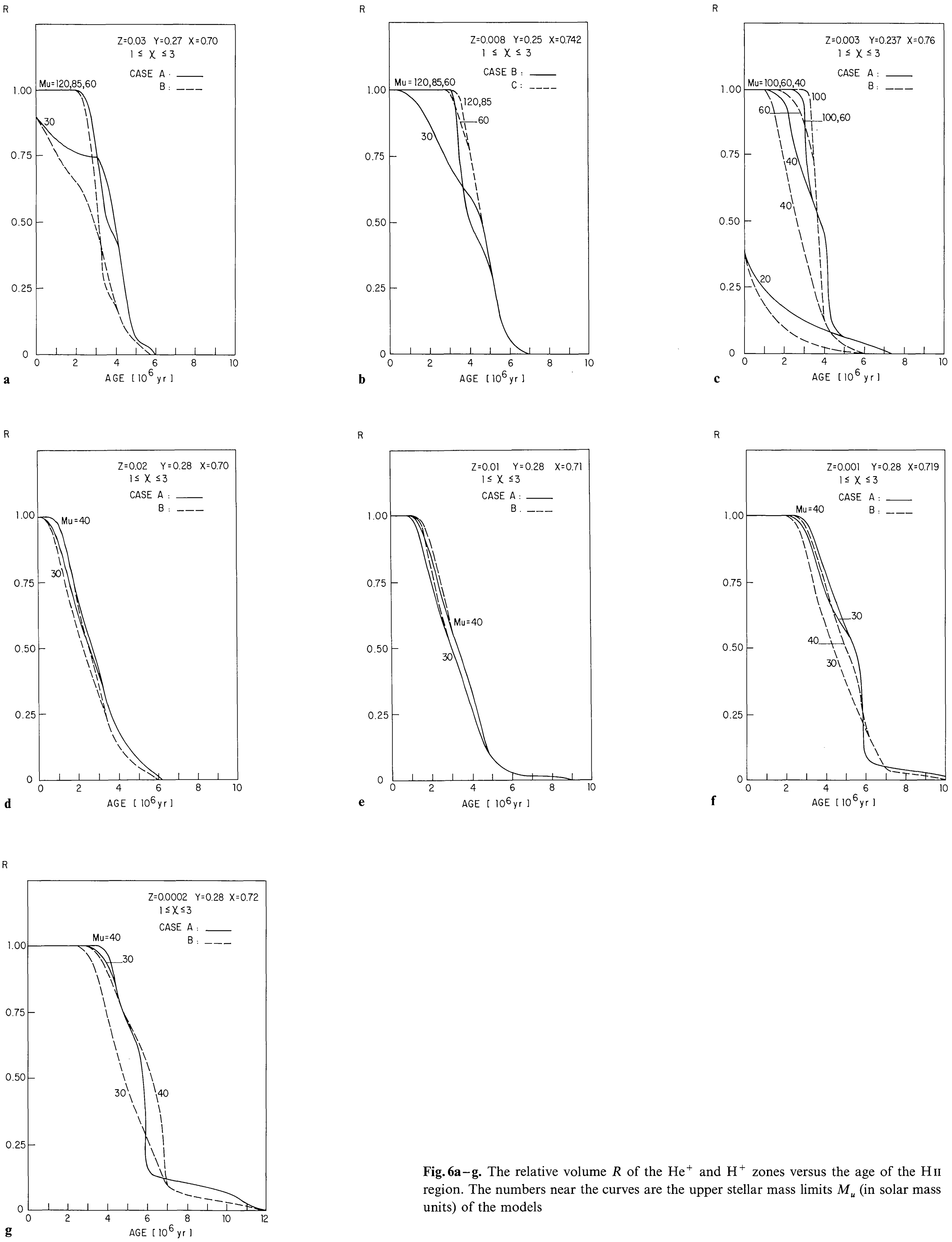


Fig. 6a–g. The relative volume R of the He$^+$ and H$^+$ zones versus the age of the H II region. The numbers near the curves are the upper stellar mass limits M_u (in solar mass units) of the models

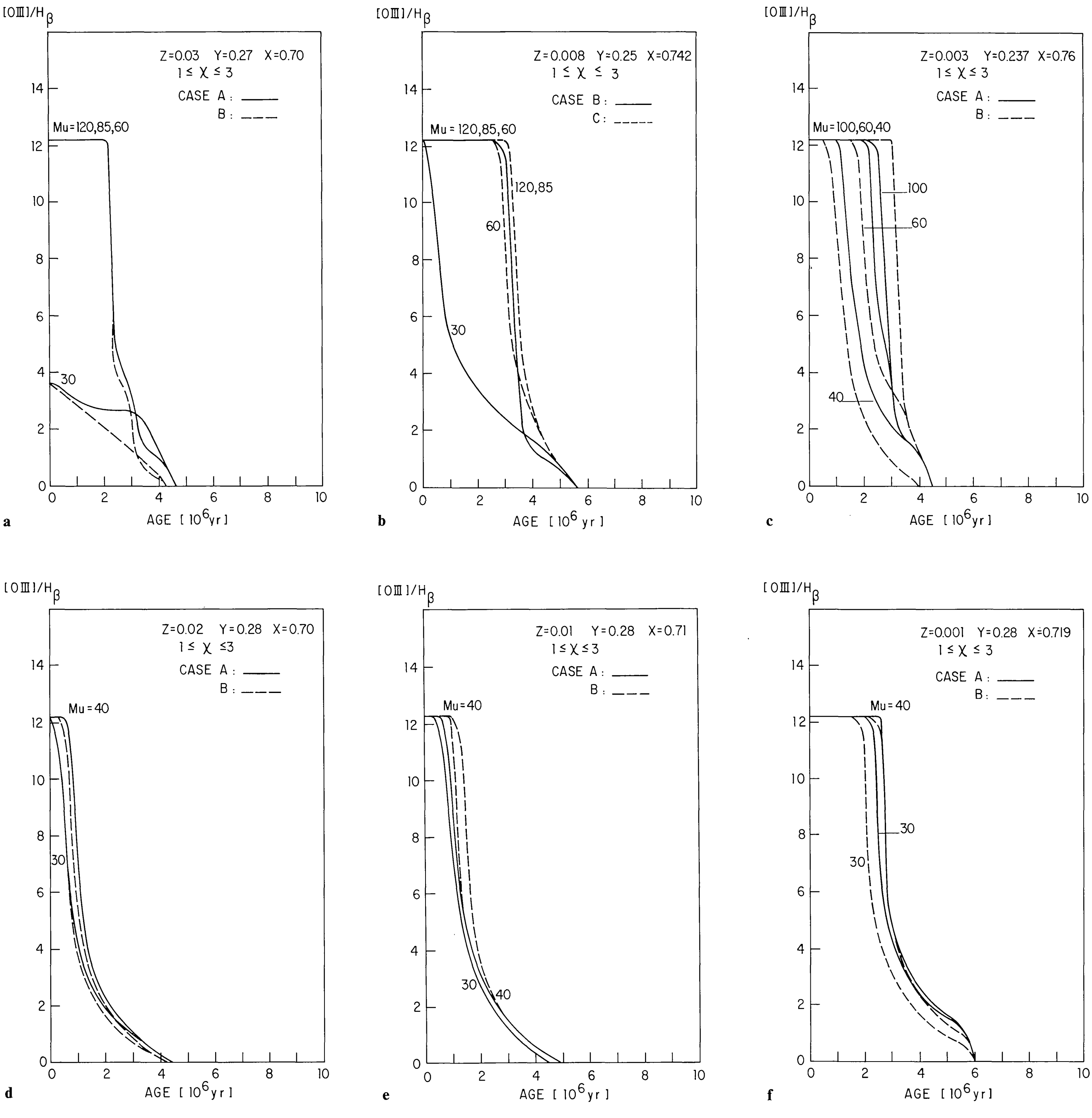


Fig. 7a–f. [O III]/H_β vs age of the H II region. The numbers near the curves are the upper stellar mass limits M_u (in solar units) of the models

6. Conclusion

This work shows that the evolution changes dramatically the properties of an H II region ionized by a group of early type stars in a few million years. The behavior of the equivalent width W_{H_β} of the H_β emission line, the relative volume R of the He^+ and H^+ zones, and the ratio [O III]/H_β of the lines [O III] λλ 4959, 5007 to H_β were studied as a function of the H II region age. It is emphasized that for a meaningful estimate of W_{H_β} and [O III]/H_β the radiation emitted by the whole volume of the ionized region must be included. The models developed show that it is possible to determine the stage of evolution of an H II region by measuring the mentioned parameters, in spite of the influence of the initial mass spectrum of the ionizing association, the upper stellar mass limit, the mass loss and the chemical composition. It is important to remark that W_{H_β}, R and [O III]/H_β are intensive properties so that they do not depend neither on the absolute dimensions of the H II regions nor on the total number of exciting stars.

The results obtained here show that the evolution should be taken into account when H II regions are used as distance indicators and in the so called "empirical" determination of abundances (Pagel et al., 1979; Alloin et al., 1979) due to the influence of changes in the nebula excitation on the line intensity ratios.

Acknowledgements. This work was partially supported by the Brazilian institutions FINEP and CNPq. The numerical computations were performed in the CPD of the UFRGS. The thank Mr. Cleto Tartarelli for the drawings and Denise Coutinho for typing the manuscript.

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