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On the meaning of a minimization procedure applied to a degenerate astrophysical problem

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Summary. We apply the MINOS optimization system to the population synthesis of galaxy nuclei using as basis a grid of star cluster spectral features as a function of age and metallicity. In inverse numerical problems, a minimization procedure is usually applied to provide a single optimal result which ignores, as a rule, a multitude of other equally good solutions whenever intrinsic errors are to be considered. We avoid this drawback by transforming the population synthesis into a series of optimization problems, each one corresponding to a fixed contribution of an individual basis component. In this way we sweep the vector space of solutions, generated by the basis components, rejecting the minimization results for which the differences between observed and synthetic equivalent widths are larger than the observational uncertainties. Consequently we map the space of mathematically similar solutions, determining the degeneracy degree of the problem.

The astrophysical implications of the synthesis results are discussed. In particular, a noteworthy result is that for some classes of red galaxy nuclei, our constraint free population synthesis, consisting of 35 components which are widely distributed in the plane age versus metallicity, reproduces the chemical evolution scenario predicted by theoretical evolutionary models. The results for bluer nuclei tend to scatter more in the plane age versus metallicity, suggesting that additional information from complementary spectral ranges is necessary to produce a better focused solution in the plane. However, strong bursts of star formation are easily detected.

1 Introduction

Much effort has been devoted to synthesize galaxy spectral properties from those of simple components such as stars (e.g. Spinrad & Taylor 1971; Alloin, Andrillat & Souffrin 1971;

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Faber 1972; Joly & Andrillat 1973; O'Connell 1976; Turnrose 1976; Pritchet 1977; Pickles 1985; Dottori & Pastoriza 1986). Most of these studies have employed minimization procedures as an algorithm to search for solutions. Throughout the years the data used in population synthesis has been considerably improved with the help of the technological progress achieved for detectors. However, not as much as has been done to increase the power of the mathematical tools used in the analysis.

Considering that we are dealing with a typical inverse problem we have to stress, first of all, the existence of non-unique solutions, based on two unavoidable facts: we do not know all the essential variables involved that specify the problem and observations have intrinsic errors. Furthermore it is known that one may particularize an inverse problem too much by arbitrarily choosing this or that feature or by adopting a linear or non-linear function to be minimized. Consequently, one should not simply search for a single 'optimal solution' in a synthesis minimization task, but rather face it as degenerate problem since it has, in fact, a variety of equally good solutions and all of them, in principle, are important. From this point of view the frequency and strength of a given component contribution to the set of acceptable solutions is obviously more significant that the value it assumes in the best mathematical solution that can be found. Thus a statistical approach certainly provides a more realistic picture of the astrophysical problem in question. Recently, a statistical approach based on direct combinations has been applied to the population synthesis problem (Bica 1988, hereafter B88). The latter method sweeps the vector space of solutions with a discrete step by generating and testing possible solutions. However, this search is computationally very time-consuming and it becomes prohibitive to test the complete set of combinations in 10 per cent steps for more than eight components. A 35 component space would generate, at 10 per cent contribution steps, around 2.5×10^9 combinations for which the summation amounts to 100 per cent. This set of combinations should be picked out among the 11^{35} total combinations.

In the present paper we present a more efficient algorithm, based on a minimization procedure, to synthesize the galaxy spectral groups in B88. The method combines the power that direct combinations have to statistically map the vector space of solutions with the efficiency that a minimization procedure has to produce a particular solution. We use the same basis as B88, consisting of a grid of star cluster spectral features as a function of age and metallicity (Bica & Alloin 1986a, b, 1987). The advantages of this two-parameter arrangement over traditional ones, based on stellar libraries, are that it includes implicitly the stellar evolution and the IMF as found in nature. This fact allows a constraint-free formulation to be employed in this synthesis. Consequently we dispose of ideal conditions to explore a minimization method in one of nature's interesting inverse problems.

In Section 2 we describe the algorithm used, which basically replaces a minimization problem by a series of minimization tasks in order to map the vector space of acceptable solutions. The detailed analysis of the results for a selection of spectral classes of galaxy nuclei from B88 is presented in Section 3. The conclusions of this work are given in Section 4.

2 The method

The basis contains 35 components, each one representing a star cluster evolutionary stage for a particular metal content. The array spans wide ranges in the plane age versus metallicity. Each component has an associated set of six metallic features and three Balmer lines. Balmer lines suspected of emission contamination in a galaxy spectrum are not used in the computations. The H II region component is represented by a featureless continuum derived from real H II regions, affected by internal reddening, which was used independently of metallicity as in B88.

As usual in population synthesis, we minimize a function which is the difference between the observed and the synthetic equivalent widths for the spectral features:

$$F(\mathbf{X})=\sum_{i=1}^{9}[W_{\mathrm{obs}}(i)-W_{\mathrm{syn}}(i)]^2, \tag{1}$$

where $W_{\mathrm{obs}}(i)$ and $W_{\mathrm{syn}}(i)$ are the observed and synthetic i spectral feature. $W_{\mathrm{syn}}(i)$ is given by

$$W_{\mathrm{syn}}(i)=\sum_{j=1}^{35}\mathbf{X}(i)\cdot W(i,j)\cdot f(i,j)\Big/\sum_{k=1}^{35}\mathbf{X}(k)\cdot f(i,k) \tag{2}$$

where $W(i,j)$ is the equivalent width of the i feature in the j component and $f(i,j)$ is its corresponding continuum normalized at 5870 Å. The presence of the proportion components of the vector **X** in the numerator and in the denominator of equation (2) makes the function $F(\mathbf{X})$ highly non-linear. The square in the difference of equivalent widths in equation (1) accelerates the search for a minimum. Among all possible vectors **X** generated by its 35 components, we restrict our search in the subspace where **X** has physical meaning:

$$0\leqslant\mathbf{X}(i)\leqslant 1 \qquad 1\leqslant i\leqslant 35, \tag{3}$$

$$\sum_{i=1}^{35}\mathbf{X}(i)=1, \tag{4}$$

where (3) are the natural bounds of the components and expression (4) is a constraint consistent with equation (2).

The minimization algorithm applied was the MINOS 5.1 system (Murtagh & Saunders 1987) which is an optimization package for the solution of linear and non-linear problems with smooth F functions. Basically it consists of a conjunction of fundamental algorithms such as the simplex, quasi-Newton, reduced gradient and projected Lagrangian methods and can be programmed to solve a series of close related problems. It makes explicit use of the F function gradient with respect to its variables $\mathbf{X}(i)$, which in our case can be given analytically, to find local minima. For the present case, the non-linear problem as established in equation (1) with the linear constraint (4), is solved by MINOS using the reduced gradient method in conjunction with the quasi-Newton algorithm (Davidon 1959; Murtagh & Saunders 1978).

Following the ideas exposed in Section 1, we do not simply apply the minimization procedure to derive a single optimal solution. We compute, rather, a series of minimization tasks. Each task starts from an initial vector $\mathbf{X}=0$ except for one of its components which is kept at a fixed value during the search for a minimum. The analysis of an object consists, then, of 350 tasks which correspond to each component assuming values from 0.1 to 1.0 with a step of 0.1. Consequently, each solution obtained expressed the local minimum around the fixed component. We also compute the classical totally free optimal solution (no fixed component). From these 351 'mathematical' solutions we reject those with the difference between observed and synthetic equivalent widths for individual spectral features violating a maximum allowed residue (window). In practice these windows are set slightly wider than the expected observational errors. Therefore we get a large number of acceptable solutions in a first phase. In a second phase we control their quality with an additional criterion based on the χ^2 of the residues.

Finally, we calculate an average solution with the acceptable set. This average itself is not only a representative solution, but also each of its components expresses, as a consequence of this statistical approach, its probability of being present at any other acceptable solution. A minimization procedure used in the present manner, with respect to a direct combination method which generates with the same step (0.1) all vectors **X** satisfying (3) and (4), presents the advantage of determining an equally representative number of acceptable solutions from a relatively small number (351) of possibilities (Section 3).

In the present method the discrete values assumed by the fixed component in each performed task are counterbalanced by the continuous values between 0 and 1 that all other components can assume and by the average carried out with the acceptable solutions. We have also tested steps of 0.2 and 0.05 and the results are essentially the same as a step of 0.1, which presents the best performance for time versus accuracy.

The multi-minimization method is an efficient tool to search for a statistically representative solution to the synthesis problem for a large number of components. On the other hand, the direct combination method, although limited in the number of components, has the advantage of estimating quantitatively the uniqueness degree of the result, simply by the ratio number of solutions over the number of generated combinations. In the multi-minimization procedure the number of solutions over the number of tasks is, rather, a measure of the efficiency of the algorithm.

The computations were carried out in the COBRA 1400 (Eclipse 8000 II) computer of the Instituto de Fisica da Universidade Federal do Rio Grande do Sul, Porto Alegre. The typical CPU time for each project (351 minimization tasks) was 20 min.

3 Results

We apply the multi-minimization method to the galaxy spectral groups defined in B88 and make a comparison with the results therein, which were obtained by means of direct combinations. The essential results are given for the following selection of spectral groups: E1, E4, S3 and E8, which are shown in Tables 1 to 4, respectively. Each table contains the observed galaxy equivalent widths (W_λ) of the features used in synthesis, which are the strongest metallic lines and molecular bands and Balmer lines, and also the respective residues and windows (Section 2). We emphasize that we are not using the spectra themselves in the computations, rather we use as a basis the grid of star cluster W_λ as a function of age and metallicity (Bica & Alloin 1986a,b, 1987). We give in each table the number of acceptable solutions found and the reduced χ^2 of the residues for the average solution (Section 2). Finally, each table presents the synthesis results throughout the plane age versus metallicity, expressed in terms of flux fractions at the normalization wavelength 5870 Å.

The group E1 consists of the strongest-lined red elliptical and lenticular nuclei. The dominant population found is by far the very metal rich old component (Table 1a). Within the old age bin there is a steeply increasing contribution towards high metallicities. The scenario, taken from this observational approach using the whole plane age versus metallicity with no astrophysical constraint whatsoever, is basically the same as that predicted by the theoretical approach of Arimoto & Yoshii (1987) for massive elliptical galaxies. A similar picture was found in Group S1, the strongest lined spiral group. These results are essentially the same as those derived earlier by B88 using direct combinations in paths.

Assuming that the gas available for star formation in a galaxy nucleus has been chemically homogeneous at any time one can expect a simple evolution path for $t \leqslant 5 \times 10^9$ yr. However, the multi-minimization method allows small contributions to appear spread all over the plane. For this reason, in order to avoid these spurious contributions, we give alternative solutions where the multi-minimization procedure is applied to subspaces in the plane age versus metallicity. These subspaces are the L-shaped paths from B88 where all the metal enrichment occuring in the old age bin (paths A), and another type with an additional enrichment between

Table 1a. The whole plane solution for the spectral group E1.

Features	Observed W_λ (A)	Synthetic W_λ (A)	Residue (A)	Window (A)
CaII K	16.8	17.0	- 0.2	2.5
CN 4216	14.5	14.2	0.3	3.0
CH G	9.3	9.5	- 0.2	1.5
Mg+MgH	10.3	9.1	1.2	1.8
CaII 8542	6.1	6.5	- 0.4	1.2
CaII 8662	4.9	5.7	- 0.8	1.2
Hδ	5.7	5.2	0.5	2.5
Hγ	4.8	5.3	- 0.5	1.2
Hβ	3.4	4.0	- 0.6	1.2

Solution in flux fractions (%) at 5870 A:

RHII	E7	5E7	E8	5E8	E9	5E9	> E10	AGE $[Z/Z_\odot]$
1.7	0	0	0	3.6	0.6	10.0	59.8	+ 0.6
	0	0	0	0.1	0.4	4.9	11.2	+ 0.3
	0	0	0	0.1	0.4	1.4	1.4	0.0
	0	0	0	0.3	0.4	0.8	0.8	- 0.5
					0.1	0.4	0.4	- 1.0
						0.4	0.4	- 1.5
							0.1	- 2.0

Acceptable solutions: 73 $\chi^2 = 0.409$.

Table 1b. The path solution 0.6A for the spectral group E1.

Features	Observed W_λ (A)	Synthetic W_λ (A)	Residue (A)	Window (A)
CaII K	16.8	16.9	- 0.1	2.5
CN 4216	14.5	14.2	0.3	3.0
CH G	9.3	9.5	- 0.2	1.5
Mg+MgH	10.3	9.1	1.2	1.8
CaII 8542	6.1	6.5	- 0.4	1.2
CaII 8662	4.9	5.7	- 0.8	1.2
Hδ	5.7	5.1	0.6	2.5
Hγ	4.8	5.2	- 0.4	1.2
Hβ	3.4	4.0	- 0.6	1.2

Solution in flux fractions (%) at 5870 A:

RHII	E7	5E7	E8	5E8	E9	5E9	> E10	AGE $[Z/Z_\odot]$
2.0	0	0	0	4.7	1.0	15.2	52.5	+ 0.6
	0	0	0	0	0	0	19.1	+ 0.3
	0	0	0	0	0	0	2.3	0.0
	0	0	0	0	0	0	1.4	- 0.5
					0	0	0.8	- 1.0
						0	0.7	- 1.5
							0.2	- 2.0

Acceptable solutions: 43 $\chi^2 = 0.407$.

Table 2a. The whole plane solution for the spectral group E4.

Features	Observed W_λ (A)	Synthetic W_λ (A)	Residue (A)	Window (A)
CaII K	13.7	12.3	1.4	2.5
CN 4216	3.4	6.2	- 2.8	3.0
CH G	6.8	6.7	0.1	1.5
Mg+MgH	6.3	4.7	1.6	1.8
CaII 8542	5.2	4.2	1.0	1.2
CaII 8662	4.7	3.9	0.8	1.2
$H\delta$	3.8	4.7	- 0.9	2.5
$H\gamma$	4.6	5.1	- 0.5	1.2
$H\beta$		3.8		1.2

Solution in flux fractions (%) at 5870 A:

RHII	E7	5E7	E8	5E8	E9	5E9	> E10	AGE [Z/Z$_\odot$]
0.2	0.1	0	0	0	0	0.2	4.2	+ 0.6
	0	0.2	0	0	0.2	0	0.5	+ 0.3
	0	0.2	0	0	0.2	0.2	1.0	0.0
	0	0.2	0	0.2	1.0	5.7	45.2	- 0.5
					0.5	6.3	28.8	- 1.0
						2.4	2.4	- 1.5
							0.5	- 2.0

Acceptable solutions: 63 $\chi^2 = 2.152$.

Table 2b. The path solution −0.5A for the spectral group E4.

Features	Observed W_λ (A)	Synthetic W_λ (A)	Residue (A)	Window (A)
CaII K	13.7	12.4	1.3	2.5
CN 4216	3.4	6.2	- 2.8	3.0
CH G	6.8	6.7	0.1	1.5
Mg+MgH	6.3	4.7	1.6	1.8
CaII 8542	5.2	4.2	1.0	1.2
CaII 8662	4.7	3.9	0.8	1.2
$H\delta$	3.8	4.8	- 1.0	2.5
$H\gamma$	4.6	5.1	- 0.5	1.2
$H\beta$		3.9		1.2

Solution in flux fractions (%) at 5870 A:

RHII	E7	5E7	E8	5E8	E9	5E9	> E10	AGE [Z/Z$_\odot$]
0.5	0	0	0	0	0	0	0	+ 0.6
	0	0	0	0	0	0	0	+ 0.3
	0	0	0	0	0	0	0	0.0
	0.1	0.4	0	0.4	2.5	18.2	50.2	- 0.5
					0	0	25.0	- 1.0
						0	2.2	- 1.5
							0.4	- 2.0

Acceptable solutions: 24 $\chi^2 = 2.141$.

the old age bin and that at 5×10^9 yr (paths B). The path solution for E1 is given in Table 1b where it reaches up to $[Z/Z_\odot] = 0.6$ (path 0.6A).

We show in Table 2a the whole plane results for E4, the weakest-lined red nuclei in lenticular and elliptical galaxies in the sample. The dominant population is the old component at $[Z/Z_\odot] = -0.5$. We also find a steeply increasing contribution in the range $-2.0 \leqslant [Z/Z_\odot] \leqslant -0.5$. This picture, obtained with our completely unconstrained method, is compatible with the predictions of Arimoto & Yoshii (1987) for low mass galaxies. As in all red nuclei that we have computed we find ≈ 10% contribution of intermediate ages and an almost negligible contribution from the young components. In Table 2b we show the path solution −0.5A, which is similar to that in Table 2a and also compatible to that in B88.

In Table 3a we show the results for the red group S3 in the whole plane. In this case we have obtained a complex combination of metal-rich and metal-poor components. A complicated interpretation for this solution could be worked out in terms of, for example, mergers of different populations. Instead we interpret this result as internal compensations of the component contributions which lead to better solutions, from a mathematical point of view, than those astrophysically acceptable. It should be noted that the previous cases E1 and E4 presented solutions at the edges of the adopted age versus metallicity space while S3, with W_λ for metallic features intermediate between E1 and E4, had the possibility of multiple compensations. Possibly this degeneracy will be raised with new information from complementary spectral ranges. The best approach for the moment is to use the path solutions. In Table 3b we show the path solution 0.0A that has the best χ^2. We also present in Table 3c the best B path solution (0.3B) which is an alternative result with similar χ^2. It is interesting to compare Table 3b and c to see the effects on the component contributions.

Table 3a. The whole plane solution for the spectral group S3.

Features	Observed W_λ (A)	Synthetic W_λ (A)	Residue (A)	Window (A)
CaII K	15.5	14.5	1.0	2.5
CN 4216	8.5	9.9	- 1.4	3.0
CH G	8.5	8.1	0.4	1.5
Mg+MgH	7.9	6.8	1.1	1.8
CaII 8542	5.7	5.3	0.4	1.2
CaII 8662	4.6	4.8	- 0.2	1.2
Hδ	3.1	4.5	- 1.4	2.5
Hγ	4.4	5.0	- 0.6	1.2
Hβ	3.8	3.9	- 0.1	1.2

Solution in flux fractions (%) at 5870 A:

RHII	E7	5E7	E8	5E8	E9	5E9	> E10	AGE $[Z/Z_\odot]$
1.8	0.1	0	0	0.1	0.3	10.2	20.0	+ 0.6
	0.1	0	0	0.1	0.3	4.2	5.3	+ 0.3
	0.1	0	0	0.1	0.3	4.2	4.5	0.0
	0.1	0	0	0.1	0.3	2.6	34.1	- 0.5
					0.3	1.5	5.8	- 1.0
						0.9	2.0	- 1.5
							0.9	- 2.0

Acceptable solutions: 108 $\chi^2 = 0.858$.

Table 3b. The path solution 0.0A for the spectral group S3.

Features	Observed W_λ (A)	Synthetic W_λ (A)	Residue (A)	Window (A)
CaII K	15.5	14.8	0.7	2.5
CN 4216	8.5	9.8	- 1.3	3.0
CH G	8.5	8.2	0.3	1.5
Mg+MgH	7.9	6.5	1.4	1.8
CaII 8542	5.7	5.1	0.6	1.2
CaII 8662	4.6	4.7	- 0.1	1.2
Hδ	3.1	4.4	- 1.3	2.5
Hγ	4.4	4.9	- 0.5	1.2
Hβ	3.8	3.7	0.1	1.2

Solution in flux fractions (%) at 5870 A:

RHII	E7	5E7	E8	5E8	E9	5E9	› E10	AGE $[Z/Z_\odot]$
1.7	0	0	0	0	0	0	0	+ 0.6
	0	0	0	0	0	0	0	+ 0.3
	0	0	0	0	0.9	17.5	51.4	0.0
	0	0	0	0	0	0	24.1	- 0.5
					0	0	3.1	- 1.0
						0	0.9	- 1.5
							0.3	- 2.0

Acceptable solutions: 32 $\chi^2 = 0.819$.

Table 3c. The path solution 0.3B for the spectral group S3.

Features	Observed W_λ (A)	Synthetic W_λ (A)	Residue (A)	Window (A)
CaII K	15.5	14.6	0.9	2.5
CN 4216	8.5	9.9	- 1.4	3.0
CH G	8.5	8.1	0.4	1.5
Mg+MgH	7.9	6.6	1.3	1.8
CaII 8542	5.7	5.1	0.6	1.2
CaII 8662	4.6	4.7	- 0.1	1.2
Hδ	3.1	4.4	- 1.3	2.5
Hγ	4.4	4.9	- 0.5	1.2
Hβ	3.8	3.8	0.0	1.2

Solution in flux fractions (%) at 5870 A:

RHII	E7	5E7	E8	5E8	E9	5E9	› E10	AGE $[Z/Z_\odot]$
2.1	0	0	0	0	0	0	0	+ 0.6
	0.2	0	0	0	0.7	24.6	0	+ 0.3
	0	0	0	0	0	0	38.8	0.0
	0	0	0	0	0	0	24.0	- 0.5
					0	0	5.7	- 1.0
						0	2.4	- 1.5
							1.5	- 2.0

Acceptable solutions: 41 $\chi^2 = 0.866$.

Table 4a. The whole plane solution for the spectral group E8.

Features	Observed W_λ (A)	Synthetic W_λ (A)	Residue (A)	Window (A)
CaII K	6.4	5.6	0.8	2.5
CN 4216	0.3	2.1	-1.8	3.0
CH G	2.5	2.6	-0.1	1.5
Mg+MgH	2.6	2.8	-0.2	1.8
CaII 8542	4.6	4.3	0.3	1.2
CaII 8662	3.9	4.1	-0.2	1.2
Hδ	9.2	8.8	0.4	2.5
Hγ	6.6	7.0	-0.4	1.2
Hβ		6.3		1.2

Solution in flux fractions (%) at 5870 A:

RHII	E7	5E7	E8	5E8	E9	5E9	> E10	AGE $[Z/Z_\odot]$
1.7	4.1	2.2	0.4	0.5	0.2	0.2	0.4	+0.6
	1.0	2.8	0.6	0.4	0.2	0.2	0.4	+0.3
	0.9	2.8	0.4	0.6	0.4	0.4	0.4	0.0
	8.3	2.8	5.6	41.5	0.9	1.2	0.9	-0.5
					3.2	2.3	2.8	-1.0
						0.7	1.2	-1.5
							7.7	-2.0

Acceptable solutions: 161 $\chi^2 = 0.626$.

Table 4b. The path solution -0.5A for the spectral group E8.

Features	Observed W_λ (A)	Synthetic W_λ (A)	Residue (A)	Window (A)
CaII K	6.4	5.6	0.8	2.5
CN 4216	0.3	2.0	-1.7	3.0
CH G	2.5	2.7	-0.2	1.5
Mg+MgH	2.6	2.8	-0.2	1.8
CaII 8542	4.6	4.2	0.4	1.2
CaII 8662	3.9	3.9	0.0	1.2
Hδ	9.2	8.8	0.4	2.5
Hγ	6.6	7.0	-0.4	1.2
Hβ		6.3		1.2

Solution in flux fractions (%) at 5870 A:

RHII	E7	5E7	E8	5E8	E9	5E9	> E10	AGE $[Z/Z_\odot]$
1.3	0	0	0	0	0	0	0	+0.6
	0	0	0	0	0	0	0	+0.3
	0	0	0	0	0	0	0	0.0
	14.4	9.4	7.3	44.0	3.3	3.3	2.7	-0.5
					0	0	9.4	-1.0
						0	1.8	-1.5
							3.2	-2.0

Acceptable solutions: 56 $\chi^2 = 0.584$.

Table 4a contains the results for group E8 which is the blue nucleus of the lenticular galaxy NGC 5102. Blue nuclei tend to scatter more in the plane, probably because we use spectral data in the visible and near-infrared ranges only. Additional data in the near-ultraviolet will certainly produce better focused solutions in the plane. However, the strong burst of star formation at 5×10^8 yr with $[Z/Z_\odot] = -0.5$ is easily detected by the whole plane approach. In Table 4b we present the -0.5A path solution that has the best χ^2.

The remaining spectral groups of B88 have also been synthesized with the multi-minimization procedure. However, the groups presented above describe the basic situations faced during the application of the method.

4 Conclusions

We apply a multiple minimization procedure to the population synthesis of galaxy nuclei, using a grid of star cluster equivalent widths as a function of age and metallicity. This method has proved to be a powerful tool to explore the space generated by the grid elements, in order to map solutions for this degenerate inverse problem. The statistical treatment applied to the acceptable solutions gives an astrophysically more realistic scenario with respect to single optimal solutions from classical methods.

For some classes of red galaxy nuclei, this observational method, which is free of any astrophysical constraint, has produced similar results to those predicted by the theoretical approach of Arimoto & Yoshi (1987). The synthesis results for blue nuclei tend to scatter more in the age versus metallicity plane, probably because we use only visible and near-infrared spectral data. Additional information at shorter wavelengths will possibly produce better focused solutions in the plane. Never-the-less, strong bursts of star formation are easily detected.

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