Optimum Truss Design under Failure Constraints Combining Continuous and Integer Programming

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1. Abstract

This paper studies the minimum weight design of space trusses under failure criteria, combining continuous and integer programming. The optimization of space truss is first performed by using a continuous optimization algorithm, the sequential linear programming (SLP). The linear programming sub problem is obtained by a linearization process of the original nonlinear problem, requiring the first order derivatives of both objective and constraint functions with respect to design variables. The linearization process and linear solution is repeated until convergence is achieved.

However, in most practical problems, discrete values of the design variables are required, due to availability of standard member sizes and accuracy limitations in construction and manufacturing. Due to this limitation, this work uses a Genetic Algorithm (GA) to solve the integer optimization problem by selection of a number of standard cross sections. The optimization results obtained from the first method (SLP) are the input data for the second stochastic global search method (GA).

The constraints imposed to the optimization process are the member stresses, nodal displacements and minimum member size. The epsilon-relaxation technique is used to avoid the singular behavior of the stress constraint when the member areas goes to zero.

The benchmark results shows competitive performance of the approach with respect to the literature.

2. Keywords: Truss Optimization, Failure Constraint, Genetic Algorithms

3. Introduction

This paper presents a truss optimization method, combining algorithms that use continuous and discrete design variables. Continuous variable can assume any value in an interval, whereas discrete variable are limited to a finite set of values. For example, the engineer when constructing a truss could use circular steel tubes in its construction. The bars cross sections are discrete variable since they are found only in some commercial sizes. However the length can be considered a continuous variable, because it can be adjusted in any value.

Problems that deals with discrete variable, in general, use computer-intensive combinatorial algorithms that might take a long time to reach a solution. In the industry, it is usual to treat a discrete variable as continuous and at end of the continuous optimization process, adopt the closest discrete values to the continuous solution. However, this procedure does not work well, specially when the discrete values are too spaced, because the design can move away from the optimum and might violate some of the restrictions imposed to the problem.

In the present work, the structural design problem is the weight minimization of space trusses subject to stress, displacement and minimum member size constraints. The design variable is the cross section areas of each bar. First this work presents a continuous optimization procedure [1], requiring first order derivatives of objective and constraints. The result obtained in the first optimization stage are used as start point for the second stage that uses genetic algorithms [4] to perform the integer programming, yielding results using only a set of commercially available cross sectional areas.

4. The optimization problem

The optimization problem is the minimization the weight of the structure subject to stress, displacement and minimum member size constrains.
The objective function is:

$$W = \sum_{i=1}^{nel} \rho_i L_i A_i \tag{1}$$

where L_i is the length, ρ_i is the material specific weight, and Ai is the cross sectional of the i-th bar. Note that the objective function is linear on the design variables Ai.
The problem is subject to tensile and compressive stress constraints, bounds on displacements, and side constraints on the areas, as follows:

$$\sigma_i^C \leq \sigma_i \leq \sigma_i^T \qquad i=1,..nel$$

$$U_i^{min} \leq U_i \leq U_i^{max} \qquad i=1,..sdof$$

$$A_i^{min} \leq A_i \leq A_i^{max} \qquad i=1,..nel$$

where

σ_i = i-th tensile stress;

σ_i^C = i-th bar limit compressive stress (lower bound);

σ_i^T = i-th bar limit tensile stress (upper bound);

U_i = Displacement of the i-th degree of freedom,

U_i^{min} = i-th degree of freedom minimum displacement (lower bound);

U_i^{max} = i-th degree of freedom maximum displacement (upper bound);

A_i^{min} = i-th bar Minimum cross sectional area;

A_i^{max} = i-th bar Maximum cross sectional area.

Stress and displacement constraints are non-linear, while the side constraints are easily dealt with.

5. Sequential linear programming (SLP)

Standard structural truss analysis by the finite element method is used to calculate the structural responses (displacements and tensions) and the sensitivity of the system. Optimization is achieved by Sequential Linear Programming, consisting in linearizing objective and all constraints functions by truncating the Taylor series at the current design [1] and applying Linear Programming to find the next design. Suitable move limits for each design variable are added to keep design changes within the validity of the linearization and to control the convergence. In this work, the adaptation of the move limits to improve convergence is based in Pedersen [10].

The stress constraint can be written as:

$$\sigma_i^C \leq \sigma_i^0 + \sum_{j=1}^{nel} \frac{d\sigma_i}{dA_j}(A_j - A_j^0) \leq \sigma_i^T \tag{2}$$

and the displacement constraint as:

$$U_i^{min} \leq U_i^0 + \sum_{j=1}^{sdof} \frac{dU_i}{dA_j}(A_j - A_j^0) \leq U_i^{max} \tag{3}$$

where *nel* is the number of elements (Cross section areas of bars) and *sdof* is the number of degrees of freedom in the model;

6. Sensitivity analysis

The objective of the sensitivity analysis is to compute the derivatives of the objective function and constrains with respect to the design variable. These derivatives are the sensitivities of the function response (objective function and constrains) over small variations in design variables.

The sensitivity analysis of structure is often a major computational cost in the optimization process and imprecisions affects adversely the convergence.

This work uses direct analytical derivatives to calculate the sensitivities.

Assuming that the external load vector *f* does not depend on the design variable, the equation for the displacement sensitivities can be written as

$$\frac{\partial u}{\partial A_j} = -K^{-1}\frac{\partial K}{\partial A_j}u \tag{4}$$

Finite element analysis provides the nodal displacements and the stiffness matrix inverse. The derivatives of the stiffness matrix with respect to a area is simply the correspondent elemental stiffness matrix divided by the area

The stress derivative can be written as:

$$\frac{\partial \sigma_e}{\partial A_j} = \frac{E}{L_e}\left(\frac{\partial u_J}{\partial A_j} - \frac{\partial u_I}{\partial A_j}\right) \tag{5}$$

where the indices J and I are the final and initial nodes of each bar.

7. Genetic Algorithms

The genetic algorithms (GA) differs from other classic optimizations process in many ways, since it is inspired in natural evolution. The genetic algorithm is a part of evolutionary computation technique of global search methods and has been preferred in wide range of the optimization problems by researchers [4] for its simplicity and the availability of algorithms.

Instead other methods, an initial randomly population of possible solutions is created. This population can be seen as points in the set of admissible solutions.

As in a biological problems submitted to external environmental pressure, chosen members of the initial population with the best (traits) have the chance to reproduce and transmit part of their genetic heritage to the next generation. A new population is then created by recombination of parents genes. The process is repeated over and over, until a satisfying solution for the problem is obtained, ensuring that certain design criteria are satisfied or the maximum number of the generations are reached.

The methodology [3] used in this work is easy to code and can be applied to a wide range of optimization problems.

The mathematical model of minimum weight design is shown in equation (1). In addiction, the problem subjected to a stress and displacement constrains can be expressed as follows:

$$\frac{|\sigma_i|}{\sigma_{adm}} - 1 \leq 0 \quad i=1,.. \text{nel} \tag{6}$$

$$\frac{|u_j|}{u_{max}} - 1 \leq 0 \quad j=1,.. \text{ngrl} \tag{7}$$

8. The Fitness Function

As the equation (1) is an unconstrained problem, it is necessary to transform it into a constrained problem. There are many possible alternatives. In this work, an exterior penalty approach is used [3]. The transformed model is expressed as follows:

$$W = \sum_{i=1}^{nel} \rho_i A_{j(i)} L_i + P \left(\sum_{i=1}^{nel} \left(\left[\frac{|\sigma_i|}{\sigma_{max}} - 1 \right]^+ \right) + \sum_{j=1}^{ngl} \left(\left[\frac{|u_j|}{u_{max}} - 1 \right]^+ \right) \right) \tag{8}$$

where

$[x]^+ = x$ if $x > 0$
$[x]^+ = 0$ otherwise

and P is a penalty coefficient set to 10^5 for all experiments

9. Numerical examples

The problems below are used to show the efficiency of the combining continuous and integer algorithms in the optimization of trusses. The algorithm was coded in Matlab, using the genetic algorithm and linear programming toolboxes. Standard benchmark problems were chosen for the comparison sake.

9.1. Weight optimization of a 10 – Bars plane truss

The geometry of 10-bar plane truss structure is show in figure 1. This optimization problem has been studied by many researchers, and solutions by many different optimization approaches are available in the literature.
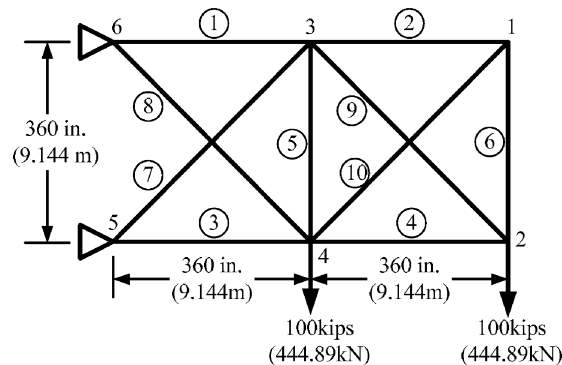
Figure 1 10-bars truss

The objective function of the problem is to minimize the weight of the structure combining techniques based on linear programming (SLP), and genetic algorithms (GA). The optimization of the truss is first performed by using sequential linear programming (SLP), and the result obtained from first method (SLP) is the input data for second stochastic global search method (GA) using available members sizes. The input data for this problem are Young's modulus, $E = 10^4$ Ksi (6.89×10^4 MPa), material density, $\rho = 0.1$ lb/in^3 (2,770 kg/m^3) and vertical downward loads of 100 Kips (445.347 kN) at joints 2 and 4. The allowable displacement is limited to 2 in (50.8 mm) in both x and y directions at all nodes, and the allowable stress=± 25 ksi (172 MPa) for all members.

Forty two shapes taken from AISC (American Institute of Steel Construction) manual are available for design variables, give in the list [1] S: {1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50} (in^2) (1 in^2= 6.45 cm^2).

Since each bar can assume any of the 42 values of the available sections, the number of combinations is 10^{42}.

Table 1 presents the results obtained for a continuous variable problem using sequential linear programming (SLP) and minimum area of 0.1 in^2. The results are compared with others studies found in literature, see [2] for details.

Table 1 Ten-Bars truss designs with SLP, in^2

Method	Weight (lb)	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
Haftka	1514	7.90	0.10	8.10	3.90	0.10	0.10	5.80	5.51	3.68	0.14
Present work	1509	7.90	0.10	8.09	3.90	0.10	0.10	5.78	5.52	3.86	0.13

Table 2 presents the results obtained with the combined SLP and GA technique introduced here, comparing the values produced with others structural optimization methods. The continuous problem differs from the previous one, since the lower bound for the area was chosen as the smallest available cross section.

Table 2 Ten-Bars truss designs with GA, in^2

Method	Weight lb (N)	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
1	5491.1	33.50	1.62	22.90	15.50	1.62	1.62	7.97	22.00	22.00	1.62
2	5613.8	33.50	1.62	22.00	15.50	1.62	1.62	14.20	19.90	19.90	2.62
3	5586.5	30.00	1.62	22.90	13.50	1.62	1.62	13.90	22.00	22.00	1.62
4	5541.1	30.00	1.62	26.50	15.50	1.62	1.62	7.97	22.00	22.90	1.62

Notes
1- Improved Penalty Function Method (Cai and Thiereu, 1993)
2- Genetic Algorithms (Rajeev and Krishnamoorthy, 1992)
3- Genetic Algorithms (Coello, 1994)
4- Present work

By comparing the results obtained with SLP and GA, the SLP produces the best reduction in the structure final weight. However, due to limitations in construction and manufacturing, engineers consider discrete values as design variables. Thus, this work used the results obtained with SLP as initial design points for GAs to accelerate the convergence. The result of the present work is comparable to the literature.

9.2. Weight optimization of a 25 bars space truss
The second example is the 25 bars space truss, shown in figure 2. There is a total of 8 design variables used for 25 truss size members, given in the list S={ 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7,1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 3.0, 3.2, 3.4} (1 in2=6.45 cm2).

Loading conditions for this space truss are given in table 4. The member groupings are given in table 3. The assumed data are: Young's modulus, $E= 10^4$ Ksi (6.89×10^4 MPa), material density, $\rho = 0.1$ lb/in^3 (2,770 kg/m^3). The allowable displacement is limited to 0.35 in (8.89 mm) in x, y and z directions at all nodes, and the allowable stress=± 40 ksi (275.8 MPa) for all members.
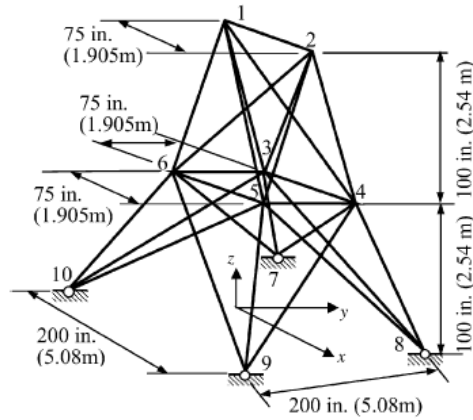
Figure 2 – Twenty-five-bar truss

Table 3 Design variable of the 25 bars truss

Design Variable Number	End Nodes of members
1	(1,2)
2	(1,4),(1,5),(2,3),(2,6)
3	(1,3),(1,6),(2,4),(2,5)
4	(3,6),(4,5)
5	(3,4),(5,6)
6	(3,10),(4,9),(5,8),(6,7)
7	(3,8),(4,7),(5,10),(6,9)
8	(3,7),(4,8),(5,9),(6,10)

Table 4 -Loading Data

Nodal Number	Px (kips)	Py (kips)	Pz (kips)
1	1.0 (4.454 kN)	-10.0 (-44.53 kN)	-10.0 (-44.53 kN)
2	0	-10.0 (-44.53 kN)	-10.0 (-44.53 kN)
3	0.6 (2.227 kN))	0	0
4	0.5 (2.672 kN)	0	0

Table 5 Optimum design for twenty-five-bar truss with SLP, in2

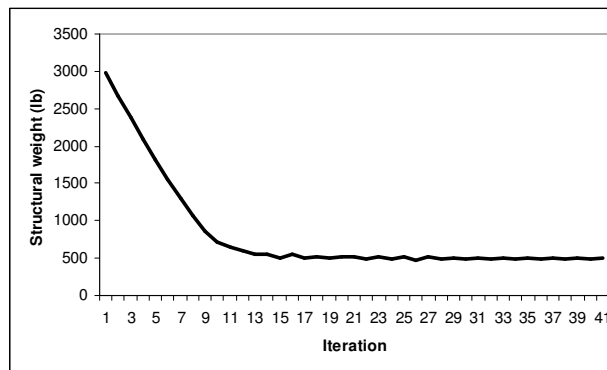| Method	Weight (lb)	A1	A2	A3	A4	A5	A6	A7	A8
Haftka	542.22	0.010	1.987	2.991	0.010	0.012	0.683	1.679	2.664
Present work	498.78	0.010	0.192	3.590	0.010	1.270	0.819	0.846	3.660

Figure 2 SLP convergence history for twenty-five-bar truss.

Table 7 Twenty-five-bar truss designs with GA, in^2

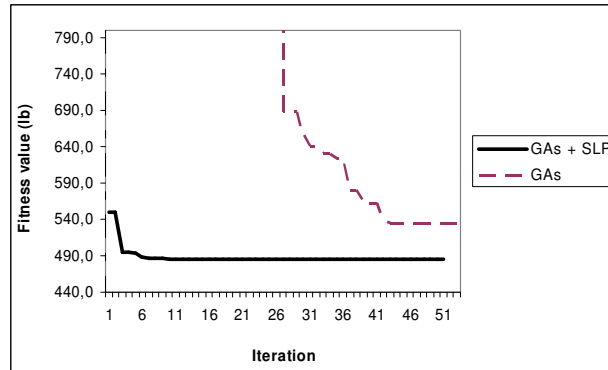| Method	Weight (lb)	A1	A2	A3	A4	A5	A6	A7	A8
Cai and Thiereut	487.41	0.1	0.1	3.4	0.1	2.0	1.0	0.7	3.4
Rajeev and Krishnamoorthy	546.01	0.1	1.8	2.3	0.2	0.1	0.8	1.8	3.0
Duan	562.93	0.1	1.8	2.6	0.1	0.1	0.8	2.1	2.6
Present work	485.5	0.1	0.2	3.4	0.1	1.5	0.9	0.8	3.4

Figure 3 Convergence history (GA) for the twenty-five-bar truss.

The solution with the SLP starting point takes 23% more time than using only the GA; however, the result is better.

10. Conclusions

This research presented an optimization method for space truss optimum design, combining the traditional sequential searching with GAs algorithms, considering displacement, stress and minimum size members constrains.

From the results of the two structures studied in this paper is possible to show that this kind of optimization method is competitive with other search methods.

The genetic algorithms allowed to solve, in a easy way, the optimization problem with discrete design variables. The convergence speed of GAs method can be accelerated using initial design points from the continuum search space, while the additional computational time of the continuous search is partially compensated with the gains in the integer search.

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