A Topological Approach to the Identification of Critical Measurements in Power-System State Estimation

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***Abstract*—This paper presents a new topological methodology for critical measurements identification in observable networks. A measurement is said to be critical, in an observability sense, if its removal from the measurement set makes the associated system lose observability. The proposed methodology is based on the properties of both, observable measurement subnetworks (OMS) and redundant branch sets (RBS), for the first time proposed. To reduce the combinatorial bluster, the proposed method divides the measurements into two groups and classifies them into two phases. It allows identifying the critical measurements without any numerical calculation. Indeed, it is simple and fast. To clarify the proposed method and to demonstrate its simplicity, two examples are provided. The proposed method is successfully tested in the IEEE-14 bus system as well as in two realistic systems of Brazilian utilities. The first is a 121-bus system by ELETROSUL, and the other is a 383-bus system by Companhia Hidroelétrica do São Francisco (CHESF).**

***Index Terms*—Critical measurements, graph theory, network observability, state estimation.**

I. Introduction

The performance of power-system control actions depends mainly on the process of state estimation. On the other hand, the performance of the state estimator depends on the quality and availability of the existing measurements. From the point of view of measurement availability, it is necessary to know if the available measurements are enough to obtain an estimation of all the system states. In this case, the system is said to be observable. Otherwise, it is unobservable.

Although the observability of the system is a necessary condition to obtain a state estimation, it is not sufficient to guarantee a reliable state estimation. From the quality point of view, in order to obtain a reliable state estimation it is necessary to guarantee that the estimation is not affected by gross errors in the measurements. Thus, the ability to detect measurements with gross errors is one of the most important functions of the state estimation process.

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The identification of the critical measurements in a measurement set is very important basically due to two aspects: the unavailability of a critical measurement, if it exists, make the system unobservable and the fact that it is impossible to detect gross errors in critical measurements [1].

Therefore, it is a good practice to allocate new measurements in the network in order to transform the existing critical measurements into redundant ones.

To guide this reinforcement of the measurement set, several methods to identify critical measurements have been reported in literature. In a general way, they can be divided in two groups: the topological [1]–[4] and the numerical methods [5], [6]. Those of the first group are based in graph concepts and have combinatorial nature, while those of the second group are based on statistical concepts and are conceptually simpler, but need some computer work and also may present some numerical difficulties, since they require the analysis of the measurement residuals.

The method to identify critical measurements as proposed in [7] is a mixed numerical–symbolic method based on both a reduced model [8] and graph theory. In [9] using information concepts, a mapping of the system states is made, finding a new state space where the identification of critical measurements is obtained in a quite straightforward manner. In these methods, the numerical calculation required to identify the critical measurements is reduced.

In this paper, a fast and simple topological method that allows the identification of critical measurements requiring no numerical calculation is proposed. The idea is to explore the intrinsic nature of the measurements (flows or injections) in such a way that the number of search possibilities is dramatically reduced, mitigating the problem of combinatorial explosion. The method was tested and has shown to be suitable for application in systems of large dimension.

This paper is organized as follows. Section II presents a review of some topological concepts. The method, the algorithm, and two examples are in Section III. Section IV presents the performed tests, and the conclusions are reported in Section V. In Appendixes A and B, the theoretical support of the proposed method is developed.

II. Topological Concepts Review

Remark 1: Some of the nomenclature and definitions presented in this paper are borrowed from [1].

Let X be a measured network, that is, a power system with a specified measurement set; $G(X)$ is the graph of the l-line diagram, $G^0(X)$ denotes the set of nodes of $G(X)$ (or buses of X) and $G^l(X)$ denotes the set of edges or branches of $G(X)$ (lines or branches of X). In the same way, the measurement set $M(X)$ on the measured network X may be described by the union of $M^0(X)$ and $M^l(X)$, where $M^0(X)$ denotes the set of nodes of $G^0(X)$ at which the real and reactive bus injection powers are measured and $M^l(X)$ denotes the set of branches of $G^l(X)$ on which the real and reactive flow powers are measured. The nodes of $M^0(X)$ are called measured nodes, and the branches of $M^l(X)$ are called measured branches.

Definition 1: The tree T of the measured network X is a subgraph $G(T)$ of $G(X)$ that is connected and loop-free. Tree T is called a spanning tree if T contains every node of X: $G^0(T) = G^0(X)$.

Definition 2: A measurement assignment "a" on a spanning tree T of X is a function that associates a measurement $a(b)$ of $M(X)$ to each branch b of $G^l(T)$ and that satisfies the following three conditions:

1) $b_1 \neq b_2 \Longrightarrow a(b_1) \neq a(b_2)$.
2) If $a(b)$ is an injection measurement, the node at which $a(b)$ occurs is an end point of branch b.
3) If $a(b)$ is a flow measurement, then $a(b) = b$.

Definition 3: A measured network is said to be *topologically observable* if there exists a spanning tree T of X and a measurement assignment "a" associated to T.

Definition 4: A measurement "m" of $M(X)$ is a *critical measurement* of X if X is made unobservable when "m" is deleted from $M(X)$. If "m" is not critical, then it is said to be *redundant*.

Topologically, a measurement "m" is *critical* if every measurement assignment "a" for each tree T of X assigns a branch to "m".

Definition 5: A measured subnetwork Y of the measured network X is described by a subgraph $G(Y)$ of $G(X)$ and a subset of measurements $M(Y)$ of $M(X)$ that satisfies the following properties:

1) Each flow measured branch "b" of $M^l(Y)$ is a branch of $G^l(Y)$.
2) Each measured node "y" of $M^0(Y)$ is a node of $G^0(Y)$.
3) Every branch "b" of $G^l(X)$ that is incident to a measured node of $M^0(Y)$ is a branch of $G^l(Y)$.

Definition 6: A measurement assignment on a fundamental loop L is a function "a" that associates a measurement of $M(X)$ to each branch of L and that satisfies the following properties:

1) $b_1 \neq b_2 \Longrightarrow a(b_1) \neq a(b_2)$
2) If $a(b)$ is an injection measurement, the node at which $a(b)$ occurs is an end point of branch b.
3) If $a(b)$ is a flow measurement, then $a(b) = b$.
4) The elimination of one branch of loop L generates a path with a measurement assignment which is part of some measurement assignment on a spanning tree T.

Remark 2: Property 4) guarantees that every measurement which is assigned to any branch of the fundamental loop L is redundant.

III. Proposed Method

The main difficulty of the existing topological methods to identify critical measurements is their combinatorial nature, which makes these methods unsuitable for application in very large systems. Aiming at the reduction of this combinatorial explosion, a methodology is proposed in this paper. Considering that the system in analysis is observable, the proposed methodology reduces the combinatorial bluster by dividing the measurements into two groups and the analysis into two phases.

In the first phase, the criticality of the injection measurements is analyzed through the concept of observable measured subnetworks (OMS). In the second phase, the criticality of the flow measurements is analyzed through the concept of redundant branch sets (RBS). Thus, before presenting these phases, it is necessary to know the following definitions.

Definition 7: A measured subnetwork Y of the measured network X is an *observable measured subnetwork* (OMS) if it is composed of a *connected subgraph* $G(Y)$ of $G(X)$ and there exists a spanning tree T of Y and a measurement assignment "a", formed only with measurements of $M(Y)$ associated to T.

Definition 8: A redundant branch set (RBS) is a measured subnetwork composed of a *connected subgraph* $G(Y)$ of $G(X)$ and a *subset of redundant measurements* $M(Y)$ of $M(X)$ that satisfies the following properties:

1) Each branch of $G^l(Y)$ is either a flow measured branch of $M^l(Y)$ or a branch incident to an injection measurement of $M^0(Y)$.
2) Every branch which is incident to some measurement of $M(Y)$ belongs to $G^l(Y)$.
3) $G^0(Y)$ contains all the nodes which are end points of $G^l(Y)$.

The RBS is a concept originally created in this paper. To clarify what an RBS is, let us identify the RBSs associated to the 6-bus power system with the measurement set as indicated in Fig. 1.

Considering that M1, M2, M3, M5, and M7 were previously classified as redundant measurements, there are three RBS associated to that system:

Y_1 (RBS 1):
- $G^0(Y_1) = [1, 2, 3]$
- $G^l(Y_1) = [(1\text{–}2)(1\text{–}3), (2\text{–}3)]$
- $M^0(Y_1) = \emptyset$
- $M^l(Y_1) = [M1, M2, M3]$

Y_2 (RBS 2):
- $G^0(Y_2) = [4, 5]$
- $G^l(Y_2) = [(4\text{–}5)]$
- $M^0(Y_2) = [M5]$
- $M^l(Y_2) = [M7]$

Y_3 (RBS 3):
- $G^0(Y_3) = [6]$
- $G^l(Y_3) = \emptyset$
- $M^0(Y_3) = \emptyset$
- $M^l(Y_3) = \emptyset$.

Remark 3: Unlike OMSs, the redundant branch sets can form unobservable measured subnetworks (see Fig. 2).

The following Lemma demonstrates some important properties of the branches of an RBS.

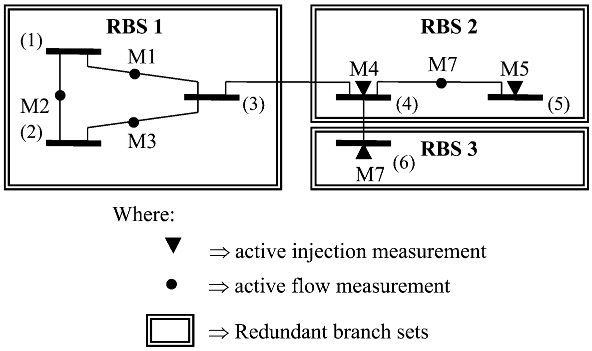


Fig. 1. RBSs of the 6-bus power system.

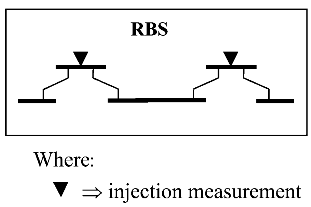


Fig. 2. RBS that forms an unobservable measured subnetwork.

Lemma 1: Every branch contained in a redundant branch set is said to be a redundant branch and satisfies one or both of the following properties:

1) The branch can be assigned to two or more redundant measurements.
2) The branch can be put into a fundamental loop L with a redundant associated measurement assignment.

Proof: Every branch "b" of an RBS is incident to at least one redundant measurement "m" of $M(Y)$. Therefore, the measured network X remains observable even after the elimination of "m", which means that there exists a measurement assignment on a spanning tree T which does not assign "m" to any branch.

If the spanning tree T contains the branch in analysis, another measurement different from the eliminated one will be assigned to this branch.

On the other hand, if the spanning tree T does not contain the branch in analysis, there will exist a path in T whose two end nodes are the end nodes of the branch in analysis. Each branch of this path is assigned to a measurement, therefore, it is possible to associate the measurement "m" to the branch in analysis forming a measurement assignment on a fundamental loop L. ■

Remark 4: Buses which are not incident to any redundant branch are considered to be an RBS composed of only one bus.

The advantage of using the OMSs and the RBSs is the dramatical reduction of the number of searches in the algorithm. This occurs because the OMSs and RBSs can be roughly viewed as supernodes which contain many nodes of the network. As a consequence, the search effort to verify the connection of the network is much less than the search effort to verify the network connection through all the nodes.

A. Phase 1

At the beginning of this phase, the OMSs are built using only the flow measurements. During the process, the OMSs are updated through the injection measurements as shown in the algorithm below.

The advantage of this phase is that, in order to build the spanning tree T of X, each OMS is considered a supernode of T.

The algorithm of phase 1 tries to classify as many measurements as possible in the first five steps of the algorithm. These steps are easily carried out and the searches involved are very simple. In general, most measurements of a measured network are classified into these five steps. As a consequence, a small number of injection measurements remains to be classified in Step 6, which requires a more complex search. Therefore, this algorithm reduces significantly the problem of combinatorial explosion.

1) Algorithm (Phase 1):

Step 1) Form all the possible OMSs using all the flow measurements (see Remark 5).

Step 2) (Lemma 2) Classify every nonclassified injection measurement which relates only nodes of a unique OMS as redundant. If there is any nonclassified injection measurement, go to the next step. Otherwise, stop; the analysis is complete.

Step 3) (Lemma 3) Let r be the number of OMSs. If there still are $(r-1)$ nonclassified injection measurements, all these injection measurements are classified as critical. The analysis is complete. Otherwise, go to the next step.

Step 4) (Lemma 4 and Remark 6) If there still exists some OMS that is incident only to one nonclassified injection measurement, this injection is classified as critical. The corresponding OMS is coalesced with an OMS connected to it through the measurement in consideration in a unique OMS, and go to Step 2. Otherwise, go to the next step.

Step 5) (Lemma 4 and Remark 6) If there still are two or more nonclassified injection measurements which relate nodes from only two OMSs Y_i and Y_j, then coalesce Y_i and Y_j into a unique OMS and return to Step 2. Otherwise, go to the next step.

Step 6) (Lemma 5) Connect all the r remaining OMSs using only $(r - 1)$ nonclassified injection measurements without forming loops. The nonclassified injection measurements not used to connect the r OMSs are classified as redundant. As these measurements are redundant, the branches incident to them are redundant branches. Thus, the OMSs connected by these branches are coalesced into a unique OMS. Return to Step 2.

Remark 5: An isolated bus, i.e., a bus which is not an end point of any flow measurement, will be considered as an OMS composed of one bus only.

The Lemmas and Remarks mentioned in the algorithm are presented in Appendix A.

B. Phase 2

In Phase 2, all the injection measurements have already been classified. Now the information about the redundancy of these measurements will be used for the analysis of the flow measurements. To carry out this phase, the RBSs are formed. First, these RBSs are formed using only the injection measurements classified as redundant in Phase 1.

At the beginning, two buses connected by a branch will belong to the same RBS if there is a redundant injection measurement in at least one of them. The main idea of this phase is to form redundant branch sets and to augment them every time two or more flow measurements that connect two or more RBSs are classified as redundant.

Through Definition 4, it can be verified that the critical measurements are incident neither to branches that can be assigned to more than one measurement, nor to branches that can be put into a fundamental loop L which has a measurement assignment associated to it. Based on these properties and the properties of Lemma 1, the proposed method analyzes the criticality of the flow measurements.

Similarly to Phase 1, the Phase 2 algorithm tries to classify as many measurements as possible into the first three steps. Step 4 requires a more complex search algorithm.

1) Algorithm (Phase 2):

Step 1) Form the RBSs using only the injection measurements previously classified as redundant (see Remark 3 and 4). Go to the next step.

Step 2) (Lemma 6, 7, and 8) Classify all the nonclassified flow measurements incident only to nodes of a unique RBS as redundant. If there is any nonclassified flow measurement, go to the next step. Otherwise, stop; the analysis is complete.

Step 3) (Lemma 9) If there are two or more flow measurements incident only to two RBSs Y_i and Y_j, coalesce these RBSs into a unique larger RBS and return to Step 2. Otherwise, go to the next step.

Step 4) (Lemma 10) Let $Y_1, \ldots, Y_r$ be r RBSs. Try to find r flow measurements which relate nodes from only these r RBSs and try to assign these measurements $m_1, \ldots, m_r$ to branches $b_1, \ldots, b_r$ in such a way the RBSs are connected through these branches into a loop. If this is possible, coalesce $Y_1, \ldots, Y_r$ into a larger RBS and return to Step 2. Otherwise, go to the next step.

Step 5) All unclassified flow measurements are critical ones. The analysis is complete.

The theoretical background of the previous algorithm is presented in Appendix B.

C. Example 1

In this example, the proposed method is applied to the 6-bus power system with the measurement set as indicated in Fig. 3. Note that this system is observable with this measurement set.

Phase 1:

Step 1) Using the flow measurements, three OMSs are obtained: [1, 2, 3]; [4, 5]; and [6].

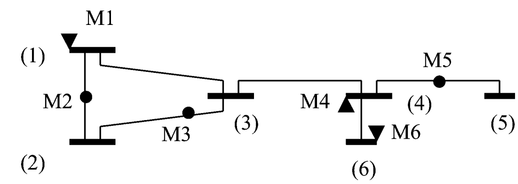


Fig. 3. 6-bus system.

Step 2) The injection measurement M1 relates only nodes of the OMS [1, 2, 3], so it is classified as redundant.

Step 3) There are three OMSs and two injection measurements which are not classified yet. Thus, the injection measurements M4 and M6 are classified as critical and the analysis of Phase 1 is finished.

Phase 2:

Step 1) Through the injection measurement classified as redundant, four RBSs are obtained: [1, 2, 3]; [4]; [5]; and [6].

Step 2) The flow measurements M2 and M3 are incident only to nodes of the RBS [1, 2, 3]. Thus, these measurements are classified as redundant.

Step 3) There are not two or more flow measurements which relate only to two RBSs.

Step 4) It is not possible to form any fundamental loop L with the nonclassified flow measurement.

Step 5) The flow measurement M5 has not been classified yet, thus this measurement is classified as critical.

The results obtained are:

— Critical measurements: M4, M5, and M6;
— Redundant measurements: M1, M2, and M3.

D. Structure of the Measurement Set

The method to identify critical measurements as previously presented takes into account measurement sets formed only by injection and flow measurements. Thus, it can be used only to analyze the active model.

To analyze the reactive model, the proposed method transforms the voltage magnitude measurements into equivalent flow measurements. Those flow measurements are put on fictitious branches that connect the ground to the buses having the voltage measurement [10]. Another necessary change is to consider that all reactive injection power measurements connect the buses where they occur to the ground. With these adaptations, the algorithm, as previously presented, can be applied to analyze the reactive model. To clarify the method, the next example is presented.

E. Example 2

The proposed method is applied to the 6-bus power system with the measurement set as shown in Fig. 3, but now with reactive power measurements. Consider also voltage magnitude measurements at buses 3 and 5.

Phase 1:

Step 1) Through the flow measurements, two OMSs are formed (recall that the voltage magnitude measurements are considered flow measurements that connect the ground to the buses having the voltage measurement): [1, 2, 3, 4, 5, G] and [6].

Step 2) The injection measurement M1 relates only nodes of the OMS [1, 2, 3, 4, 5, G]. Thus, this measurement is classified as redundant.

Step 3) There are two OMSs and two injection measurements which are not classified yet. Go to the next step.

Step 4) There is no OMS related only by one nonclassified injection measurement.

Step 5) The injection measurements M4 and M6 relate nodes from only the two OMSs. Thus, these OMSs are coalesced into a unique OMS. Thus, now there exists only one OMS composed of all the buses of the system. Return to Step 2.

Step 2) The injection measurements M4 and M6 relate only nodes of a unique OMS. Thus, these measurements are classified as redundant and the analysis of Phase 1 is finished.

Phase 2:

Step 1) Through the injection measurements classified as redundant, only one RBS is formed: [1, 2, 3, 4, 5, 6, G].

As all the buses of the system are in the same RBS, the flow and voltage magnitude measurements will be identified as redundant by the second step.

As a result, the proposed method identifies all measurements as redundant.

IV. Tests and Analysis of Results

To verify the efficiency of the proposed method, it was implemented under the C compiler to UNIX on a Pentium 166 Hz and applied to the IEEE-14 bus system and two realistic systems of Brazilian utilities. One system is a 121-bus system by ELETROSUL (Fig. 4), and the other is a 383-bus system by CHESF.

Using these systems, various scenarios were tested and their results are given in Table I. They are as follows.

Scenario 1: The IEEE-14 bus system with nine injection measurements of active power at buses 1, 2, 3, 4, 9, 11, 12, 13, 14, and six flow measurements of active power on branches (1–2), (1–5), (4–7), (4–9), (7–8), (6–10).

Scenario 2: The IEEE-14 bus system with the measurement set presented in case 1, but now as reactive power measurements. Consider also 2-voltage magnitude measurements at buses 8 and 10.

Scenario 3: The 121-bus system by ELETROSUL, considering 69 injection measurements and 65 flow measurements of active power, as shown in Fig. 4.

Scenario 4: The 121-bus system by ELETROSUL, considering 69 injection measurements, 65 flow measurements of reactive power, and eight voltage magnitude measurements, as shown in Fig. 4.

Scenario 5: The 383-bus system by CHESF, with 132 injection measurements and 396 flow measurements of active power.

TABLE I
Obtained Results

Scenario	Number of Critical Measurements			CPU time (sec)
	Injections	Flows	Voltage magnitudes	
1	**5**	**2**	**---**	**0,0000**
2	**0**	**0**	**0**	**0,0000**
3	**40**	**19**	**---**	**0,04**
4	**33**	**9**	**1**	**0,04**
5	**14**	**101**	**---**	**0,12**
6	**13**	**73**	**0**	**0,11**

Scenario 6: The 383-bus system by CHESF, with 132 injection measurements, 396 flow measurements of reactive power, and 19 magnitude voltage measurements.

The results obtained always indicated the correct answer, requiring a short computing time and reducing dramatically the number of search to classify the measurements. To illustrate this reduction, consider the test related to Scenario 3. The proposed method classifies all the injection measurements reaching Step 6 of Phase 1 only twice (this step requires a lot of search).

When the method reached that step by the first time, 36 injection measurements had already been classified, which corresponded to 52% of the number of available injection measurements. The number of OMSs at this stage was 30, which means a reduction of 75% of the system size (from 121 buses to 30 OMSs).

When that happened by the second time, 48 injection measurements had already been classified and there were only 21 OMSs.

To check whether the results were correct, the observability program developed by Bretas *et al.* [11] was used.

V. Conclusion

In this paper, a new topological method to identify critical measurements in observable power networks was developed. The proposed method does not require any numerical calculation and provides interesting performance in terms of speed, efficiency, and robustness.

The main advantage of the proposed method is to reduce the combinatorial nature of the problem of identifying critical measurements, dividing the measurements into two groups and the analysis into two phases. As a consequence, the method is very fast and of easy implementation.

These divisions are supported by a solid background on graph theory.

Several new concepts were proposed and new results were proved in order to justify both phases of the algorithm. The first phase takes advantage of the properties of observable measured subnetworks (OMS), while the second explores the properties of redundant branch sets (RBS), for the first time proposed. Using the OMSs and the RBSs, the search for verifying the network connection is dramatically reduced.

Many networks available in literature and used as a test for observability purpose were also tested in this research.

The proposed method can be useful to indicate to the system operator the critical measurements in the monitoring scheme. It

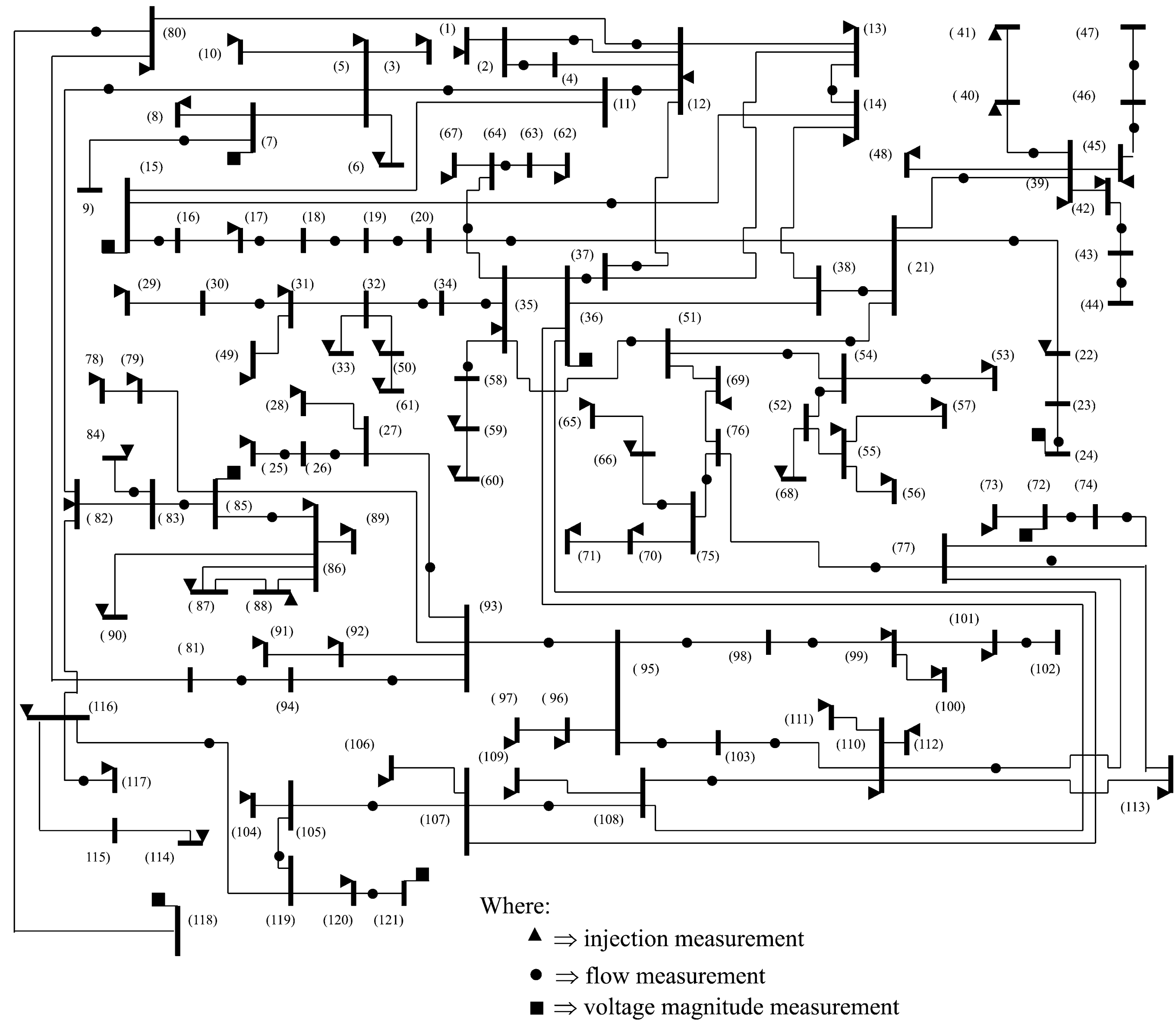


Fig. 4. 121-bus system by ELETROSUL (BRAZIL).

can also be applied for the design of a new measurement set, as well as for the evaluation of an existent one.

APPENDIX A
PHASE 1

This Appendix aims to support theoretically phase 1 of the algorithm proposed in this paper.

Lemma 2: Let Y be an observable measured subnetwork composed of a connected subgraph $G(Y)$ of $G(X)$ and a subset of measurements $M(Y)$ of $M(X)$.

Suppose a measurement "m" is added to the set $M(Y)$ and suppose "m" is incident only to branches of $G^l(Y)$. Then the pair $G(Y), M'(Y)$, where $M'(Y) = m \cup M(Y)$, forms an observable measurement subnetwork (OMS) and "m" is a redundant measurement.

Proof: It is evident from the definition of measured subnetwork (Definition 5) that the pair $G(Y)$ and $M'(Y)$ forms a measurement subnetwork. As $G(Y)$ and $M(Y)$ forms an OMS, there exists a measurement assignment on a spanning tree T of $G(Y)$. It is clear that this tree is also a tree of the new measured subnetwork and the same measurement assignment can be done without using measurement "m". Then "m" is a redundant measurement and the new measured network is observable. ■

Lemma 3: Let $Y_1, \ldots, Y_r$ be r OMSs of an observable network X. If there are only $(r-1)$ measurements of $M(X)$ possible to assign to $(r-1)$ branches of $G^l(X)$, in such a way that all OMSs become interconnected, then $(r-1)$ measurements are critical.

Proof: As $Y_1, \ldots, Y_r$ are r OMSs, then, for each Y_i, there is a spanning tree T_i, with a measurement assignment associated to it. As these OMSs are subnetworks of an observable network X, there is a spanning tree T with a measurement assignment associated to T that can be formed by $\cup_{i=1}^{r} T_i$, plus a set of additional measurements connecting all the OMS.

To connect those r spanning trees, at least $(r-1)$ measurements are necessary. As there are only $(r-1)$ additional measurements possible to connect these r spanning trees, X is made unobservable when any of these $(r-1)$ measurements is deleted. Therefore all these measurements are critical. ■

Lemma 4: Let Y_1 and Y_2 be two OMSs of an observable network X. Suppose an injection measurement "m" is incident only to branches which connect a node i of $G^0(Y_1)$ to a node j of $G^0(Y_2)$ and possibly incident to branches of $G^l(Y_1)$ or $G^l(Y_2)$, then it is possible to obtain a greater observable measured subnetwork represented by

$$G^0(Y_1) \cup G^0(Y_2)$$
$$G^l(Y_1) \cup G^l(Y_1) \cup B$$
$$M(Y_1) \cup M(Y_2) \cup m$$

where B is the set of all branches which connect Y_1 to Y_2.

Proof: As Y_1 and Y_2 are OMSs, there are spanning trees T_1 and T_2 with a measurement assignment associated. The measurement "m" can be assigned to one of its incident branches which connect both OMSs. Let "b" be this branch, then $T_1 \cup T_2 \cup \mathrm{b}$ forms a spanning tree for the larger measured subnetwork and each branch of this tree has a measurement assigned to itself, therefore, the larger subnetwork is observable. ■

Remark 6: If two or more measurements $m_1, \ldots, m_r$ are in the situation of Lemma 4, then all these measurements are redundant because one of them can be used to coalesce the OMSs into a larger one while the others are redundant by Lemma 2. As the choice of the measurement to coalesce the OMSs is arbitrary, all of them are redundant.

Otherwise, if there is only a measurement m in the situation of Lemma 4, and there is no other path to connect the OMSs Y_1 and Y_2, m will be a critical measurement. This occurs because, with the lost of the measurement "m", the spanning tree T will not exist any more, consequently the large OMS associated to T will be divide in two OMSs, the OMS Y_1 and the OMS Y_2, and the network X will be made unobservable.

Lemma 5: Let $Y_1, \ldots, Y_r$ be r observable measured subnetworks. Consider the existence of $(r-1)$ additional measurements $m_1, \ldots, m_{r-1}$, which are incident to $(r-1)$ branches that connect a node i of $G^0(Y_i)$ to a node j of $G^0(Y_j)$ and incident to branches of $\bigcup_{i=1}^{r} G^l(Y_i)$. If it is possible to assign these $(r-1)$ branches in such a way that all OMSs become interconnected, then it is possible to obtain a greater observable measured subnetwork represented by

$$\cup_{i=1}^{r} G^0(Y_i)$$
$$\cup_{i=1}^{r} G^l(Y_i) \cup B$$
$$\cup_{i=1}^{r} M(Y_i) \cup_{i=1}^{(r-1)} m_i$$

where B is the set of all branches which connect Y_i to $Y_j, i, j = 1, \ldots, r$.

Lemma 5 is a generalization of Lemma 4. The proof is very similar, therefore, it will be omitted.

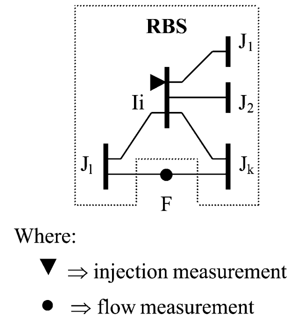


Fig. 5. Flow measurement incident to nodes which are incident to a unique redundant injection measurement.

Appendix B
Phase 2

This Appendix aims to support theoretically Phase 2 of the algorithm proposed in this paper.

Lemma 6: The addition of a pseudo-flow in a redundant branch does not change the criticality of any measurement.

Proof: In Lemma 1, it was proved that a redundant branch satisfies one or both of the following properties:

1) The branch can be assigned to two or more redundant measurements.
2) The branch can be put into a fundamental loop L with an associated redundant measurement assignment.

Thus, if a pseudo-flow is added to a redundant branch, there will be an increase in the redundancy only of the redundant measurements incident to this branch or of the redundant measurements incident to branches that form a fundamental loop L with the redundant branch where the pseudo-flow was added.

Through the definition of critical measurement (Definition 4), it can be verified that critical measurements are incident only to branches that can be neither associated to more than one measurement, nor put into a fundamental loop L. Then the addition of a pseudo-flow in a redundant branch does not change the criticality of a critical measurement. ■

Lemma 7: If a flow measurement is incident to nodes that are incident to a unique redundant injection measurement, then the flow measurement is redundant (see Fig. 5).

Proof: First, note that the set of branches incident to the redundant injection measurement plus the set of nodes incident to these branches constitute the more elementary redundant branch set which can be formed with only redundant injection measurements.

Consider now the flow measurement F and suppose its associated measured branch belongs to $G^l(Y)$. Then this branch can be assigned to another measurement, as it was proved in Lemma 1.

Now suppose the measured branch associated to this flow does not belong to $G^l(Y)$. For example, imagine that the flow occurs at the branch $(J_l - J_k)$, as shown in Fig. 5. It is necessary to prove that there exists an alternative path to connect J_l to J_k without using the flow measurement in consideration.

As proved in Lemma 6, the addition of a pseudo-flow to a redundant branch changes the redundancy of only measurements already redundant. Then, as the branches $(J_l - I_i)$ and

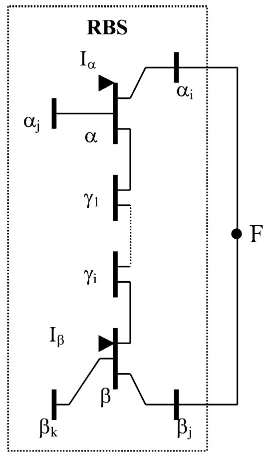


Fig. 6. Flow measurement incident to nodes which are not incident to a unique redundant injection measurement.

$(I_i - J_k)$ are redundant, one pseudo-flow can be added to each one of these branches to verify the criticality of the flow measurement F.

Considering these two pseudo-flows, J_l can be connected to J_k through the redundant branches $(J_l - I_i)$ and $(I_i - J_k)$, without using flow F. Then the flow measurement F is redundant and Lemma 7 is proved. ■

Lemma 8: If a flow measurement is incident to nodes contained in the same RBS, then this flow measurement is redundant.

Proof: Consider flow F, shown in Fig. 6, that connects node α_i to node β_j, which are contained in the same RBS. Note that the branch $(\alpha_i - \beta_j)$ in consideration does not belong to this RBS and these nodes are not incident to the same injection measurement, which was already proved in Lemma 7.

To prove that F is redundant, it is necessary to demonstrate that there exists at least one path to connect α_i to β_j without using flow F.

As nodes α_i and β_j are in the same RBS, there exists a path formed only with redundant branches that connects α_i to β_j. This path can be described by nodes $\alpha_i, \alpha, \gamma_1, \ldots, \gamma_i, \beta$, and β_j.

Through Lemma 6, pseudo-flows can be added to redundant branches without affecting the redundancy of the critical measurements. Therefore, adding pseudo-flows to every redundant branch of path $\alpha_i, \alpha, \gamma_1, \ldots, \gamma_i, \beta, \beta_j$, flow F can be assigned to a branch in a fundamental loop L. Then this measurement is redundant and Lemma 8 is proved. ■

Lemma 9: Let Y_i and Y_j be two RBSs of an observable measured network X. If there are two flow measurements, F_1 and F_2, incident to branches that connect a node of $G^0(Y_i)$ to a node of $G^0(Y_j)$, as shown in Fig. 7, these measurements are redundant and the two RBSs can be coalesced into a unique RBS represented by

$$\begin{array}{l} G^0(Y_i) \cup G^0(Y_j) \\ G^l(Y_i) \cup G^l(Y_j) \cup b_1 \cup b_2 \\ M(Y_i) \cup M(Y_j) \cup F_1 \cup F_2 \end{array}$$

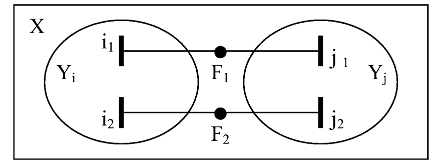


Fig. 7. Two RBSs connected by two flows measured in two different branches.

where b_1 and b_2 are the branches that connect buses i_1 to j_1 and i_2 to j_2, respectively.

Proof: To prove this Lemma it is necessary to prove that measurements F_1 and F_2 are redundant. First, measurement F_1 will be analyzed.

To prove that F_1 is redundant, it is necessary to demonstrate that there exists at least one path to connect i_1 to j_1 without using measurement F_1. As Y_i is an RBS, there exists at least one path which connects nodes i_1 to i_2 without using flow F_1. In the same way, as Y_j is an RBS, there exists at least one path to connect nodes j_1 to j_2 without using flow F_1.

Therefore, through measurement F_2, which connects node i_2 of $G^0(Y_i)$ to node j_2 of $G^0(Y_j)$, it is possible to obtain a path to connect node i_1 of $G^0(Y_i)$ to node j_1 of $G^0(Y_j)$, without using measurement F_1. Thus this measurement is redundant.

Following the same steps, it is possible to prove that F_2 is also redundant.

As branches b_1 and b_2 are in a fundamental loop L, they are redundant branches and the RBSs Y_i and Y_j can be coalesced into a unique RBS. ■

Lemma 10: Let $Y_1, \ldots, Y_r$ be r RBSs of an observable measured network X. If there are r flow measurements, $F_1, \ldots, F_r$, which are incident to r branches that connect the r RBSs into a fundamental loop L, then these r flow measurements are redundant and the r RBSs can be coalesced into a unique larger RBS represented by

$$\begin{array}{l} \cup_{k=1}^{r} G^0(Y_k) \\ \cup_{k=1}^{r} G^l(Y_k) \cup B \\ \cup_{k=1}^{r} M(Y_k) \cup_{k=1}^{r} F_k \end{array}$$

where B is the set of all branches which connect Y_i to $Y_j, i, j = 1, \ldots, r$, and F_ks are the flow measurements incident to r branches that connect the r RBSs into a fundamental loop L.

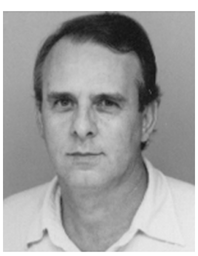
Proof: As the RBSs are in a fundamental loop L, the r branches and the r flow measurements, used to connect these RBSs are redundant branches and redundant measurements, respectively. Thus, the r RBSs can be coalesced in a unique RBS. ■

REFERENCES

[1] K. A. Clements, G. R. Krumpholz, and P. W. Davis, "Power system state estimation residual analysis: An algorithm using network topology," *IEEE Trans. Power App. Syst.*, vol. 100, no. 4, pp. 1779–1787, Apr. 1981.

[2] A. Simões Costa, T. S. Piazza, and A. Mandel, "Qualitative methods to solve qualitative problems in power-system state estimation," *IEEE Trans. Power Syst.*, vol. 5, no. 3, pp. 941–949, Aug. 1990.

[3] E. D. Crainic, H. P. Horisberger, X. D. Do, and D. Mukhedkar, "Power network observability: The assessment of the measurement system strength," *IEEE Trans. Power Syst.*, vol. 5, no. 4, pp. 1267–1285, Nov. 1990.
[4] N. G. Bretas and J. B. A. London Jr., "Network observability: The critical measurement identification by the symbolic Jacobian Matrix," in *Proc. Int. Conf. Power Syst. Technol.*, vol. 2, Beijing, China, Aug. 1998, pp. 1222–1226.
[5] G. N. Korres and G. C. Contaxis, "A reduced model for bad data processing in state estimation," *IEEE Trans. Power Syst.*, vol. 6, no. 2, pp. 550–557, May 1991.
[6] M. B. Coutto Filho *et al.*, "Identifying critical measurements and sets for power-system state estimation," in *Proc. IEEE Porto Power Tech. Conf.*, Porto, Portugal, Sep. 2001, CD: paper no. PP2001_147.
[7] G. N. Korres and G. C. Contaxis, "Identification and updating of minimally dependent sets of measurements in state estimation," *IEEE Trans. Power Syst.*, vol. 6, no. 3, pp. 999–1005, Aug. 1991.
[8] G. C. Contaxis and G. N. Korres, "A reduced model for power-system observability analysis and restoration," *IEEE Trans. Power Syst.*, vol. 3, no. 4, pp. 1411–1417, Nov. 1988.
[9] J. B. A. London Jr., G. L. R. Brito, and N. G. Bretas, "Method for meter and RTU placement for state estimation purposes," in *Proc. IEEE Bologna Power Tech. Conf.*, Bologna, Italy, Jun. 2003, CD: paper no. 329.
[10] H. Mori and S. Tsuzuki, "A fast method for topological observability analysis using a minimum spanning tree technique," *IEEE Trans. Power Syst.*, vol. 6, no. 2, pp. 491–499, May 1991.
[11] N. G. Bretas, "Network observability: Theory and algorithms based on triangular factorization and path graph concepts," in *Proc. IEE Gener., Transm., Distrib.*, vol. 143, 1996, pp. 123–128.



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