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ORIGINAL PAPER

Reliable and straightforward PID tuning rules for highly underdamped systems

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Abstract
Proportional-Integral-Derivative (PID) controllers reign absolute when automatic control is applied. There is an expressive number of tuning rules for these controllers in literature. However, for highly oscillatory (or highly underdamped) systems, such as the ones found in oil production and polymerization reactors, the available methods provide poor closed-loop performance and robustness. Besides, most of these tuning rules are developed for systems based on a first-order with pure time delay (FOPTD) transfer function and for parallel form PID controllers. Therefore, the focus of this paper is the development of appropriate tuning rules for highly underdamped systems through non-cancellation of dominant poles and easily adjustable robust performance, making them applicable for both series and parallel PID controllers, since the proposed tuning only places the controller zeros at the real axis. The new tuning rules were developed for these systems and were tested on 15,000 different transfer functions described by a second-order with pure time delay (SOPTD) expression. Additionally, a recommendation interval is also provided in which the controller gain can be varied online or by simulations to achieve the desired trade-off between performance and robustness. The proposed rules are also validated using two case studies: the suppression of slugging in oil production and the temperature control of an industrial gas phase polyethylene reactor.

Keywords Tuning rules · PID series · Highly underdamped systems · Slugging control · Polyethylene reactor control

Introduction

The application of automatic control in process industries is justified for many reasons, such as: reducing the product variability, increasing the process and energy efficiencies, as well as ensuring a safe operation (Isermann 2011). A controller that is widely used in industries due to its simple structure, operating easiness and acceptable robustness and performance is the Proportional-Integral-Derivative (PID) controller (Liu and Daley 2001; Shardt et al. 2012).

The PID controller was born in 1939, developed by the Taylor Instrument Companies (Bennett 2001). The first implementation of this controller algorithm, adopted in many digital controllers, was the series (or interacting) form (Bennett 2001). Later, its implementation in parallel (or non-interacting) form arose as well. In both forms, this controller has three basic tuning parameters or constants related to the proportional, integral, and derivative actions (Seborg et al. 2011). The main difference between the two parametrizations is that, in the series form, the derivative constant influences the integral constant, which makes manual tuning easier (Åström and Hägglund 1995).

The first systematic methodology for adjusting PID controllers emerged only in 1942, proposed by Ziegler and Nichols (1942), based on the knowledge of two characteristic process parameters, namely, the ultimate gain and period. From 1942 to the present day, a large number of tuning rules for PID controllers have been proposed for different system types. These tuning rules can be classified into methods based on: (i) a first-order with pure time delay (FOPTD) transfer function; (ii) a second-order with pure time delay (SOPTD) transfer function; (iii) the ultimate gain and the

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ultimate period; among others (O'Dwyer 2009). As it is shown in O'Dwyer (2009), most tuning rules are based on FOPTD, and they are developed for the parallel form of PID controllers only.

Motivated by the rich literature available regarding tuning rules of PID controllers, as well as the broad application of this controller in the industrial scenario, we were interested in applying this controller to solve the problem of operational instability associated with the multiphase flow known as slugging, and also to increase oil well production. This problem has great industrial interest. It is characterized by a limit cycle behavior that causes a wide pressure variation, which can lead to operational risks to the systems such as fatigue of equipment and, consequently, an increase in production costs. Moreover, this pressure variation decreases the average oil production and the remaining useful life of equipment (Godhavn et al. 2005; Di Meglio et al. 2012).

It is worth mentioning that some papers have studied the use of PID controllers for slugging suppression, such as those reported by Jahanshahi and Skogestad (2013), Jahanshahi et al. (2014), and Diehl et al. (2019). However, evaluating these papers, a common feature in all of them, concerning the adjustments of the PID controller, is that they were performed at operation points that present little or no oscillations, differently from what we typically find in these systems due to the slugging, which is a highly oscillatory behavior. Thus, a question arises: is it not possible to achieve better results, such as increased oil production, with an appropriate tuning for highly oscillatory systems?

Before diving into this question, it is important to quantify how much a small increase in oil production can mean financially. Consider the average oil production from the well, named 4BRSA-711-RJS, in May, 2020, whose production values were 17,158 bbl/d (ANP 2020), and also consider the oil barrel value equal to US$ 40. A small 10% increase in production of this well would represent a profit increase of US$ 2,058,960 per month and US$ 24,707,520 per year, for just one oil well.

We have tested many tuning rules available in literature for the PID controller to obtain better results through the appropriate tuning for this controller. After extensive research in the literature, it was noted that for highly oscillatory systems, i.e., systems with a tiny damping factor ($\zeta \in [0, 0.1]$), such as the ones found in oil production, few of them could stabilize these systems, such as the rules proposed by Rivera et al. (1986), Lee et al. (1998), Marchetti and Scali (2000), and Grimholt and Skogestad (2018). Nevertheless, all these rules exhibit poor performance.

This poor performance obtained by the tuning rules mentioned above occurs because these methods have the premise of plant inversion and desirable performance as a FOPTD, leading to the cancellation of the dominant poles. There is nothing wrong with this premise since the dominant poles are not near the origin of the imaginary axis, which is the case for highly oscillatory systems (or highly underdamped systems). Seborg et al. (2011) evaluated the effect of this premise for the FOPTD with the pole near the origin, i.e., an integrator system. This study showed that this approach does not provide suitable tuning, being incredibly slow for disturbance rejection.

Highly oscillatory systems are also present in other industrial scenarios in addition to oil production, such as polymerization reactors (Salau et al. 2009; Isakova and Novakovic 2017) and ethanol production (Trierweiler and Diehl 2009), which corroborates the need and importance of obtaining appropriate tuning rules for PID controllers applied to this class of systems.

Starting from such motivations, the main contribution of this paper is not merely the development of an additional method for PID adjustment, but the development of appropriate tuning rules for highly underdamped systems with non-cancellation of dominant poles and easily adjustable robust performance, which makes it possible to apply them for both series and parallel PID controllers, since the proposed tuning only places the controller zeros at the real axis.

This paper is structured as follows: "Methodology—development of tuning rules" describes how the new tuning rules of PID controllers for highly underdamped systems were developed. "Quality assessment of the proposed simple PID tuning rules for highly underdamped systems" shows the quality assessment of the proposed tuning rules. "Slugging control—a practical case study" and "An industrial gas-phase polyethylene reactor—another practical case study" show the potential application of the new tuning rules for a well of an offshore oil production system and in an industrial gas-phase polyethylene reactor, respectively. "Root Locus comparison of the proposed tuning rules with IMC tuning rules" shows the root locus corresponding to the proposed tuning and the Internal Model Control (IMC) tuning rules for the same industrial gas-phase polyethylene reactor. Finally, "Conclusions" discusses the main conclusions.

Methodology—development of tuning rules

The essence of the tuning rules presented here for PID controllers is based on the optimal solution of selected optimization problems, which are divided into four steps.

The first step is to obtain optimal parameters of the PID controller for a group of highly underdamped plants, considering performance and robustness metrics. The second step is to obtain simple equations that relate the optimal parameters obtained in the first step to parameters of a SOPTD transfer function, being subjected to a fit criterion.

In the third step, the equations obtained in the previous step for the parameters τ_I and τ_D of the PID controllers are used to tune them, and this adjustment is used in another optimization problem in order to obtain the optimal solution for the parameter K_p, with another group of highly underdamped plants, considering the same metrics as the first stage. Finally, simple equations that relate the optimized parameter K_p of the PID controllers obtained in the third step with SOPTD parameters are developed, using the same criterion as in the second step.

Thereby, the details of each step performed for the development of the simple PID tuning rules for a highly underdamped system are presented here. The criteria for selecting the plants and these tuning rules are described below.

Problem formulation

In the first step, the Integrated Squared Error (ISE) was chosen as the performance metric, i.e., the cost function of the optimization problem. The ISE is given by

$$ISE = \int_0^\infty \left[y_{set}(t) - y(t)\right]^2 dt \tag{1}$$

where $y_{set}(t)$ is the setpoint, and $y(t)$ is the actual value of the controlled variable in the time domain (or t). The ISE is a criterion that penalizes larger values of the difference between the setpoint and the actual value (Marlin 2000). Thus, it is a useful metric for obtaining tuning of controllers that are better at rejecting process disturbances concerning setpoint changes.

The ISE was calculated for a closed-loop system, consisting of a controller and a plant, as shown in Fig. 1. This closed-loop was first subjected to a change in the setpoint, and later, after the system reached a new steady-state, a load disturbance, both with magnitude equal to 1.

In Fig. 1, $C(s)$ and $G(s)$ are the controller and plant transfer functions, respectively, $Y_{set}(s)$ is the setpoint, $Y(s)$ is the actual value of the controlled variable, and d is the load disturbance.

The plant transfer function used here to represent highly underdamped systems is a SOPTD given by

$$G(s) = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1} exp(-\theta s) \tag{2}$$

where K is the gain, τ is the time constant, ζ is the damping coefficient, and θ is the time delay of the plant. It is relevant

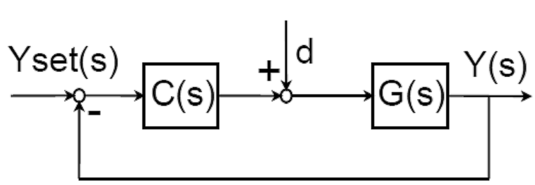

Fig. 1 Block diagram of the closed-loop system

to note that highly oscillatory SOPTD are the transfer functions with ζ values between 0 and 0.1.

The controller transfer function used here is of a PID controller in its parallel form with a first-order filter in the derivative action, which is given by

$$C(s) = K_p \left[\left(Y_{set}(s) - Y(s)\right) + \frac{\left(Y_{set}(s) - Y(s)\right)}{\tau_I s} - \frac{Y(s)\tau_D s}{\frac{\tau_D}{N}s + 1} \right] \tag{3}$$

where K_p is the controller gain, τ_I is the integral time, τ_D is the derivative time, N is the first-order filter proportion factor of the derivative time constant, $Y_{set}(s)$ is the setpoint, and $Y(s)$ is the actual value of the controlled variable in the Laplace domain (or s). In this paper, the parameter N is set to 10 so the derivative action can be of a decade in the frequency domain. Although the purpose of this tuning is also to apply it to the series form of the PID controller, the parallel form of the controller was used to develop the new tuning rules because this approach also encompasses the series form.

The robustness metric used in the first step as the constraint was the sensitivity peak or maximum sensitivity, M_S, limited to 2. According to Rivera et al. (1986), this feature guarantees at least 1.5 of gain margin and 29 degrees of phase reserve, approximately.

The sensitivity transfer function is given by

$$S(s) = \frac{1}{1 + G(s)C(s)} \tag{4}$$

where $C(s)$ and $G(s)$ are the controller and plant transfer functions, respectively.

Thus, with all the main elements defined, the formal formulation of the optimization problem for the first objective is given by

$$\begin{aligned} K_p, \tau_I, \tau_D &= \underset{K_p, \tau_I, \tau_D}{arg\ min}\ (ISE) \\ &sj.\ to\ M_S \leq 2 \end{aligned} \tag{5}$$

where the parameters K_p, τ_I and τ_D are the PID controller parameters and decision variables of the optimization problem, ISE is the cost function, and M_S is the constraint.

In a preliminary analysis, we observed that the optimization problem described in Eq. 5 for the class of highly underdamped systems does not have an easy solution, since it has multiple local minima. Therefore, this problem requires global optimization methods. The drawback is that global optimization methods usually require a considerable computational effort, which could be translated into many hours or days to obtain the optimal solution.

Regarding these observations, the Improved Stochastic Ranking Evolution Strategy (ISRES), a global optimization method that is available in the NLopt nonlinear-optimization package (Johnson 2019) for Python 2.7, was used for the solution of the optimization problems. The evolution strategy of the ISRES method is based on the combination of a mutation rule and a differential variation (Johnson 2019). In these problems, a population of 80 individuals was used.

The optimal PID parameters that were obtained in this first stage, from the optimum tunings of the PID controllers for a set of highly underdamped plants, were dimensionless. A non-linear equation was fitted for each one and will be presented as the result of these adjustments in "Simple PID tuning rules for highly underdamped systems". The Mean Squared Error (MSE) was used as the fit criterion in this step.

In the third step, an optimization problem similar to that presented in Eq. 5 was set up. This problem is given by

$$\begin{aligned} K_p &= \underset{K_p}{arg\ min}\ (ISE) \\ &sj.\ to\ M_S = 2 \end{aligned} \tag{6}$$

where K_p is the PID controller parameter and decision variable, ISE is the cost function, and M_S is the constraint.

The main objective of this optimization problem is to obtain optimal K_p values for another group of highly underdamped systems, since this parameter is the main performance button of a closed-loop. The difference here concerning the previous problem (i.e., Eq. 5) is that the parameters τ_I and τ_D are tuned from the equations obtained in the second stage, and the parameter K_p is the only decision variable.

In the fourth step, the optimal values of the parameter K_p obtained in the previous step were normalized by the parameter K_p values obtained from the rule developed in the second step for the respective evaluated systems. Then, the equations were adjusted to these normalized values. These equations will also be presented as the result of these adjustments in "Simple PID tuning rules for highly underdamped systems". They are named as second gain of the simple PID tuning rules, as presented in this paper. The MSE was used as the fit criterion in this step, as it was in the second stage.

Thus, there are two expressions to calculate the parameter K_p of the controller for the same plant. These values define the limits of the range of recommended values, allowing the user to find a good compromise between performance and robustness quickly just by selecting the most appropriate value of K_p in the calculated interval, as it will be seen later.

Plant selection criteria

As discussed so far, the new tuning method presented here aims to provide adequate tunings to PID controllers for highly underdamped plants. Thus, the first plant selection criterion is the use of SOPTD transfer functions with $\zeta \in [0, 0.1]$ for the development of these new tuning rules, as already mentioned. Another very relevant issue is associated with the non-minimum phase factor, which for a SOPTD is related to the pure time delay, and, therefore, is another criterion taken into consideration for the selection of plants. This is an important criterion because the pure time delay is a limiting factor for the achievable performance in a closed-loop.

Based on these premises, the new tuning rules proposed here were developed using 104 SOPTD transfer functions. The plant gains were set equal to 1, the damping coefficients ranged from 0.025 to 0.1, and the ratios between the time constant and the time delay ranged from 1 to 50,000. These plants can be found in Tables 1 and 2 of the supplementary material of this paper.

This set of SOPTD was divided into two groups: the first group (70 transfer functions) contemplates the systems whose dominant dynamics and pure time delay ratio is not high, i.e., $0 < \tau/\theta \leq 10$, and the second group (44 transfer functions) mainly consists of systems whose pure time delay is negligible in comparison to the dominant dynamics, i.e., $\tau/\theta > 10$. Thus, the first and second groups were used in the development of the first and third stages described in the previous section, respectively.

Simple PID tuning rules for highly underdamped systems

The simple PID tuning rules presented here were obtained as results of the second and fourth stages described in "Problem formulation". As a result of the second stage, the tuning rules for PID controllers in the parallel form are presented in Table 1, and their respective conversions for PID controllers in the series form are shown in Table 2.

The expressions that determine the second K_p value obtained as a result of the fourth step are shown in Table 3.

In Tables 1, 2, and 3, K is the gain, τ is the time constant, ζ is the damping coefficient, and θ is the time delay of a SOPTD.

It is emphasized that to arrive at the expressions presented in Tables 1 and 3, several equations such as those available in McConville (2008) and their combinations were tested. These expressions were selected based on the smallest MSE. Besides, the *min* function was incorporated

Table 1 Simple tuning rules of parallel PID controllers for highly underdamped systems

Parallel PID controller's parameters	Equation
$K_{p,1st}$	$\frac{1}{K}\exp\left[9.1\zeta + (2.2 - 2.7\zeta)\ln(\min(\tau/\theta, 10)) - 3.4\right]$
τ_I	$\tau\left[(0.2 + 0.5\zeta)\min(\tau/\theta, 10) + \exp(-33\zeta) + 0.2\right]$
τ_D	$\tau\exp\left[1.3 - 0.2\min(\tau/\theta, 10) - 2.9\zeta\right]$

Table 2 Simple tuning rules of series PID controllers for highly underdamped systems

Series PID controller's parameters	Equation
$K^{\#}_{p,1st}$	$\frac{\exp[9.1\zeta+(2.2-2.7\zeta)\ln(\min(\tau/\theta,10))-3.4]}{2K}\left[1+\sqrt{1-4\frac{\exp[1.3-0.2\min(\tau/\theta,10)-2.9\zeta]}{(0.2+0.5\zeta)\min(\tau/\theta,10)+\exp(-33\zeta)+0.2}}\right]$
$\tau^{\#}_{I}$	$\frac{\tau[(0.2+0.5\zeta)\min(\tau/\theta,10)+\exp(-33\zeta)+0.2]}{2}\left[1+\sqrt{1-4\frac{\exp[1.3-0.2\min(\tau/\theta,10)-2.9\zeta]}{(0.2+0.5\zeta)\min(\tau/\theta,10)+\exp(-33\zeta)+0.2}}\right]$
$\tau^{\#}_{D}$	$\frac{\tau[(0.2+0.5\zeta)\min(\tau/\theta,10)+\exp(-33\zeta)+0.2]}{2}\left[1-\sqrt{1-4\frac{\exp[1.3-0.2\min(\tau/\theta,10)-2.9\zeta]}{(0.2+0.5\zeta)\min(\tau/\theta,10)+\exp(-33\zeta)+0.2}}\right]$

Table 3 Second gain relations for series and parallel PID controllers

Ratio	Equation
$\frac{\tau}{\theta}\leq 3$	$K^{\#}_{p,2nd}\left(or\ K_{p,2nd}\right)=\left[1.3-0.2\frac{\tau}{\theta}\right]K^{\#}_{p,1st}\left(or\ K_{p,1st}\right)$
$3<\frac{\tau}{\theta}\leq 10$	$K^{\#}_{p,2nd}\left(or\ K_{p,2nd}\right)=\left\{1-2.3\exp\left[-0.6\frac{\tau}{\theta}\right]\right\}K^{\#}_{p,1st}\left(or\ K_{p,1st}\right)$
$10<\frac{\tau}{\theta}\leq 500$	$K^{\#}_{p,2nd}\left(or\ K_{p,2nd}\right)=\left\{14.7-14.5\exp\left[-0.009\frac{\tau}{\theta}\right]\right\}K^{\#}_{p,1st}\left(or\ K_{p,1st}\right)$
$500<\frac{\tau}{\theta}\leq 1000$	$K^{\#}_{p,2nd}\left(or\ K_{p,2nd}\right)=\left\{\exp\left[\frac{3.5\frac{\tau}{\theta}}{135.3+\frac{\tau}{\theta}}\right]\right\}K^{\#}_{p,1st}\left(or\ K_{p,1st}\right)$
$\frac{\tau}{\theta}>1000$	$K^{\#}_{p,2nd}\left(or\ K_{p,2nd}\right)=25.3K^{\#}_{p,1st}\left(or\ K_{p,1st}\right)$

into the equations with the ratio τ/θ limit equal to 10 (Tables 1 and 2), since, in this stage, a group of the highly underdamped plants used, whose ratios higher than 10 tended to infinity, resulted in high values of K_p and τ_I, and small values of τ_D, which produces an inappropriate control action. Therefore, to avoid this problem, the maximal τ/θ ratio was saturated in 10.

It is noteworthy that the use of this mathematical artifice does not reduce the ability to generalize the simple tuning rules proposed here, as will be seen in the next section of this paper. On the contrary, as previously described, there is a second equation for the parameter K_p (Table 3), which provides a second value for it and thus defines a range of values for this parameter that transitions between a more robust tuning and one with a better performance.

The main advantage of obtaining tuning rules as simple expressions is linked to the computational time associated with getting the optimal parameters of the controller. In the development of the tuning rules proposed here, for example, the computational time of hours or days was necessary in several cases. Now with these tuning rules, it is not necessary to go through this process again to obtain a suitable tuning for the PID controllers.

In this paper, it is worth mentioning that different computational tools were used, namely: Python 2.7, Python 3.7, Statistica 8.0, and Matlab 2012b, as well as different packages for Python, namely: NumPy (NumPy Developers 2019), Python Control Systems (Python-control.org 2018), NLopt (Johnson 2019), Matplotlib (Hunter et al. 2018), PyFMI (Python Software Foundation 2019), and SciPy (Jones et al. 2001).

Quality assessment of the proposed simple PID tuning rules for highly underdamped systems

The quality assessment terms in this dedicated section are related to the adjustments made in the previous section and the generalizability of these rules.

Quality assessment of the fit obtained

In order to assess the quality of the adjustment presented in Tables 1, 2, and 3, the Pearson's correlation coefficient between the optimal values of the optimal tunings and the values obtained by the proposed tuning rules in this paper (predicted values) was calculated. This coefficient is a measure of how strong the linear relationship between two variables is (Hauke and Kossowski 2011), where this value is ranged from − 1 to 1. The closer to the extremes this coefficient is, it indicates that the linear relationship is strong between the two variables.

The values of Pearson's correlation coefficients obtained for the first K_p ($K_{p,1st}$), the parameters τ_I and τ_D, and the second K_p ($K_{p,2nd}$) are shown in Table 4. In addition, the values of the optimal tuning parameters of the PID controller and those calculated by the simple tuning rules presented in Tables 1 and 3 for the 104 plants that were used for their development are available in Tables 1 and 2 of the supplementary material of this paper.

Table 4 indicates that there is a strong linear correlation between the optimal values and the values obtained from the simple tuning rules proposed here, which indicates an excellent quality for these rules. However, this is a result regarding only the 104 plants that were used to develop the tuning method presented here. A more interesting assessment is to evaluate whether the proposed tuning rules can be used on any highly underdamped systems with guaranteed performance and robustness, i.e., whether the method can be generalized for this system class. This analysis will be seen below.

Evaluation of the generalization of the simple tuning rules for highly underdamped systems

The evaluation of the generalization of the tuning rules presented here was performed based on the test of servo and regulatory control, i.e., a change in the closed-loop setpoint and an application of additive disturbance at the plant input also in closed-loop after the system reaches the new setpoint, respectively. The maximum sensitivity was also evaluated as a result of the validation for the two K_p parameters that the proposed method provides.

Thus, 15,000 different and highly underdamped plants represented by SOPTD transfer functions were tested. These plants were generated following a uniform distribution, where τ and the ratio τ/θ were determined in the range of 1–1000 and of 1–10,000, respectively. The plant gain, K, was always equal to 1.

The set of plants was equally divided into three subsets in terms of the damping coefficient, this being the least to the most highly underdamped system, namely here: Group A, Group B, and Group C, as shown in Table 5. The main reason for this subdivision is to specifically evaluate also how the tuning rules proposed here will behave in terms of performance and robustness, highlighting that all tuning rules available in the literature have poor performance already in Group A.

Table 4 Pearson's correlation coefficient, ρ, between the values of the optimal tuning and the values obtained by the proposed tuning rules

PID controller's parameters	ρ
$K_{p,1st}$	0.9992
τ_I	0.9977
τ_D	0.9934
$K_{p,2nd}$	0.9961

Table 5 Classification of the set of plants used for validation in terms of the damping coefficient

Group	Damping coefficient (ζ)
Group A	0.01–0.1
Group B	0.001–0.01
Group C	0–0.001

The tuning rules proposed here were able to provide appropriate tuning for these 15,000 plants so that the servo and regulatory control objectives were met. Also, the average closed-loop maximal sensitivity values obtained for both versions of the simple tuning rules, i.e., with first and second K_p, for each Group, are shown in Table 6. The individual values of the parameters of these plants, the performance metrics, and the closed-loop maximal sensitivity are available in Tables 3, 4, and 5 of the supplementary material of this paper.

Table 6 shows that the average closed-loop maximal sensitivity presented by the proposed tuning with $K_{p,1st}$, and with $K_{p,2nd}$, provides, respectively, a robust tuning and a performance tuning for all Groups. Moreover, the standard deviations around the averages were small, which indicates that the samples are well distributed around the means.

From the results achieved in this section, it can be concluded that the simple PID tuning rules proposed in this paper can be generalized for the entire class of highly underdamped systems. Besides that, it is proved that the rules provide a reliable tuning with a recommended interval for the controller gain, for which the desired trade-off between performance and robustness can be achieved.

From now on in this paper, the tuning with the smallest K_p of the recommended tuning interval will be called the proposed robust tuning version (or proposed tuning—robust version), and the tuning with the highest K_p of the recommended tuning interval will be called the proposed

Table 6 The average closed-loop maximal sensitivity values obtained for two versions of the simple tuning rules

Group	Closed-loop maximal sensitivity (M_S)	
	Proposed tuning with $K_{p,1st}$	Proposed tuning with $K_{p,2nd}$
Group A	1.24 ± 0.03	2.09 ± 0.12
Group B	1.23 ± 0.06	2.18 ± 0.13
Group C	1.23 ± 0.04	2.19 ± 0.13

performance tuning version (or proposed tuning—performance version).

Slugging control—a practical case study

The case study addressed in this paper was the slugging control in a well on an offshore oil production system. For this, the Fast Offshore Well Model (FOWM) was used, which reproduces the slugging phenomenon in gas-lifted wells. FOWM is a non-linear model, and it is the first simplified model that combines a complete production system setup, i.e., reservoir + production column + gas lift annular + flowline + riser (Diehl et al. 2017). More details of the model used in our case study are described in the paper of Diehl et al. (2017).

These oil production systems are known to have regions of stable and also unstable operation (slugging). The point where the system exhibits a change in its regions is denominated Hopf's point. Another essential characteristic of these systems is that, as the opening value of the Choke valve gets closer to Hopf's point, the systems exhibit highly oscillatory behavior.

The parameters of a real oil well, identified as "Well A" in the paper of Diehl et al. (2017), were used to simulate this case study. In these settings, the Hopf's point is located at 21% opening of the Choke valve.

Choke valves in these oil production systems are valves that are manipulated through a step actuator, which have slow dynamics, on the order of 5 up to 10 min to move from a fully closed position (0% opening) to a fully open position (100% opening) and vice versa, according to Diehl et al. (2019). Therefore, in order to get closer to a real system, a first-order dynamic was considered for the manipulated variable (MV) in our simulations, which had a rate of 0.24%.s^{-1}, i.e., the MV takes 7 min to move from a fully closed position to a fully open position, approximately.

For the slugging control, a control-loop was used with the opening of the Choke valve as the manipulated variable and the pressure in the Permanent Downhole Gauge (PDG) as the controlled variable (CV). In this loop, the PID controller (Eq. 3) was used. The parameter N is equal to 10, as in "Methodology—development of tuning rules".

In order to tune the other parameters (K_p, τ_I and τ_D) of this controller, the proposed rules and the Internal Model Control (IMC) approach for a SOPTD (Rivera et al. 1986) were used. It is worth mentioning that the IMC tuning has an adjustable parameter. Thus, to make a fair comparison between the tunings mentioned above, this parameter was determined optimally, similar to what was done for the development of the proposed tuning described in Eq. 6.

The goal of this case study was to verify the capacity of the tuning rules presented in this paper to stabilize an oil production system by closing the control-loop at an unstable operating point, i.e., slugging control. In addition, a compromise between servo and regulatory control objective, i.e., changes in the closed-loop setpoint and an application of additive disturbance at the gas-lift flowrate, also in closed-loop after the system reaches the new setpoint, was evaluated after the respective stabilization.

In the analysis of servo control, the maximum achievable choke opening capacity was examined, without destabilizing the system, which means a larger oil production. In the analysis of regulatory control, the ability to reject disturbances was evaluated by adding disturbances in the gas-lift flowrate of − 10%, + 20%, and − 10% concerning the initial gas-lift value of the system (1.43 kg s^{-1}).

The metrics used to evaluate the performance comparison were the ISE (Eq. 1) and the Integrated Absolute Error (IAE) criteria, which is given by

$$IAE = \int_0^{\infty} \left| y_{set}(t) - y(t) \right| dt \tag{7}$$

where $y_{set}(t)$ is the setpoint and $y(t)$ is the present value of the controlled variable in the time domain.

The controllers are designed at three different operating points in the stable region that correspond to 10%, 16%, and 19% of the Choke valve opening. For this, SOPTD transfer functions were identified from a step test for each one of these points. These transfer functions are shown in Table 7. The identified responses and data from the nonlinear model (FOWM) in the step test are shown in Fig. 2.

From the transfer functions in Table 7, the controllers were designed using the proposed tuning (Tables 1 and 3) and the IMC tuning methods. It is noteworthy that, even though there is no pure time delay in the system response, the dynamic consideration in the manipulated variable is, in this case, viewed as a delay of the system. Thus, a delay equal to 20 s was considered for the controllers' design. A demonstration of how to obtain the adjustment of the controller using the proposed tuning is illustrated in Appendix A.

The parameter values of the PID controllers obtained from the simple tuning rules and the IMC methods used in

Table 7 Transfer functions identified for three operation points

Operation point	Choke valve opening	Transfer function
1	10%	$G_{10}(s) = \frac{-9619099.5}{383.6^2 s^2 + 2(0.23)(383.6)s + 1}$
2	16%	$G_{16}(s) = \frac{-6834359.5}{461^2 s^2 + 2(0.082)(461)s + 1}$
3	19%	$G_{19}(s) = \frac{-5692909.9}{473^2 s^2 + 2(0.012)(473)s + 1}$

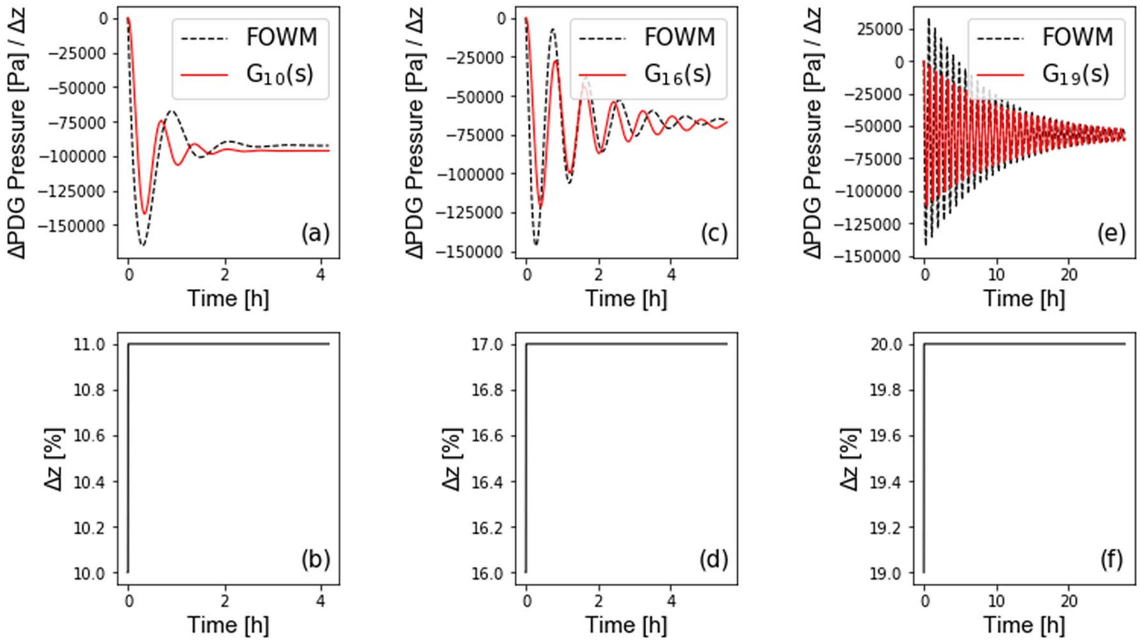

Fig. 2 Step response identified for opening choke valve (Δz) equal to 10% in subplots (**a**) and (**b**),16% subplots (**c**) and (**d**), and 19% subplots (**e**) and (**f**)

Table 8 Performance comparison based on IAE and ISE, and PID controller parameters obtained by the proposed tuning rules and IMC for the slugging control

Operating point	Tuning rule	IAE		ISE		PID controller's parameters			Maximum sensitivity
		Servo	Regulatory	Servo	Regulatory	K_p [Pa^{-1}]	τ_I [s]	τ_D [s]	
$G_{10\%}(s)$	Proposed tuning—robust version	2.36	2.85	2.47	7.09	$-1.06 \cdot 10^{-6}$	1285.8	97.7	1.69
	Proposed tuning—performance version	1.74	1.18	1.82	1.37	$-2.67 \cdot 10^{-6}$	1285.8	97.7	2.02
	IMC	4.14	2.51	4.56	8.92	$-2.16 \cdot 10^{-7}$	176.7	833	2.00
$G_{16\%}(s)$	Proposed tuning—robust version	1.97	2.85	2.07	7.09	$-9.81 \cdot 10^{-7}$	1234.3	180.4	1.49
	Proposed tuning—performance version	1.38	1 $(3.73 \cdot 10^{7})$[a]	1.45	1 $(1.38 \cdot 10^{11})$[a]	$-2.86 \cdot 10^{-6}$	1234.3	180.4	2.00
	IMC	Unstable	Unstable	Unstable	Unstable	$-1.00 \cdot 10^{-8}$	75.8	2904.1	2.00
$G_{19\%}(s)$	Proposed tuning—robust version	1.84	3.15	1.88	8.15	$-9.62 \cdot 10^{-7}$	1387.9	226.9	1.45
	Proposed tuning—performance version	1 $(1.80 \cdot 10^{10})$[a]	1.08	1 $(4.26 \cdot 10^{16})$[a]	1.10	$-2.87 \cdot 10^{-6}$	1387.9	226.9	2.08
	IMC	Unstable	Unstable	Unstable	Unstable	$-6.51 \cdot 10^{-11}$	11.3	19,824	1.02

[a]Reference value

the comparative analysis, the maximum sensitivity values, and the servo and regulatory performances obtained by each controller are shown in Table 8.

In Table 8, the absolute IAE and ISE values for the two control situations are presented inside the parenthesis and are used as reference values for the other tuning methods. Thus, all other IAE (and ISE) values described in this table represent how many times higher they were relative to the reference value of the respective control situation. This is also valid for Table 9.

Table 9 Performance comparison based on IAE and ISE, and PID controller parameters obtained by the proposed tuning rules and IMC for the temperature of an industrial gas phase polyethylene reactor

Tuning rule	IAE		ISE		PID controller's parameters			Maximum sensitivity
	Servo	Regulatory	Servo	Regulatory	K_p [%.°C^{-1}]	τ_I [min]	τ_D [min]	
Proposed tuning—robust version	1.2	1.5	1.17	2.2	− 4.8	22.7	3.3	1.94
Proposed tuning—performance version	1 (6.3)[a]	1 (3.2)[a]	1 (3.0)[a]	1 (2.1)[a]	− 7.4	22.7	3.3	2.21
IMC	15.8	82.2	11.9	720.7	$-3.12\cdot10^{-2}$	1.4	52	2.0

[a]Reference value

The proposed tuning rules in robust and performance versions were able to stabilize the system at the three operating points in which the controllers were designed. In contrast, the IMC tuning was only able to stabilize the system at operating point 1, in which the damping factor of the transfer function is greater than 0.1 (see Table 8). Table 8 shows that the maximum sensitivity of the tunings obtained by the proposed tuning at each operation point is consistent with a good compromise between robustness and performance, i.e., values between 1.2 and 2.2.

In the analysis of servo control, Table 8 shows that the Proposed tuning—performance version at the operation point 3 has better performance than the other tunings, since lower values of the IAE and the ISE criterion were obtained. In the analysis of regulatory control, Table 8 shows that the Proposed tuning—performance version at the operation point 2 presented better performance than the other tunings. However, it should be noted that the Proposed tuning—performance version at operation point 3 achieved similar performance.

In order to illustrate the dynamic behavior of the system responses under controller action, Fig. 3 shows the open-loop and closed-loop responses under the action of the best tuning in terms of servo performance for the proposed tuning rules in robust and performance versions and IMC tuning, the respective control actions, the gas-lift flowrate (load disturbance) and the oil production.

It can be observed in Fig. 3 that the evaluated tunings were able to stabilize the system in a limit cycle region. Another important observation is that the Proposed tuning—performance version demanded more of the manipulated variable to stabilize the system than the Proposed tuning—robust version, due to its higher K_p. However, the settling times for both tunings are similar. The IMC tuning presents a slow and more significant variation of the MV for the stabilization of the system in relation to the others, as well as a longer settling time.

It is also shown in Fig. 3 that the Proposed tuning—performance version presented the best servo performance. This tuning allowed a higher opening level for the Choke valve. That is, it reached a lower PDG pressure setpoint in relation to the other tunings, keeping the system stable. Thus, with this tuning, it was possible to increase oil production by 20% in relation to the IMC tuning (60 kg s^{-1} versus 40.9 kg s^{-1}). Recalling the example mentioned in the introduction to this paper, this increase in production may represent a gain of US$ 4,117,920 per month for this well. Besides that, the Proposed tuning—performance version also presented the best regulatory performance, since this tuning resumes the controlled variable to the respective setpoint, which allows a smaller deviation.

Based on Table 8 and Fig. 3, it is observed that better performances are achieved as the operating point used for the design of the controllers is closer to Hopf's point for the proposed tuning rules. However, the opposite behavior is observed for the IMC tuning. It is worth mentioning that other methodologies for tuning PID controllers that are capable of working with oscillatory systems, such as those proposed by Lee et al. (1998), Marchetti and Scali (2000), and Grimholt and Skogestad (2018) were also compared in Barreiros (2019) for highly underdamped systems. However, with these approaches, similar performance to the IMC was achieved and that is why only the IMC tuning was presented in this paper.

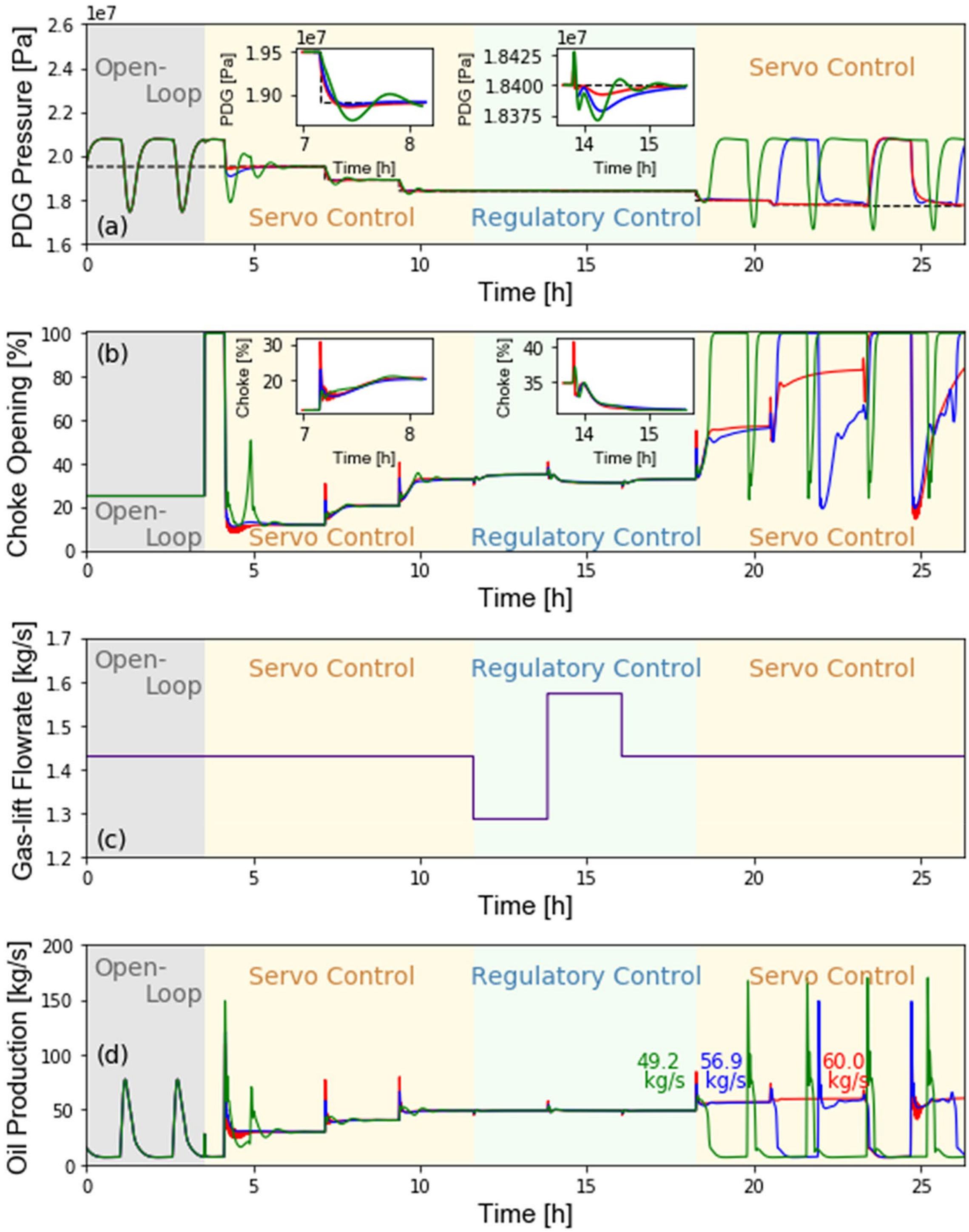

Fig. 3 Dynamics of **a** PDG Pressure, **b** Choke valve opening, **c** Gas-lift flowrate, and **d** closed-loop oil production, under servo-regulatory control analysis. (− −) Setpoint, (red dash) Proposed tuning—performance version, (blue dash) Proposed tuning—robust version and (green dash) IMC tuning

An industrial gas-phase polyethylene reactor—another practical case study

Another industrial case study that can be discussed is the temperature control of a gas-phase polyethylene reactor. For this, a transfer function identified from operational data by Salau et al. (2009) is used. This transfer function is given by

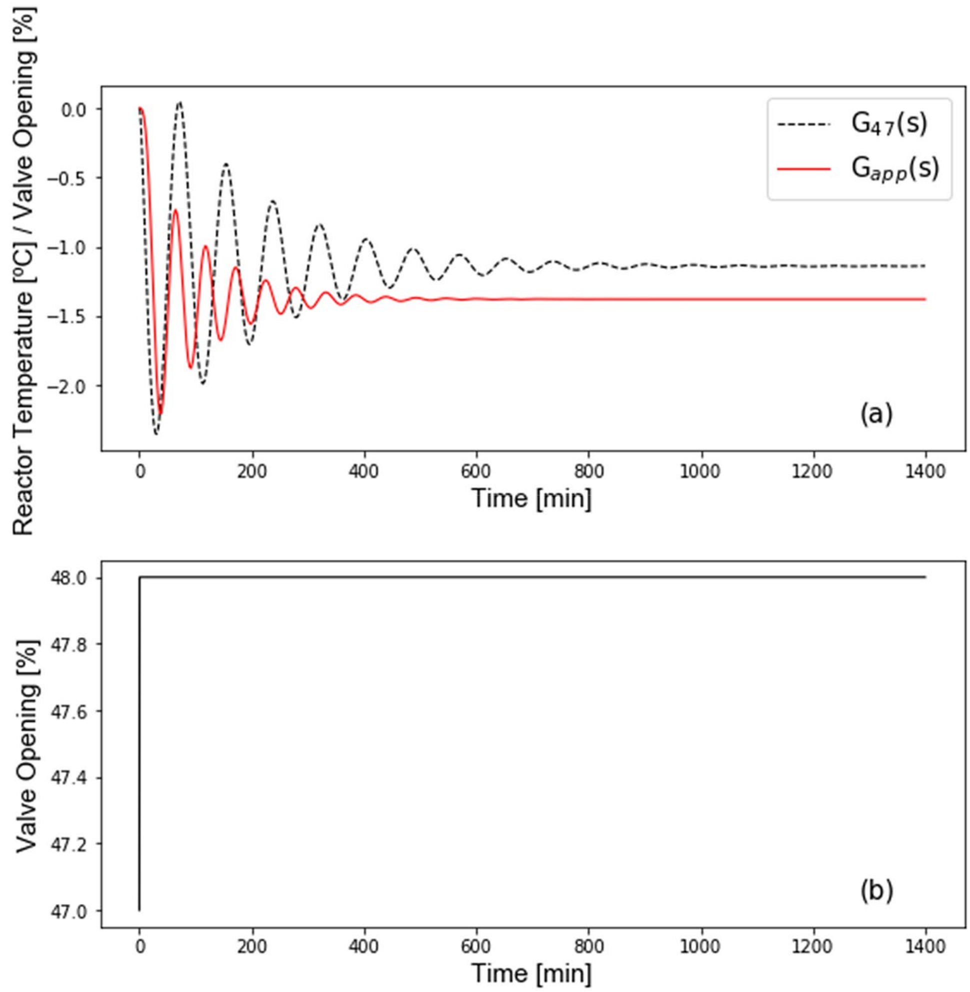

Fig. 4 Step response of the system for a model of an industrial gas-phase polyethylene reactor $G_{47}(s)$ and with the SOPTD identified $G_{app}(s)$

$$G_{47}(s) = \frac{-5.3 \cdot 10^{-2}s^4 - 9.4 \cdot 10^{-1}s^3 - 5.5s^2 - 3.4 \cdot 10^{-1}s - 5.3 \cdot 10^{-2}}{s^6 + 15.4s^5 + 102.5s^4 + 52.7s^3 + 1.9s^2 + 3 \cdot 10^{-1}s + 4.6 \cdot 10^{-3}} exp(-0.5s) \tag{8}$$

which relates the variation of the reactor temperature for a given variation of the cooling water valve position.

Since the transfer function used is a high-order model, it is necessary to perform an approximation by a SOPTD transfer function to use the tuning rules presented here. This approximation was performed by a step response, and its result is given by Eq. 9, also being illustrated in Fig. 4.

$$G_{app}(s) = \frac{-1.38}{8.49^2 s^2 + 2(0.082)(8.49)s + 1} exp(-0.8s) \tag{9}$$

From Eq. 9, the PID controller was tuned by Table 1 and Table 3 rules, and also by the IMC tuning rules. As in "Methodology—development of tuning rules", the PID controller was used with the N equal to 10. It is worth mentioning that the adjustable parameters of the IMC tuning rules were defined based on the same premise used in the previous section, i.e., the tuning parameters were provided by ISE minimization, considering servo and regulatory response.

The closed-loop was submitted to a step setpoint change in the reactor temperature and later, after the system reached a new steady-state, to a step disturbance in the cooling water valve position with the proposed tuning rules in robust and performance versions, as well as with the IMC tuning. For performance comparison, the IAE and the ISE criterion were used, as they were in the previous section. The IAE and the ISE values for the two control situations, as well as the parameter values of the PID controllers obtained from the new tuning method and the IMC methods used in the comparative analysis, are shown in Table 9.

Table 9 clearly indicates that the proposed approach is more suitable for regulatory than for servo performance since

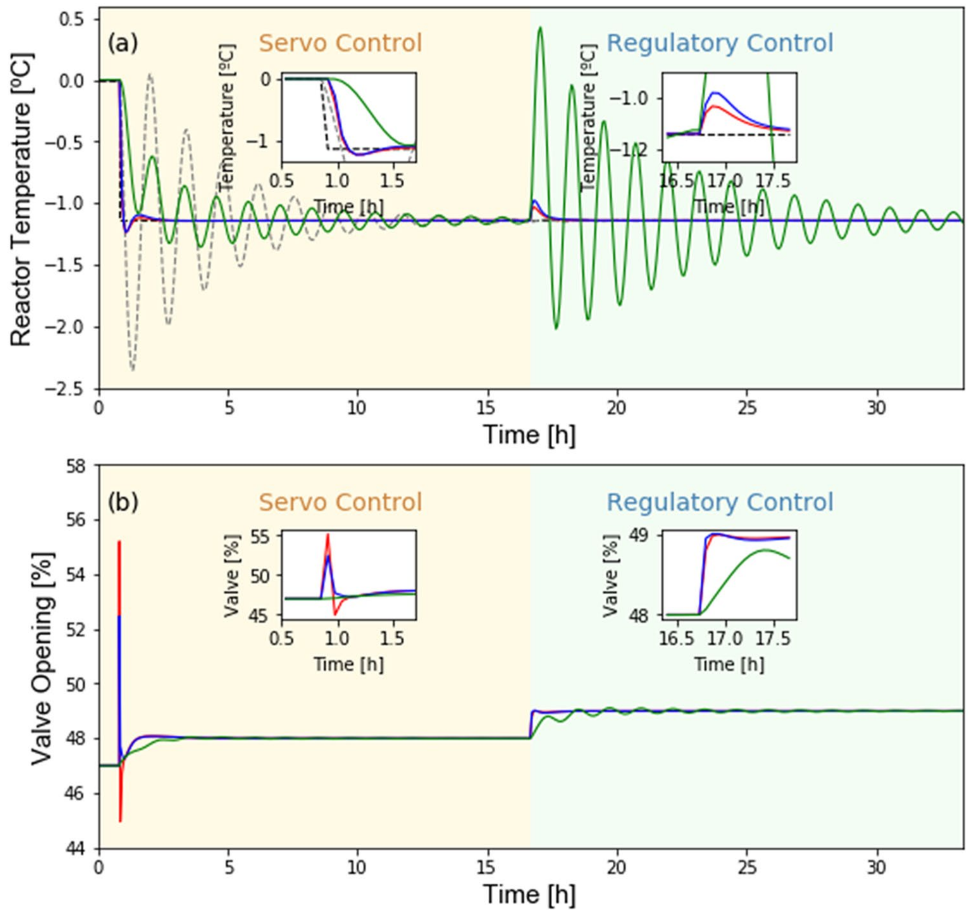

Fig. 5 Dynamics of the system for a step setpoint change in the reactor temperature and a step disturbance in the cooling water valve position, where **a** is the open-loop and the closed-loop response, and **b** is the control action. (shaded double dash) Open-loop, (− −) Setpoint, (red dash) Proposed tuning—performance version, (blue dash) Proposed tuning—robust version and (green dash) IMC tuning

the performance value obtained by the IMC tuning was 15.8, 82.2 times (11.9, 720.7 times) higher than the value obtained by the Proposed tuning—performance version, in the respective control situations in terms of the IAE (the ISE). The control loop performances are also illustrated in Fig. 5.

It can be observed in Fig. 5 that the behavior of the controlled variable signature of the performance version and robust version of the proposed tuning is similar, differing slightly in the settling time. In contrast, the servo control by the IMC tuning presents a very oscillatory behavior and also a longer settling time. Besides that, in regulatory control, the IMC tuning presents the same oscillatory behavior observed in the servo control situation. This is due to the high value of the derivative time and the small value of the integrative time of the controller.

According to Fig. 5, it is also indicated that the proposed approach is valid for the industrial case subjected to regulatory and servo actions as well. It was also observed in this practical case that the difference in performance between both adjustments is smaller compared to the robustness presented by the robust version. Therefore, for this system, the robust version is more appropriate.

A comparison of the proposed tuning against IMC tuning rules in terms of the root locus is presented in the next section.

Root locus comparison of the proposed tuning rules with IMC tuning rules

In order to evaluate the root location of the simple tuning rules in the robust version, they were compared with the Internal Model Control (IMC) methodology given by Rivera et al. (1986) for the industrial gas-phase polyethylene reactor case study presented in "An industrial gas-phase polyethylene reactor—another practical case study". It is worth pointing out that the system analyzed in this section is composed of the product of the PID controller transfer function in its ideal form, i.e.,

$$C_{ideal}(s) = K_p\left[\left(Y_{set}(s) - Y(s)\right) + \frac{\left(Y_{set}(s) - Y(s)\right)}{\tau_I s} + \left(Y_{set}(s) - Y(s)\right)\tau_D s\right] \tag{10}$$

using the previous case study, i.e., Eq. 8. It is also emphasized that the time delay was approximated from a first-order Padé approximation, i.e.,

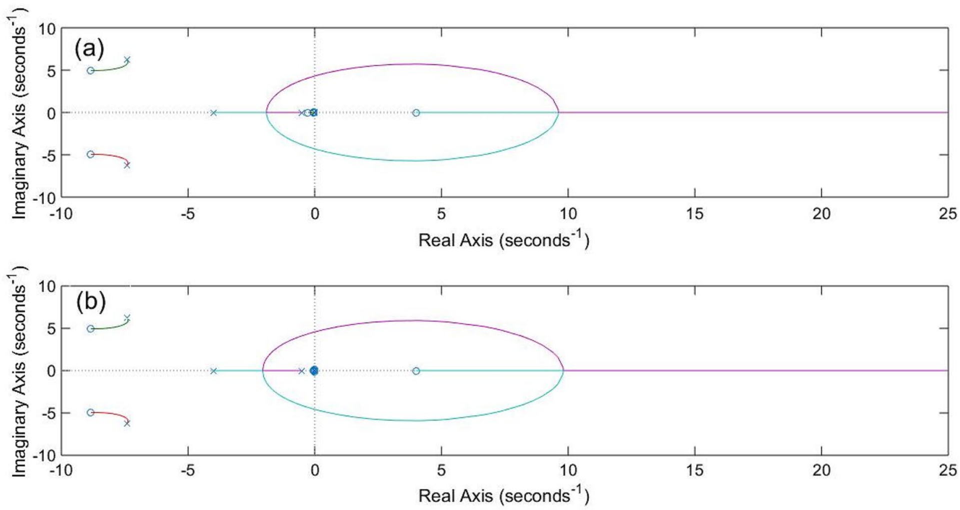

Fig. 6 Root locus comparison for $G_{47}(s)$ with **a** the proposed tuning rules and **b** the IMC tuning rules

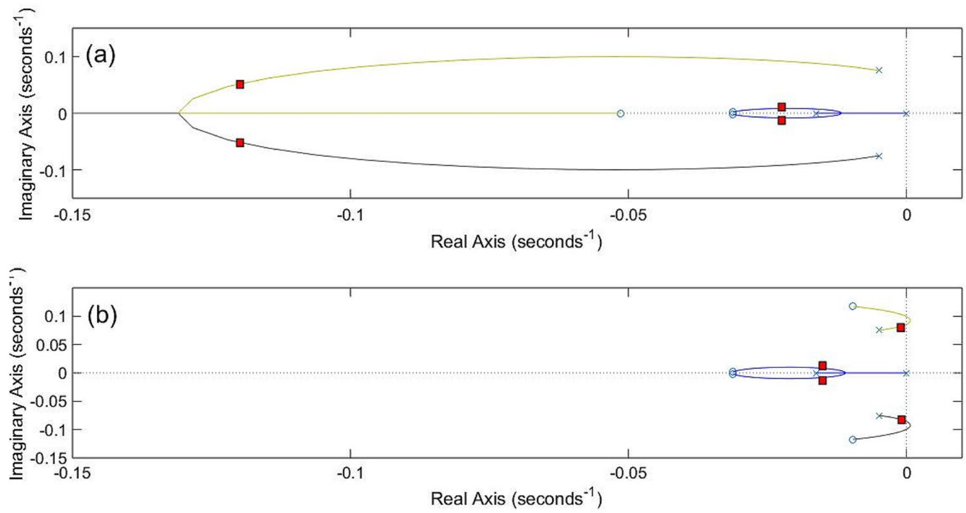

Fig. 7 Zoom of root locus comparison for $G_{47}(s)$ with **a** the proposed tuning rules and **b** the IMC tuning rules

$$exp(-\theta s) \cong \frac{1 - \frac{\theta}{2}s}{1 + \frac{\theta}{2}s} \tag{11}$$

Thus, the root locus of the systems with the proposed tuning rules and IMC tuning rules for this case study are shown in Fig. 6.

The root loci for both tuning rules compared in Fig. 6 seem to be similar. However, the region of most significant interest and the most significant difference between the tuning methodologies for this case study is near the origin of the axis, where the dominant poles and the zeros inserted by the controller are concentrated, as shown in Fig. 7.

In Fig. 7, it is observed that the proposed method has a characteristic pattern of placing the controller zeros in the negative real axis, close to the origin pole, as a result of the integral action of the controller. The IMC tuning, on the other hand, provides a characteristic pattern of cancellation of open-loop plant poles, i.e., insertion of controller zeros near open-loop plant poles. It is emphasized that the red squares in this figure indicate the position of the poles by the controller tuning, i.e., with the system gain (controller-plant) equal to the respective K_p, that they presented for the proposed tuning—robust version and IMC tuning in Table 9. For other highly underdamped systems, the same behavior pattern is also observed, as can be seen in Barreiros (2019).

This characteristic presented by the tuning rules proposed here shows advantages in terms of implementation of the PID algorithm, such as the easy application of the proposed adjustment in both parallel and series form, where the zeros inserted by the controller must be on the real axis. Besides, another feature of the method is the possibility of increasing the performance, since the dominant pole of the system is shifted closer to the origin with the increase of K_p. In contrast, the poles that confer a highly oscillatory behavior are shifted in the direction of the negative real axis.

This standard characteristic presented by the IMC tuning is problematic for highly underdamped systems, because due to the cancellation of the dynamics of these poles from the zeros inserted by the controller near the imaginary axis, it does not allow good performance for any value of K_p, since the dominant poles will always be near the imaginary axis, which constrains the effect of a variation of K_p in the closed-loop performance. It is emphasized that increasing the controller gain with no increase in the performance is something not usually expected. Another behavior exhibited due to this feature is a poor regulatory control performance compared to servo control performance, as previously reported in terms of the IAE and the ISE in Table 9.

Conclusions

This paper presents the development of appropriate tuning rules of PID controllers for highly underdamped systems. The results showed that the tuning rules proposed were able to provide a more robust or a higher performance tuning from the controller gain range, with the closed-loop maximal sensitivity restricted between 1.2 and 2.2.

A slugging control case study was evaluated, and it showed that the designed controller was able to stabilize the system in a limit cycle condition and obtain good performance in relation to the setpoint range and disturbance rejection. Furthermore, it is important to highlight that, in addition to stabilizing the system, the proposed tuning rules were able to offer an increase in oil production of up to 20% compared to the maximum production obtained by the IMC tuning.

An industrial gas-phase polyethylene reactor case study was also evaluated, and it showed that the proposed tuning rules present good performance and robustness even though the model used is a high-order system. Therefore, it highlights the potential of the tuning rules for the industrial gas-phase polyethylene reactor application.

Also, the simple tuning rules presented here showed superior performance when compared to the methods available in the literature as the IMC tuning, presenting good servo and regulatory control characteristics for the same adjustment. Another essential characteristic presented by the proposed tuning rules is the insertion of real zeros in closed-loop, which allows the implementation of PID controllers in both series and parallel forms.

Appendix A. Demonstration of how to obtain the adjustment of the controller using the new tuning rules

The plant used for the demonstration is given by:

$$G_A(s) = \frac{1}{194.6^2 s^2 + 2(0.059)(194.6)s + 1} exp(-4s)$$

The step-by-step application of the new tuning rules for $G_A(s)$ is exemplified below:

Step 1: Determination of τ/θ

$$\frac{\tau}{\theta} = \frac{194.6}{4} \Rightarrow \frac{\tau}{\theta} = 48.7$$

Step 2: Determination of the parameters $K_{p,1st}$, τ_I and τ_D from Table 1

$$K_{p,1st} = \frac{1}{1}\exp[9.1 \cdot 0.059 + (2.2 - 2.7 \cdot 0.059)\ln(\min(48.7,10)) - 3.4]$$

$$\Rightarrow K_{p,1st} = 6.3$$

$$\tau_I = 194.6\left[(0.2 + 0.5 \cdot 0.059)\min(48.7,10) + \exp(-33 \cdot 0.059) + 0.2\right]$$

$$\Rightarrow \tau_I = 513.3$$

$$\tau_D = 194.6\exp[1.3 - 0.2\min(48.7,10) - 2.9 \cdot 0.059]$$

$$\Rightarrow \tau_D = 81.4$$

Step 3: Determination of the parameter $K_{p,2nd}$ from Table 3

$$\frac{\tau}{\theta} = 48.7 \Rightarrow 10 \le \frac{\tau}{\theta} \le 500$$

$$K_{p,2nd} = \{14.7 - 14.5\exp[-0.009 \cdot 48.7]\}6.3$$

$$\Rightarrow K_{p,2nd} = 33.7$$

where $K_{p,1st}$, and $K_{p,2nd}$ provide a recommended interval for the controller gain.

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Declaration

Conflict of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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