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Comparison of the performance of different metaheuristic optimization algorithms

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Abstract. Metaheuristic algorithms are powerful tools for solving optimization problems. With the advancement of computational technology, many metaheuristic algorithms were developed to solve optimization problems quickly and accurately. Within this context, in this paper five of the main metaheuristic optimization algorithms developed in recent decades – Particle Swarm Optimization (PSO), Harmony Search (HS), Firefly Algorithm (FA), Search Group Algorithm (SGA) and Whale Optimization Algorithm (WOA) – had their performance compared in the solution of problems involving the optimization of benchmark functions and the optimization of trusses in which the design variables are the cross-sectional areas. Each algorithm was evaluated in terms of precision, computational time of operation and standard deviation among the results obtained after many executions. With the results obtained, the effectiveness of the five algorithms has been proven, although the older algorithms have a slightly lower performance. In most problems, the best results were achieved through the WOA or the SGA.

Keywords: optimization, metaheuristic algorithms, performance.

1 Introduction

Structural optimization is currently an important area of study for engineering, as there is a growing demand that structural designs be able to minimize the use of resources and the cost of the work and, at the same time, maximize the parameters related to the quality and strength of the structure. Modern metaheuristic optimization algorithms have been developed in the last decades and their application in structural optimization problems is widely used in the academic and professional environment. Within this context, this paper aims to evaluate the performance of the five metaheuristic optimization algorithms. Particle Swarm Optimization (PSO), Harmony Search (HS), Firefly Algorithm (FA), Search Group Algorithm (SGA) and Whale Optimization Algorithm (WOA) are used for the optimization of twelve benchmark functions and two truss size optimization problems.

2 Metaheuristic algorithms studied in this paper

According to Yang [1], metaheuristic algorithms are stochastic algorithms that use a certain exchange between randomization and local research. In general, these algorithms work with a trial and error process, which does not guarantee that the best solution is obtained, but rather an approximation of that solution in which the precision may depend, for example, on the complexity of the problem.

Particle Swarm Optimization (PSO) was developed in 1995 by Kennedy and Eberhart [2] and was inspired by the behavior of birds in search of food. Each search agent "remembers" the position in which he obtained the best value so far and compares it with the value obtained in the current position. In addition, each agent "knows" the best overall position a member of the group has found and the value corresponding to that position.

Harmony Search (HS) was developed in 2001 by Geem et al. [3] and was inspired by the performance of a musical group that seeks the perfect harmony produced by different instruments. Thus, in HS, the values of each of the design variables that influence the assessment of the objective function are compared to the sounds of each of the instruments that make up a musical harmony.

In the Firefly Algorithm, proposed in 2009 by Yang [4], the optimization process is compared to how fireflies use their luminescent characteristics to attract partners and possible prey. In this way, the research agents will have a relative attractiveness, according to the distance between them, and still proportional to their brightness, this being given according to the distance of the agent in relation to the global optimum of the objective function.

The Search Group Algorithm (SGA), developed in 2015 by Gonçalves et al. [5] performs in five stages: the initial population, initial search group selection, mutation of the search group, generation of the families and selection of the new search group. The main differential of the SGA is that it uses the strategy that the better the member of the research group, the more individuals it generates in a given iteration.

In the Whale Optimization Algorithm (WOA), developed in 2016 by Mirjalili and Lewis [6], the optimization process was inspired by the humpback whale's hunting strategy. In this way, WOA is governed by equations that describe the attack movements of the humpback whale and the interaction between them during hunting for prey.

3 Benchmark problems

The problems presented in this section were used to compare the performance of the algorithms studied in this paper: PSO, HS, FA, SGA and WOA. These problems have already been studied by several authors, in order to validate and compare some of the algorithms listed in this paper with other algorithms available in the literature.

3.1 Optimization of benchmark and constrained functions

The first problem solved in this paper is the minimization of the 12 benchmark functions presented in Table 1. The functions from f_1 to f_{10} were analyzed by Mirjalili and Lewis. [6] to validate their WOA algorithm and compare their performance with other algorithms different from those studied in this paper. The constrained functions, f_{11} and f_{12}, are found in Rao [7], as a proposal to be minimized through classic optimization methods.

Table 1. Benchmark and constrained functions

Function	Formula	Range
Sphere	$f_1(x) = \sum_{i=1}^{n} x_i^2$	[-100, 100]
Schwefel 2.22	$f_2(x) = \sum_{i=1}^{n}\lvert x_i\rvert + \prod_{i=1}^{n}\lvert x_i\rvert$	[-10, 10]
Rosenbrock	$f_3(x) = \sum_{i=1}^{n-1}[100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	[-30, 30]
Step	$f_4(x) = \sum_{i=1}^{n}([x_i + 0.5])^2$	[-100, 100]
Quartic function with noise	$f_5(x) = \sum_{i=1}^{n} ix_i^4 + random[0,1)$	[-1.28, 1.28]
Schwefel 2.26	$f_6(x) = \sum_{i=1}^{n} -x_i \sin(\sqrt{\lvert x_i\rvert})$	[-500, 500]
Rastrigin	$f_7(x) = \sum_{i=1}^{n}[x_i^2 - 10\cos(2\pi x_i) + 10]$	[-5.12, 5.12]
Griewank	$f_8(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	[-600, 600]
six-hump camel back	$f_9(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	[-5, 5]
Branin	$f_{10}(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{8}{\pi}\right)\cos x_1 + 10$	[-5, 5]
Constrained function 1	$f_{11}(x) = x_1^2 + x_2^2 - 2x_1 - 2x_2 + 2$ subject to: $g_1(x) = -2x_1 - x_2 + 4 \leq 0$ and $g_2(x) = -x_1 - 2x_2 + 4 \leq 0$	[-50, 50]
Constrained function 2	$f_{12}(x) = (x_1 - 1)^2 + (x_2 - 5)^2$ Subject to: $g_1(x) = -x_1^2 + x_2 \leq 4$ and $g_2(x) = -(x_1 - 2)^2 + x_2 \leq 3$	[-50, 50]

For a fair comparison, all the algorithms used to optimize the functions were executed with 200,000 evaluations, composed of 40 research agents and 5,000 iterations. The computing plataform used is an Intel Core i5 – 8^{th} generation witch 8 GB of RAM and Windows 10 Home. For functions from f_1 to f_8, without fixed dimensions, ten design variables were used. In functions f_{11} and f_{12}, constraints were treated with penalty methods. The results obtained through each algorithm are the minimum global value of the function, the standard deviation between the solutions obtained after 50 independent executions and the average computational time of each execution.

Table 2. Comparison of the global minimum generated by each algorithm

f	Exact	PSO	HS	FA	SGA	WOA
f_1	0	0	6.79E-09	0.0582	4.75E-06	0
f_2	0	1.53E-18	2.01E-04	0.0810	5.54E-04	0
f_3	0	1.1822	7.1063	8.0603	29.9913	4.4564
f_4	0	4.50E-33	6.62E-09	0.0565	4.91E-06	1.23E-11
f_5	0	3.50E-04	3.57E-04	0.0199	0.5399	2.48E-04
f_6	-4189.83	-2549.70	-4189.80	-3260.20	-2903.67	-3753.80
f_7	0	11.8798	1.38E-06	8.3482	12.8350	6.34
f_8	0	0.0804	0.0713	0.3480	0.0542	0.1844
f_9	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316	-1.0316
f_{10}	0.3979	0.3979	0.3979	0.3979	0.3979	0.3979
f_{11}	0.2222	0.2222	0.2222	0.2258	0.2223	0.2224
f_{12}	0.2539	0.2539	0.2539	0.2682	0.2540	0.2544

The global minimum of each function obtained through the studied algorithms is shown in Table 2 and compared with the exact value available in the literature. The performance of each algorithm varies according to the complexity of the objective function studied. WOA, for example, had the best performance in the analysis of almost half of the functions without fixed dimensions and a regular performance in the other functions without restrictions. However, it had a performance slightly inferior to all other algorithms in the functions with constraints.

Table 3. Comparison of standard deviation and computational time (s)

f	PSO		HS		FA		SGA		WOA	
	std	time	std	time	std	time	std	time	std	time
f_1	0	9.90	1.71E-09	10.64	0.0148	19.03	2.13E-06	4.59	0	5.67
f_2	1.08E-17	9.60	2.59E-05	10.99	0.0810	19.91	5.54E-04	4.59	0	5.63
f_3	1.4067	10.08	7.4650	11.09	1.2224	19.11	69.2543	4.91	10.4263	5.86
f_4	1.06E-32	9.40	1.65E-09	10.90	0.0149	19.47	2.18E-06	4.67	8.83E-12	5.59
f_5	2.56E-04	9.89	1.70E-04	11.51	0.0190	19.44	0.2767	4.88	2.34E-04	5.85
f_6	388.65	11.00	1.82E-10	11.68	251.14	19.47	270.64	5.26	391.67	6.12
f_7	5.6441	9.47	3.08E-07	11.08	3.6991	19.85	5.4245	4.79	6.13	5.76
f_8	0.0382	9.52	0.0235	10.92	0.0891	19.40	0.0394	4.96	0.1670	5.73
f_9	0	9.07	0	5.61	0	18.94	0	4.56	0	2.27
f_{10}	0	9.16	0	5.51	0	18.72	0	4.52	0	2.08
f_{11}	2.52E-16	9.29	4.01E-07	5.50	0.0021	18.40	2.74E-05	4.47	1.41E-04	2.15
f_{12}	1.55E-16	9.14	1.12E-06	5.55	0.0081	18.42	6.93E-05	4.51	4.70E-04	2.14

Table 3 shows the results in terms of standard deviation and computational time for each algorithm in the solution of benchmark functions. While the standard deviation varied widely in different functions analyzed by the same algorithm, the computational time underwent minor changes in different functions, so that the SGA and WOA algorithms obtained the best times in all analyzes.

3.2 Size optimization of a ten-bar plane truss

The second problem studied in this paper deals with the size optimization of a 10-bar plane truss shown in Figure 1. The truss has Young's modulus equal to 68.95 GPa , specific mass equal to 2767.99 kg/m³ and is subject to vertical loads of -444.82 KN in nodes 2 and 4. Stress constraints are $\pm$517.11 MPa for member 9 and $\pm$172.37 MPa for other members. The displacement constraints are $\pm$5.08 cm in y direction for nodes 1, 2, 3 and 4. The range of variables is 0.645 cm² to 200 cm².

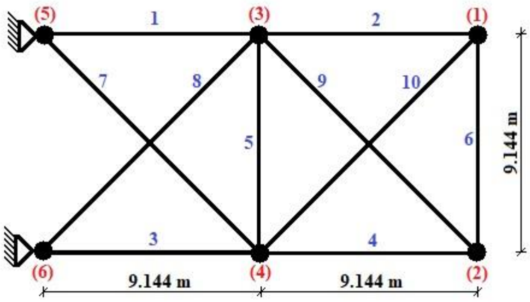

Figure 1. ten-bar plane truss

This problem has already been studied by Borges [8] who compared only the performance between HS and FA. In the present paper, 200,000 evaluations were used for each algorithm and the results obtained are shown in Table 4. The best design was obtained through the SGA algorithm. HS, WOA and FA algorithms also generated slightly better results than those found by Borges [8], however, it should be noted that in this paper, more evaluations were used than in Borges' work.

Table 4: Optimum design of cross-sections (cm²) for ten-bar plane truss

Member	Borges [8]		Present Paper				
	HS	FA	PSO	HS	FA	SGA	WOA
1	196.180	186.680	180.150	195.646	189.738	195.376	194.040
2	1.128	0.645	0.645	0.735	0.645	0.645	0.645
3	144.560	164.350	151.150	147.805	162.323	148.557	151.370
4	103.170	94.527	99.269	99.828	93.207	99.017	96.585
5	0.645	0.645	0.645	0.660	0.645	0.645	0.645
6	3.560	4.103	3.726	3.358	3.508	3.624	2.743
7	48.884	46.884	47.710	49.003	47.468	48.300	48.673
8	138.040	137.160	139.810	137.144	139.803	137.039	140.370
9	138.020	139.300	146.780	138.371	136.405	138.654	136.74
10	0.673	0.645	0.645	0.648	0.645	0.645	0.667
Mass (kg)	**2302.60**	**2301.08**	**2301.41**	**2297.01**	**2300.03**	**2295.59**	**2297.44**

Table 5 shows the standard deviation values of the minimum mass obtained after five independent executions of each algorithm and the computational time required for each execution. In this problem, a smaller number of executions was performed, due to the increase in computational time necessary to solve this problem, when compared to the previous problems. The SGA algorithm obtained an very low standard deviation value, that is, the five independent executions obtained similar values of minimum mass. The SGA was also the algorithm that obtained the shortest computational execution time.

Table 5: Standard deviation and computational time for ten-bar plane truss

	PSO	HS	FA	SGA	WOA
Std	3.65	1.12	4.64	0.54	6.94
Time (s)	202.79	204.43	208.00	185.42	208.14

Constraints were treated with penalty methods. It is important to note that the stress and displacement constraints were not violated during the optimization process, as can be seen in tables 6 and 7. Table 6 shows that member 5 activates the stress restriction in the PSO while table 7 shows node 1 activates the restriction of displacements in all algorithms.

Table 6: Stresses (MPa) obtained at the end of the optimization for ten-bar plane truss

Member	PSO	HS	FA	SGA	WOA
1	50.038	46.071	47.501	46.134	46.446
2	-10.409	-7.560	-8.034	-9.117	-8.340
3	-58.077	-59.397	-54.090	-59.098	-58.007
4	-44.877	-44.614	-47.780	-44.983	-46.110
5	172.370	169.153	172.357	172.367	171.350
6	-1.802	-1.656	-1.477	-1.622	-1.962
7	128.355	124.991	129.058	126.814	125.880
8	-46.188	-47.078	-46.174	-47.112	-45.983
9	42.923	45.520	46.172	45.430	46.061
10	14.720	12.142	11.362	12.893	11.408

Table 7: Displacements (cm) in the y direction obtained at the end of the optimization for ten-bar plane truss

Node	PSO	HS	FA	SGA	WOA
1	-5.08	-5.08	-5.08	-5.08	-5.08
2	-5.06	-5.06	-5.06	-5.06	-5.05
3	-1.89	-1.86	-1.85	-1.86	-1.84
4	-4.17	-4.10	-4.14	-4.15	-4.11

3.3 Size optimization of a 17-bar plane truss

The third problem studied in this paper deals with the size optimization of a 17-bar plane truss shown in Figure 2. The truss has Young's modulus equal to 206.84 GPa, a specific mass equal to 7418.21 kg/m³ and is subject to a vertical load of -444.82 kN in node 9. Stress constraints are $\pm$344.74 MPa for all members. The displacement constraints are $\pm$5.08 cm in x and y directions for all nodes. The range of variables is 0.645 cm² to 200 cm². The main difference of this problem in relation to the previous one is that here the number of constraints is much greater, since it includes the displacement of all the nodes of the structure.

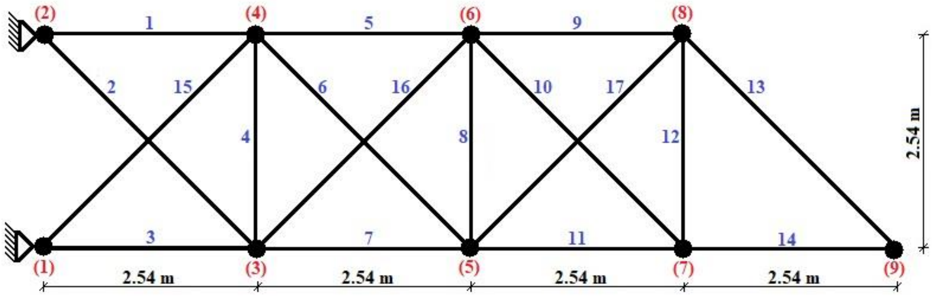

Figure 2: 17-bar plane truss

This problem has already been studied by Miguel and Fadel Miguel [9] who compared only the performance between HS, ABC and FA algorithms. In the present paper, 200,000 evaluations were used for each algorithm and the results obtained are shown in Table 8. The best design was obtained through the SGA algorithm, while the other algorithms also generate good results.

Table 9 shows the standard deviation values of the minimum mass obtained after five independent runs of each algorithm and the computational time required for each run. The SGA algorithm obtained the smallest standard deviation, indicating small differences between the results obtained in each execution. The SGA was also the algorithm that obtained the lowest computational execution time.

Table 8: Optimum design of cross-sections (cm²) for 17-bar plane truss

Member	Miguel and Fadel Miguel [9]			Present paper				
	HS	ABC	FA	PSO	HS	FA	SGA	WOA
1	104.440	102.178	101.960	99.841	102.295	105.414	103.722	102.400
2	0.785	0.645	0.645	1.220	1.022	0.925	0.645	0.686
3	78.906	74.002	76.447	82.641	77.002	79.071	77.946	77.845
4	0.711	0.645	0.648	0.648	0.664	0.645	0.645	0.654
5	51.578	51.666	53.225	53.950	54.500	54.702	53.963	51.229
6	33.995	34.700	35.220	34.389	35.807	35.241	35.845	35.143
7	75.888	75.023	76.507	76.918	77.070	74.405	75.352	76.500
8	0.676	0.645	0.645	0.645	0.651	0.669	0.645	0.886
9	49.838	50.181	51.102	52.243	49.282	49.295	50.992	52.105
10	0.995	0.645	0.645	0.646	0.831	0.645	0.645	0.975
11	27.752	30.444	25.842	26.199	25.650	26.069	25.868	26.266
12	0.662	0.645	0.645	0.645	0.802	0.645	0.645	1.077
13	36.946	39.770	37.258	37.180	35.175	35.466	36.058	36.677
14	24.954	24.393	26.921	26.721	25.167	25.763	26.739	26.476
15	35.919	35.483	36.433	33.764	37.026	35.650	35.564	37.028
16	1.408	1.193	0.645	0.933	0.800	0.717	0.645	0.988
17	36.503	38.590	36.081	34.650	37.318	36.588	35.694	34.928
Mass (Kg)	**1,173.22**	**1,174.63**	**1,171.47**	**1,172.59**	**1,172.51**	**1,172.11**	**1,171.45**	**1,172.04**

Table 9: Standard deviation and computational time for 17-bar plane truss

	PSO	HS	FA	SGA	WOA
Std	1.54	1.16	3.65	0.52	1.35
Time (s)	286.24	265.90	251.50	194.78	253.97

Constraints were treated with penalty methods. Tables 10 and 11 show, respectively, the stresses and displacements obtained after optimization, proving that the restrictions imposed were not violated.

Table 10: Stresses (MPa) obtained at the end of the optimization for 17-bar plane truss

Member	PSO	HS	FA	SGA	WOA
1	176.789	172.848	167.801	170.789	171.805
2	164.591	153.984	159.334	171.652	168.296
3	-163.195	-174.747	-170.084	-172.208	-170.765
4	-45.145	-21.433	-23.702	-2.818	-12.880
5	166.990	165.016	164.258	166.277	169.767
6	168.292	171.852	174.940	172.482	167.473
7	-172.026	-171.890	-178.156	-176.083	-169.215
8	65.208	29.177	36.764	22.253	12.610
9	168.936	178.938	179.168	173.249	175.149
10	154.726	132.789	141.012	136.197	130.541
11	-172.482	-176.464	-173.097	-174.358	-176.320
12	-109.540	-97.298	-99.711	-96.305	-101.676
13	169.196	178.838	177.372	174.459	173.527
14	-166.468	-176.747	-172.660	-166.354	-181.767
15	-180.367	-165.650	-172.327	-173.771	-174.215
16	-170.778	-171.417	-175.357	-167.667	-167.332
17	-178.666	-165.613	-169.450	-173.777	-175.659

It is important to note that none of the stresses after optimization has even approached the constraint value. In the case of displacements, the displacement in the y direction of node 9 made the displacement constraint active in all algorithms, as shown in Table 11.

Table 11: Displacements (cm) obtained at the end of the optimization for 17-bar plane truss

Node	x direction					y direction				
	PSO	HS	FA	SGA	WOA	PSO	HS	FA	SGA	WOA
1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0
3	-0.20	-0.21	-0.21	-0.21	-0.21	-0.60	-0.59	-0.60	-0.63	-0.63
4	0.22	0.21	0.20	0.21	0.17	-0.66	-0.62	-0.63	-0.64	-0.64
5	-0.41	-0.43	-0.43	-0.43	-0.42	-1.73	-1.68	-1.69	-1.70	-1.68
6	0.42	0.41	0.41	0.41	0.42	-1.64	-1.64	-1.64	-1.67	-1.66
7	-0.62	-0.64	-0.64	-0.64	-0.63	-3.07	-3.03	-3.04	-3.06	-3.04
8	0.63	0.63	0.63	0.63	0.63	-3.21	-3.15	-3.16	-3.18	-3.16
9	-0.82	-0.86	-0.85	-0.85	-0.86	-5.08	-5.08	-5.08	-5.08	-5.08

4 Conclusions

In this paper, five algorithms - PSO, HS, FA, SGA and WOA - had their performance evaluated in the optimization of benchmark functions and plane trusses. With the results obtained, it can be concluded that the studied algorithms are efficient. SGA and WOA are good tools, in terms of precision and computational time, for the optimization of mathematical functions, although WOA had performance below the expected in functions with constraints. In the optimization of plane trusses, which are problems with many restrictions, the SGA presented the best performance in all evaluated parameters.

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