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To cite this article: Sergio Eduardo Silva Duarte and Nathan Willig Lima 2020 *Eur. J. Phys.* **41** 045602

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Special relativity from classical gedanken experiments involving electromagnetic forces: a contribution to relativity without light

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Received 12 September 2019, revised 4 March 2020

Accepted for publication 17 March 2020

Published 2 June 2020



CrossMark

Abstract

Our goal is to present a derivation of Lorentz's transformations from gedanken experiments with electromagnetic forces without having to assume Einstein's postulate of the constancy of the speed of light. Instead, we propose a different second postulate, namely that electric charge is independent of the state of motion of the system, and we restrict the first postulate in order to state the relativity of electromagnetic forces only. We argue that assuming that charge is invariant is much more intuitive than assuming that light speed is constant. In this derivation, furthermore, the constancy of a limit speed c is obtained as a result of the velocity addition rule. Finally, we discuss the epistemological and didactic implications of this formulation.

Keywords: relativity without light, relativity, special theory of relativity

1. Introduction

The special theory of relativity (STR) is one of the most revolutionary theories in the history of recent science (Kuhn 1996, Renn 2004). It has led to new conceptions of space and time, including the recognition of counterintuitive phenomena such as time dilatation and space contraction, which can be derived from Einstein's two postulates through Lorentz's transformations.

Historically, the STR has its roots grounded in the development of electromagnetic theory (Giannetto 2009). Einstein considered that Lorentz's 1895 paper represented the

renunciation of the well accepted principle of relativity, established in the context of Galilean and Newtonian physics:

As long as one believed that all of physics can be founded on Newton's equations of motion, one therefore could not doubt that the laws of nature are the same without regard to which of the coordinate systems moving uniformly (without acceleration) relative to each other they are referred. However, this independence from the state of motion of the system of coordinates used, which we will call 'the principle of relativity,' seemed to have been suddenly called into question by the brilliant confirmations of H. A. Lorentz's electrodynamics of moving bodies. (Einstein 1907, p. 252)

Einstein's theory consisted, thus, of reaffirming the validity of the principle of relativity, considering not only that Newton's laws were valid in all inertial frames, but also that the laws of electrodynamics should hold in the same circumstances (Einstein 1905). As is well known, one of the major issues with this conception is that the laws of electrodynamics are not covariant under Galileo's transformations. Because of that, many attempts had been made to bypass the complications derived from studying electromagnetic phenomena in different reference frames, such as the proposition of a local time by Voigt in 1887 and of the length contraction by Lorentz in 1895 (Pauli 1981). In 1904 and 1905, Lorentz (1904) and Poincaré (1905) showed that the electromagnetic laws should be covariant under Lorentz transformations and not Galileo's. Despite that, it was Einstein who realized that, together with the principle of relativity, it was necessary only to assume that 'light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body' (Einstein 1905, p.140), to provide Lorentz's transformations, providing a new deeper interpretation of the formulation of the entire problem—and bringing about a new conception of space and time (Pauli 1981).

The postulate of the constancy of the speed of light, however, was considered very counterintuitive since its formulation. Einstein himself commented on the apparent contradiction between the two postulates on different occasions (Einstein 1917). This contradiction was not easily accepted, originating many attempts at describing electromagnetic phenomena in agreement with the first postulate but rejecting the second one, such as those led by Ritz (1908), Comstock (1910), Kunz (1910) and Tolman (1910). Moreover, even decades after Einstein's original paper, experiments were performed to test alternative theories to the second postulate (1925). Tolman (1987) presents a detailed discussion of this problem.

Despite all the corroborations of the second postulate, resistance to building a theory accepting it *a priori* has not vanished with time, and, more recently, many works have proposed relativity without (the postulate of) light: by assuming space-time isotropy and homogeneity, one may achieve the general structure of the addition law for parallel velocities (Mermin 1984) and even the structure of Lorentz transformations in which a limit speed c shows up, but its value remains undetermined (Berzi and Gorini 1969, Pal 2003, Gao 2017).

On the other hand, in the pedagogic context, the STR is introduced in undergraduate introductory courses. In these presentations, usually, time dilatation and space contraction, as well as Lorentz's transformations, are derived from Einstein's two postulates without much contextualization of the origins of the postulate or of the attempts to build relativity without having to assume *a priori* the counterintuitive idea of the constancy of the speed of light (Arriasecq and Greca 2007). We understand that this approach, despite the benefit of introducing the student to important results of the STR quickly, may present some inconveniences: it reduces the comprehension of the extension of the STR by detaching it from electromagnetic studies and by overemphasizing the kinematic implications; it introduces the

counterintuitive conception of the constancy of the speed of light without much further discussion as it was an intuitive idea; and it sometimes gives the wrong impression that the STR is a theory only of high-speed phenomena, whereas many authors have already discussed how relativity is necessary to explain electromagnetic phenomena even at low velocities (Piccioni 2007, Feynman *et al* 2013, Ackerson 2019).

We are interested in presenting a didactic alternative to introduce the theory, reattaching the STR to the study of electromagnetic phenomena (through the study of electromagnetic forces) with the advantage of escaping from considering light speed constant *a priori*. We understand that this presentation is quite intuitive, and it is suitable to be applied in introductory electromagnetic courses.

Our goal is to present a derivation of Lorentz's transformations from a gedanken experiment with electromagnetic forces highlighting the minimum assumptions that should be made in order to achieve such a goal and without having to assume Einstein's postulate of the constancy of the speed of light. Thus, we propose a more restrictive postulate of relativity (in comparison to Einstein's original one), explicitly stating only that electric and magnetic force expressions must be submitted to the postulate of relativity, and a different second postulate, i.e. 'the electric charge of a system is independent of its state of motion' (we discuss the advantages of the exchange of postulates in section 2). In this derivation, the constancy of a limit speed c is obtained as a result of the velocity addition rule. We also discuss the epistemological and didactic implications of this formulation.

2. An alternative approach: special relativity from classical gedanken experiments involving electromagnetic forces

In 1905, Einstein proposed the following two postulates:

Einstein's postulate of relativity: The same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good.

Einstein's postulate of the constancy of light speed: Light is always propagated in empty space with a definite velocity c , which is independent of the state of motion of the emitting body.

Einstein's first postulate is stated very imprecisely. In other works, Einstein has explained that its deep meaning is that there is no absolute rest, i.e. there is no empirical test (mechanical, optical or electromagnetic) able to determine if a specific reference frame is in uniform motion or at rest. In this sense, the principle of relativity states that all movement is a relative movement. As we have discussed, Einstein was interested in extending the validity of the principle of relativity to beyond Newtonian physics—giving up the notion of a privileged reference frame (such as the ether). Despite that, Einstein did not state explicitly what was implied by 'laws of electrodynamics and optics'. If one includes Maxwell's equations, for instance, into the set of laws of electrodynamics and optics, then this postulate alone implies the Lorentz transformation, as shown in 1904 by Poincaré and Lorentz. In particular, the speed of light should then be the same in all inertial frames. Furthermore, Einstein's second postulate states that the speed of light is independent of the state of motion of the emitting body, a result coming from the ether theory (Tolman 1987). The apparent contradiction between the information and theoretical assumptions of both postulates were often the object of controversy (Einstein 1917).

2.1. A possible pair of postulates for the STR without light

In this section, we have two goals. The first one is to propose a more restrictive and precise postulate of relativity. In this sense, we clearly state which set of laws we postulate as valid in

all reference frames. The second one is to present a more intuitive alternative to Einstein's second postulate.

First postulate (the principle of relativity): Electromagnetic forces will be valid for all reference frames for which equations of mechanics hold good.

Corollary 1. *The Coulomb force (electric force) will be valid for all reference frames for which the equations of mechanics hold good.*

Coulomb force expresses the force between charged particles, and it may be written as

$$\overrightarrow{dF^{(e)}} = \frac{q}{4\pi\epsilon_0} \cdot \frac{dq}{r^2} \hat{r}. \quad (1)$$

$\overrightarrow{F^{(e)}}$ is the electric force, q is the electric charge, ϵ_0 is the permittivity of free space and r is the distance between the two bodies.

Corollary 2. *Grassmann's force will be valid for all reference frames for which the equations of mechanics hold good.*

Grassmann's force (magnetic force) expresses the force between elements of current or, contemporarily, among charged particles in movement (Santos and Gardelli 2017). In contemporary notation, Grassmann's force may be written as

$$d^2 \overrightarrow{F^{(m)}} = \frac{\mu_0}{4\pi} i d\vec{l} x \frac{id\vec{l} x \hat{r}}{r^2}. \quad (2)$$

$\overrightarrow{F^{(m)}}$ is the magnetic force, μ_0 is the permeability of free space, i is the current intensity, dl is an element of length, and r is the distance between the two current elements.

Corollary 3. *The resultant force \vec{F} acting on a system that is submitted to Coulomb's and Grassmann's forces is the vector sum of the forces and it will be valid for all reference frames for which equations of mechanics hold good. This may be written as*

$$\vec{F} = \overrightarrow{F^{(e)}} + \overrightarrow{F^{(m)}}. \quad (3)$$

Summarizing, according to the principle of relativity, expressions (1)–(3) must be valid in all inertial frames.

Corollary 4. *A particle in equilibrium ($\sum \vec{F} = 0$) in an inertial reference frame is in equilibrium in any other inertial reference frame.*

If a particle is in equilibrium in an inertial frame, the resultant force acting on it is zero and its acceleration is null. Thus, the particle is at rest in relation to at least one specific frame and, consequently, it will be observed or at rest or at uniform motion at any other inertial reference frame. Since the particle is observed at rest or at uniform motion in all inertial frames, all of them agree that the particle is in equilibrium.

In this sense, we may say that if one particle is in equilibrium in one reference frame, the determination of the magnetic force, the electric force and the resultant force should also provide the information that the particle is in equilibrium in any other reference frame.

Second postulate (postulate of the constancy of the electric charge): The electric charge of a physical system is independent of the state of motion of the system.

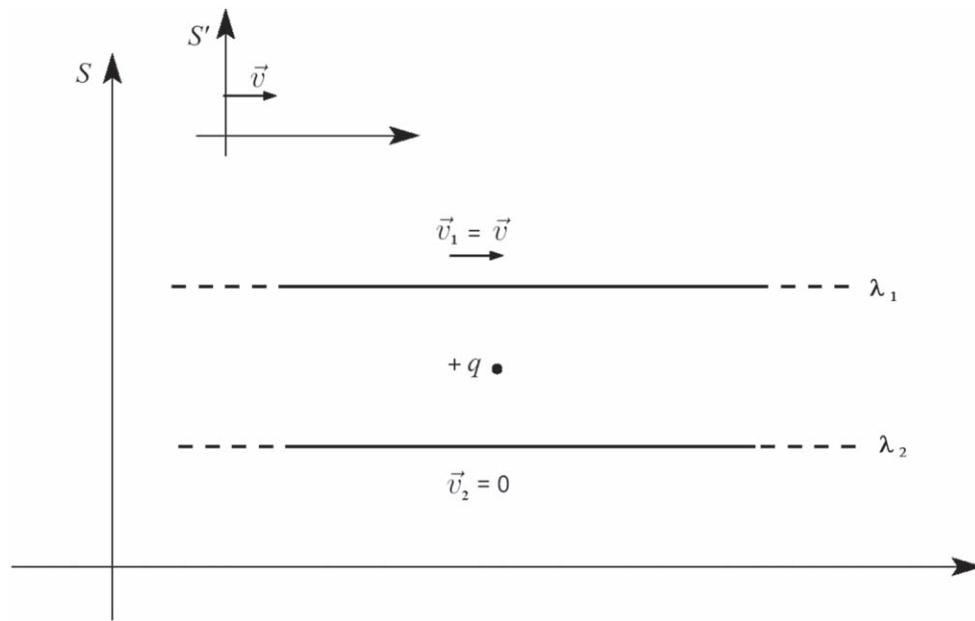


Figure 1. Reference frame S observes a particle with charge $+q$ in the middle of two infinite charged lines with charge density λ_1 and λ_2 . The particle is in equilibrium and at rest in S . In the same reference frame, the charged carriers in line 1 are observed to move to the right with velocity v . Another inertial reference frame S' moves with velocity v to the right in relation to S .

To think of electric charge as independent of the state of motion of the physical system, and invariant under reference frame transformation, is something much more intuitive and palatable than considering the speed of light constant. At first glimpse, it does not create any intellectual discomfort. It is a reasonable hypothesis. Intuition, however, may be misleading. *A priori*, there is not any information that excludes the possibility of electric charge to be a dynamical property. However, as was pointed out by Richard Feynman, if electric charge was dependent on velocity, we should observe a charge variation when heating some physical systems—which has never been observed (Feynman *et al* 2013). In this sense, combining empirical evidence with the reasonability of this statement, we promote the constancy of electric charge to the status of postulate.

2.2. Finding space contraction with a gedanken experiment with electromagnetic forces

In an inertial reference frame S , a particle with charge $+q$ is at rest and in equilibrium. Also, the particle is exactly in the middle of two infinite charged lines with charge density λ_1 and λ_2 . Still in S , the charge carriers in line 1 are observed to move to the right with speed v (figure 1).

In reference frame S , the particle is submitted only to electric forces and, since it is in equilibrium, we may state that

$$F_1^{(e)} = F_2^{(e)} \quad (4)$$

where $F_1^{(e)}$ and $F_2^{(e)}$ are the electric forces caused by lines 1 and 2 respectively in reference frame S . Using the Coulomb expression to determine the electric force caused by each charged line and considering that the particle is in the mid position between the lines,

equation (4) becomes

$$\frac{q\lambda_1}{2\pi\epsilon_0 r} = \frac{q\lambda_2}{2\pi\epsilon_0 r}. \quad (5)$$

In this case, r is the distance between the particle and each line. And we conclude that the charge density in lines 1 and 2 must be equal:

$$\lambda_1 = \lambda_2 = \lambda. \quad (6)$$

On the other hand, in the reference frame S' , which is moving with velocity v to the right in relation to the reference frame S (figure 1), the charged particle is observed to move with velocity $-v$. The charged carriers in line 1 are observed to be at rest, and the charges at line 2 are observed to move with velocity $-v$. The observed velocities in reference frame S' are a consequence of the principle of relativity. Since there is no absolute velocity, all velocity must be treated as 'relative velocity' and, since there is no privileged reference system, the description must be symmetrical.

Furthermore, in S' , the charged particle is not submitted only to electric forces but also to a magnetic force. Despite that, according to corollary 2, the particle must still be in equilibrium. After considering the direction of the electric and magnetic forces, it is possible to write the following force balance equation:

$$F_1^{(e)} + F_2^{(m)} = F_2^{(e)}. \quad (7)$$

In which $F_1^{(e)}$ and $F_2^{(e)}$ are the electric forces caused by line 1 and 2 in the reference frame S' , and $F_2^{(m)}$ is the magnetic force caused by line 2 in the reference frame S' . The introduction of a new force and the exigence of the state of equilibrium directly implies that some physical quantity must have changed from reference S to S' . The task is to find out which quantity has changed and to describe how. The only quantities that we know *a priori* that have not changed are the constants (due to the first postulate) and the particle charge (due to the second postulate). All other quantities may have changed. Using Grassmann's force and corollary 3, equation (7) turns into

$$\frac{q\lambda'_1}{2\pi\epsilon_0 r'} + \frac{\mu_0 \lambda'_2 v^2 q}{2\pi r'} = \frac{q\lambda'_2}{2\pi\epsilon_0 r'}. \quad (8)$$

Since r' appears in all terms of equation (8), any possible alteration in it would be insignificant to the force balance problem. From (8), we have that

$$\lambda'_2(1 - \epsilon_0\mu_0 v^2) = \lambda'_1. \quad (9)$$

Since the vacuum permittivity and permeability are constants, we may designate their product as a new constant $\epsilon_0\mu_0 = \frac{1}{c^2}$, thus

$$\lambda'_2 \left(1 - \frac{v^2}{c^2}\right) = \lambda'_1. \quad (10)$$

According to equation (10), we conclude that the observance of the two postulates implies directly that the value of the density charge that is measured must change. We may go even further to understand how the shift of the reference frame is affecting the physical systems. We will call the charge density of a system at rest the 'proper density charge' (λ_0), and designate the charge density of a moving system (with velocity v) as λ_v . We are interested in describing how these two quantities relate. Making the simplest possible hypothesis, we

test whether the relation between λ_v and λ_0 is linear:

$$\lambda_v = \gamma\lambda_0. \quad (11)$$

Using this definition of the system described in reference S and S' , we may say that $\lambda'_2 = \gamma\lambda_2$ and $\lambda'_1 = \frac{\lambda_1}{\gamma}$. According to equation (6), $\lambda_1 = \lambda_2 = \lambda$. Thus, equation (10) turns into

$$\gamma\lambda\left(1 - \frac{v^2}{c^2}\right) = \frac{\lambda}{\gamma} \quad (12)$$

from which we find that

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (13)$$

and

$$\lambda_v = \frac{\lambda_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma\lambda_0. \quad (14)$$

Since we are dealing with linear charge densities, and the charge is invariant (second postulate), we are led to conclude that the physical quantity that is varying is the system length, i.e. $\lambda = \frac{q}{L}$ and $\lambda_0 = \frac{q}{L_0}$. Thus, equation (14) turns into

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} = \frac{L_0}{\gamma} \quad (15)$$

which is the so-called space contraction, obtained in the original formulation of STR.

2.3. Time dilatation

From the last result we have obtained (space contraction), it is easy to show that time is also affected by movement without any further hypothesis. Suppose that there is a ruler of proper length L_0 at rest in a reference frame S . An observer, located at the origin of a reference frame S' (with velocity v relative to S), measures the length of the ruler by recording the instants of time in which he passes through both edges of the ruler. Those two events, the passage through the ends of the ruler, happen in the same position at the reference frame S' (we will call the time interval between two events in the same space position proper time t_0). Setting $t' = 0$ for the first event, the time interval between these two events is $\Delta t' = t_0 - 0 = t_0$. In S (where S' is observed moving with a velocity v) the distance between the two events is the proper length of the ruler L_0 . This is represented in figure 2.

Thus, we have that the proper time t_0 (measured in S') is equal to

$$t_0 = \frac{L}{v} \quad (16)$$

and the time t (measured in S) is equal to

$$t = \frac{L_0}{v}. \quad (17)$$

We may relate the time interval in both reference frames:

$$t = \frac{t_0 L_0}{L'} \quad (18)$$

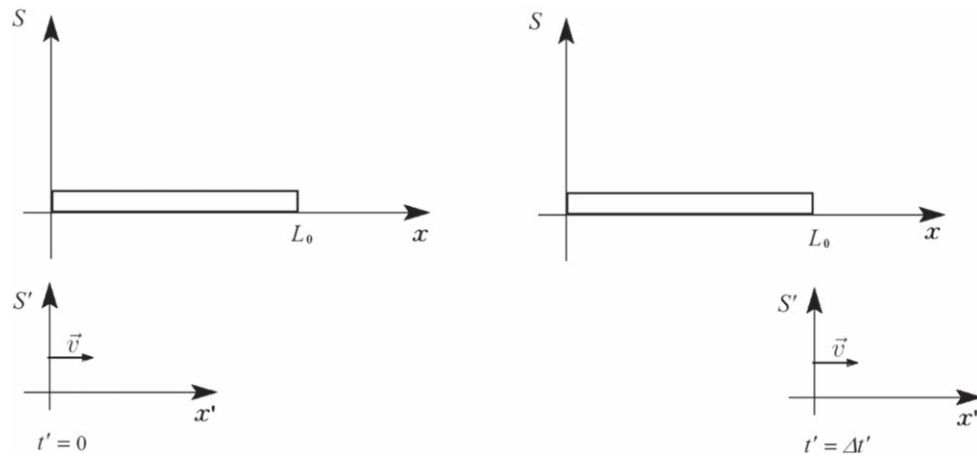


Figure 2. Two events happen in the same position at a reference frame S' . In reference frame S , where S' is observed moving with a velocity v , the distance between these two events is measured corresponding to L_0 , the proper length.

using (15):

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (19)$$

which is the so-called time dilation, obtained in the original formulation of the STR.

2.4. Lorentz's transformations

Usually, it is possible to find in textbooks how Lorentz's transformation leads to space contraction and time dilation. It is not straightforward, however, that the recognition of space contraction and time dilation will lead directly to Lorentz's transformations. In this section, we will use a new gedanken experiment to achieve that goal—which has not been presented in the literature.

By recognizing that the lengths of moving rulers change at different reference frames, it is possible to obtain the coordinate transformations that relate observed events in both frames. Two rulers with different lengths x and x' are at rest in two reference frames S and S' respectively. S' moves with velocity v in relation to S . We intend to compare the ruler lengths from the point of view of both reference frames. In order to do so, we define two events. The first one is when the left edges of both rulers are at the origin. The first event is observed as $(0, 0)$ in both reference frames. And the second one is when the right edges of both rulers coincide. The second event is observed as $(x; t)$ and $(x'; t')$ in S and S' respectively. This is represented in figure 3.

In reference frame S , the proper length x of the ruler is equivalent to the length of the ruler at rest in S' measured from S plus the displacement of S' :

$$x = vt + \frac{x'}{\gamma}. \quad (20)$$

Then,

$$x' = \gamma(x - vt). \quad (21)$$

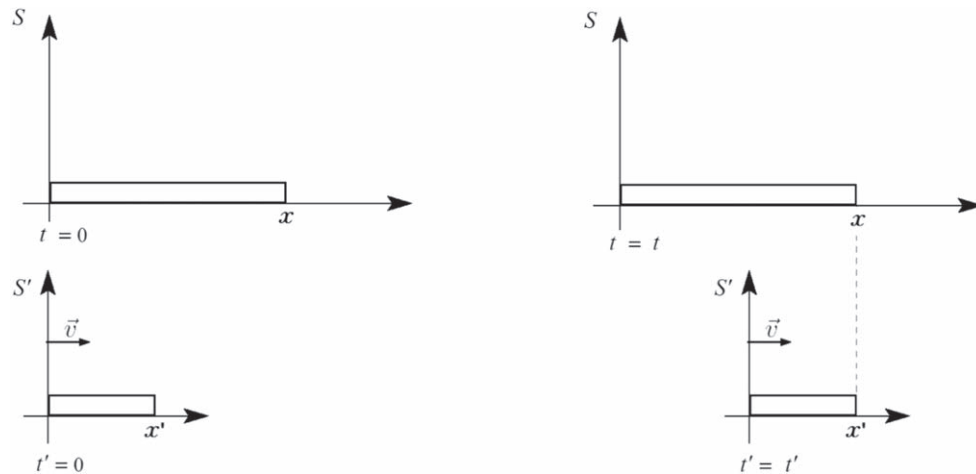


Figure 3. Two rulers with different lengths x and x' are at rest in two reference frames S and S' respectively. S' moves with velocity v in relation to S .

Furthermore, in reference frame S' , the length of the ruler at rest in S (measured from S') is equivalent to the proper length of the ruler at rest in S' plus the displacement of S :

$$\frac{x}{\gamma} = vt' + x'. \quad (22)$$

Using (21) and (22), it is possible to eliminate x' to obtain

$$t' = \gamma \left(t - \frac{vx}{c^2} \right). \quad (23)$$

Equations (21) and (23) may be recognized as the well-known Lorentz's transformations. Since the studied movement is unidimensional, we may also add that $y' = y$ and $z' = z$, completing the entire set of transformations.

2.5. Velocity transformation rule and the invariance of the speed c

In an inertial reference frame S , a physical system is observed to move with a velocity v in the x direction. In another inertial reference frame S' , moving with velocity V in the x direction, the velocity of the physical system may be observed as

$$v' = \frac{\Delta x'}{\Delta t'} = \frac{v - V}{1 - \frac{vV}{c^2}}. \quad (24)$$

If the velocity of the system is exactly equal to c , it is possible to determine that also $v' = c$ independently of the relative velocity V , i.e. if there is a system that travels with speed c in a specific inertial reference frame, it is observed to travel with speed c in all inertial reference frames.

3. Conclusions

In this work, we introduced the possibility of deriving space contraction, time dilatation and Lorentz's transformation from a more restrictive principle of relativity and from the principle

of the constancy of the electric charge. Our version of the principle of relativity demanded only to consider that electric and magnetic forces should be valid in all inertial frames (instead of all electromagnetic laws as stated by Einstein). In our approach, we analysed a gedanken experiment with electromagnetic forces in which a particle is in equilibrium applying both postulates to obtain space contraction, time dilation and, consequently, Lorentz's transformations.

After obtaining Lorentz's transformations, it was possible to derive that the speed c is invariant under the reference frame transformation. In this sense, we may say that our proposal is not only consistent with the original proposition of the STR, but also reveals that the constancy of the speed of light and the constancy of the electric charge may be considered epistemologically equivalent, i.e. when someone assumes the constancy of speed of light as a postulate, the constancy of the electric charge may be obtained as a derivation (Feynman *et al* 2013). On the other hand, when one assumes the constancy of electric charge as a postulate, the constancy of the speed of light may be obtained as a derivation. In this sense, this work corroborates the idea that a specific mathematical statement may assume different epistemological status inside of a physical theory (Karam and Krey 2015). Also, by restricting the postulate of relativity to the expression of forces we expect to show which are the minimum considerations that must be made in order to achieve Lorentz's transformation.

We understand that introducing the STR through the presented method may be an interesting approach before presenting Einstein's original postulates. It is an organized, consistent presentation, that cannot be found in any textbook (nor the epistemological debate that it promotes). The physics teacher may derive Lorentz's transformations from both approaches and discuss the epistemological status of different mathematical statements in each case. We hope that this suggestion may contribute to a physics teaching that is more concerned with discussing the fundamentals of physical theories.

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