

# SEDIMENT TRANSPORT FOR NONUNIFORM SEDIMENT MIXTURES: THE EGIAZAROFF EQUATION

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## INTRODUCTION

The comprehension of sediment yield and consequently the sediment transport in rivers is one of the main subjects of hydrosedimentology. Sediments are produced, disaggregated, transported and then deposited reaching rivers, lakes, reservoirs and seas. As they differ in size and weight depending on location where they are yielded, understanding the processes that define how sediments are transported is a big theme. Some studies (such as Shields, 1936; Egiazaroff, 1965) have been trying to describe the incipient sediment movement in rivers. For achieving them, some considerations such as uniform grain size distribution, constant gradient, mean velocity and constant shear stress have been usually done for simplifying the sediment transport. Although Shields' equation is widely used for many purposes, the Egiazaroff's equation is more suitable for nonuniform sediment transport. In Brazil, the Egiazaroff equation was neglected for many decades, thus, the objective of the present paper was to introduce the Egiazaroff's theory and equation recommending its use in Brazil, especially in mountain regions.

## SEDIMENT TRANSPORT

Sediment transport is the movement of particulate material due to the action of water. In mountain rivers where the sediments are usually in nonuniform mixtures, the transport depends on a series of complex interactions among river discharge, activation of sediment sources from different types, and river morphodynamics. Brardinoni et al. (2015) commented that there is a lack of observations in field with appropriate quantity and quality capable to allow the development of physically-based models. One of these models is that developed by Shields (1936), which consider the equilibrium between gravity and fluid forces. Egiazaroff (1965) did some considerations about transport of nonuniform sediment mixtures from Shields's theory.

## SHIELDS THEORY

The classical approach for the incipient sediment movement proposed by Shields considers the forces and the processes involved on determination of critical conditions for which the sediments will move due to the shear stress and the Reynolds number for a grain during the critical velocity (Fig. 1). If the critical conditions (considering velocity, discharge or deep) is achieved the sediments will move on. The Shields' curve is shown in Fig. 1b. If the combination of values is located below the curve, then the sediment will not be transported. If the combination is found above the curve, then the sediment will be transported.

## EGIAZAROFF ASSUMPTION

It is very hard to find sediments composed by homogeneous mixtures in natural streams, especially in mountain rivers where they vary in a large order of magnitude. Thus, Egiazaroff (1965) commented the incipient condition of motion is highly dependent on material's dimension and the

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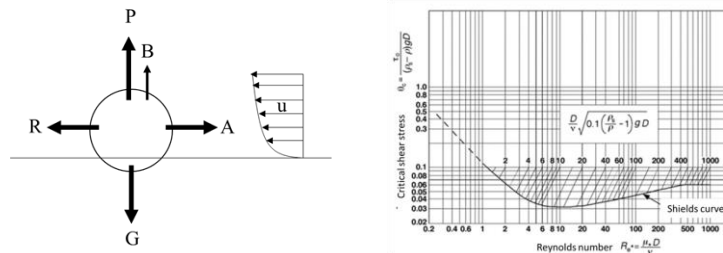


Fig. 1 – Scheme of forces considered in Shields theory and Shields' diagram

sheltering or hiding phenomena, in which small particles are protected by larger particles. Then, he proposed a hiding coefficient,  $\zeta_j$ , in order to compensate the critical shear stress in such condition:

$$\theta_{cj} = \theta_{cu} \times \zeta_j \quad (4)$$

where  $\theta_{cj}$  is the critical shear stress for the mixture;  $\theta_{cu}$  is the critical shear stress for  $d_{50}$ ; and  $\zeta_j$  is the sheltering coefficient. In his assumption, Egiazaroff (1965) proposed that the sheltering coefficient may be calculated by:

$$\zeta_j = \left( \frac{\log_{10} 19}{\log_{10} 19 \cdot \frac{d_j}{\bar{d}}} \right)^2 \quad (5)$$

where  $\zeta_j$  is the sheltering coefficient;  $d_j$  is the representative diameter of a considered fraction in the sediment distribution;  $\bar{d}$  is median diameter of the sediment mixture.

Later, Ashida and Michiue (1978) observed the expression (5) sub-estimates the mobility of small particles ( $d_j < 0.4\bar{d}$ ) although it works well for larger particles. So, they proposed a conditional function, as described below:

$$\zeta_j = 0.85 \times \left( \frac{\bar{d}}{d_j} \right)^2, \quad \text{if } \left( \frac{\bar{d}}{d_j} \right) < 0.4 \quad (6)$$

If the relation is larger than 0.4, equation (5) should be used.

The Egiazaroff's assumption was considerably neglected for some years, especially because they thought that it was not considering lift forces. However, Okazaki et al. (2001) demonstrated that lift forces were implicitly considered. Also, Okazaki et al. (2001) demonstrated that the Egiazaroff assumption has been presenting good results for nonuniform mixtures.

## CONCLUSIONS

The Egiazaroff equation was neglected for so many years, especially because it was thought the equation did not consider lift forces. However, Okazaki et al. (2001) demonstrated the lift forces were implicitly included in Egiazaroff's assumption. The equation is largely used in Asia, especially in Japan, where the most part of rivers are mountain environments presenting nonuniform mixtures. Thus, the authors strongly recommend the use of Egiazaroff equation in order to evaluate the sediment transport in mountain areas.

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