

Dynamic response analysis of truss footbridges under human load

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Abstract: Trusses are one of the most common types of structures employed industrially. Their applications go through bridges, pavilions, energy towers, footbridges, among others. The analysis of this type of structure presents several similarities, since they have in common the submission to both static and dynamic loads. The footbridges are characterized by being subjected to dynamic loads such as those of wind and those imposed by pedestrians. In this work the objective is to analyze a Warren and a Pratt truss footbridge subjected to the load caused by pedestrians. The models were analyzed by Newmark's method, in order to obtain the dynamic response of the structures in terms of vertical displacement and acceleration. The natural frequencies of the structures were also obtained. As a result of this work, an algorithm for a first analysis was generated, which will be used later in the application of heuristic methods to optimize the structure and characteristics of passive energy dissipation devices. The results shown a huge precision of the developed method based on response of the structure.

Keywords: Footbridges, Dynamic analysis, Newmark's method

INTRODUCTION

Vibration response is one of the most important information about a structure subjected to dynamic loads. In civil construction, the wind is object of special attention and it is subject of several standards that aims to orient engineers on the right way to develop a solid and safe structure. On the other hand, there is another kind of load that subject structures to a cycle, which can cause many structural problems as fatigue and even collapse: the human loads. It is a well-known type of load, and the improvement of technology has permitted to predict and even control the response of structures under this kind of load. Several researches have been published over the years approaching many topics on footbridges vibrations, among which can be cited: Soriano and Filho (1988), Živanović *et al.* (2005), Maraveas *et al.* (2015), with a literature review on footbridges under human excitation; Pedersen and Frier (2010), Qin *et al.* (2014), Miguel *et al.* (2015), Jiménez-Alonso and Sáez (2017), Van Nimmen *et al.* (2017) with different approaches, including optimization studies. In this way, this research aims to carry out a dynamic analysis of two footbridges, through the Newmark's method, which is the first step to future studies on optimization of footbridges and of characteristics of passive energy dissipation devices installed in this kind of structure in order to reduce the dynamic response.

PROBLEM FORMULATION

On this section, the theoretical approach of the problem is presented. Based on theory on human induced vibration, the load function of motion can be obtained. Next, the numerical method is implemented for calculate the vibration response of two types of structures: Warren and Pratt truss footbridges.

Human induced vibrations on footbridges

A well-known approach for human induced vibrations in structures is presented by Bachmann and Ammann (1987). The authors characterize motion for a single person as given by Eq. (1), in which: f_s is the pacing rate, considered $f_s = 2.0 \text{ Hz}$; G is the weight of the person, considered $G = 800 \text{ N}$; ΔG_1 , ΔG_2 and ΔG_3 are the load components (amplitude) of the first, second and third harmonic, respectively, where $\Delta G_1 = 0.4G$, $\Delta G_2 = 0.1G$ and $\Delta G_3 = 0.1G$; ϕ_2 and ϕ_3 are the phase angle of second and third harmonics, respectively, relatives to the first harmonic, considered $\phi_2 = \pi/2$ and $\phi_3 = \pi/2$.

$$F_p(t) = G + \Delta G_1 \sin(2\pi f_s t) + \Delta G_2 \sin(4\pi f_s t - \phi_2) + \Delta G_3 \sin(6\pi f_s t - \phi_3) \quad (1)$$

For more people walking on a footbridge, Bachmann and Ammann (1987) recommend the use of an enhancement factor $m = \sqrt{\lambda T_o}$ multiplied by Eq. (1), where λ is the mean flow rate, assumed $\lambda = 0.5 \text{ people/s}$ and

$\lambda = 1.0 \text{ people/s}$ for Warren and Pratt footbridges, respectively. T_o is the time necessary to cross the footbridge given by $T_o = L_f / v_s$.

For running, Bachmann and Ammann (1987) suggest using Eq. (2) that uses the same format as Eq. (1), i. e. as the sum of static weight G and harmonic load components.

$$F_p(t) = G + \sum_{n=0}^{\infty} \Delta G_n \cos \left[2\pi n f_s t \left(t - \frac{t_p}{2n} \right) \right] \quad (2)$$

In this equation, t_p represents the contact duration given by relation $t_p = 1.9744 f_s^{-2.151}$, which is an empirical equation proposed by Purroy (2017) based on studies of Bachmann and Ammann (1987).

Equations (1) and (2) are both presented in Fig. 1, representing the load-time function of a single person walking and running.

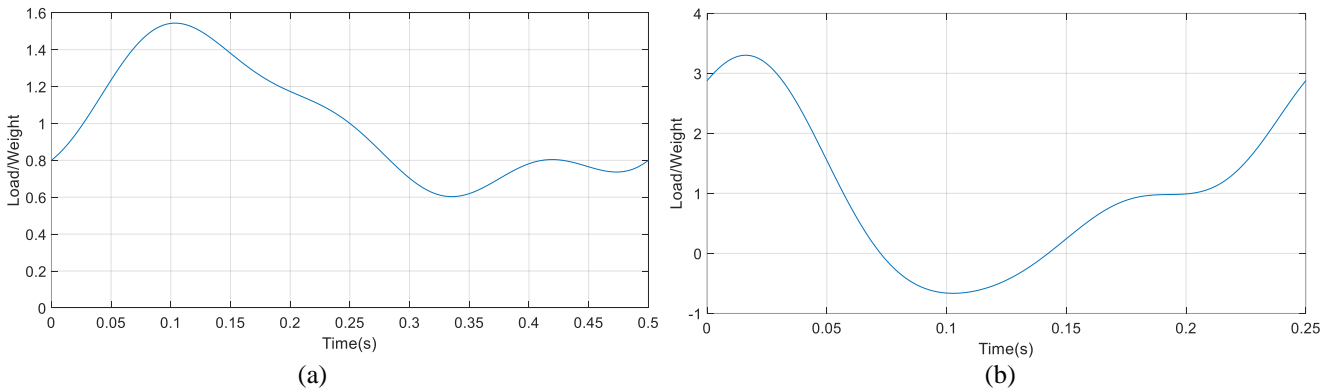


Figure 1 – Representation of the load function for a single person (a) walking and (b) running

Newmark’s method

The Newmark’s method (1959) is an implicit method used in numerical analysis for obtaining the approximating solutions of ordinary or partial time-dependent differential equations. The stability of the method in linear problems is obtained by setting two constant parameters: $\alpha = 0.25$ and $\delta = 0.50$. Starting with the differential equation of motion given by Eq. (3) of Rao (2011) for a damped system, which is written in matrix form, the implementation of Newmark’s method for the vibration problems were developed, while the mass, damping and stiffness matrices are obtained through finite element method.

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \quad (3)$$

Modal and dynamic analyses

Modal analysis can be used for determining both natural frequencies (eigenvalues) and mode shapes (eigenvectors), while a dynamic analysis can be used to obtain the response of the system under free or forced vibrations.

The structures taken as example on this paper have 58 and 64 degrees of freedom (D.O.F.), respectively. The global matrix will have their dimension changed proportionally when the number of D.O.F increase or decrease. The final global matrix is obtained by applying constraint conditions to the rows and columns corresponding to each D.O.F.

RESULTS AND DISCUSSION

Following previously approach, the method was implemented and its results can be seen in the next subsection. First, the focus is concentrated on modeling the structure. On this case, the footbridges studied by Miguel *et al.* (2015) are analyzed: a Warren truss footbridge (Fig. 2) and a Pratt truss footbridge (Fig. 3).

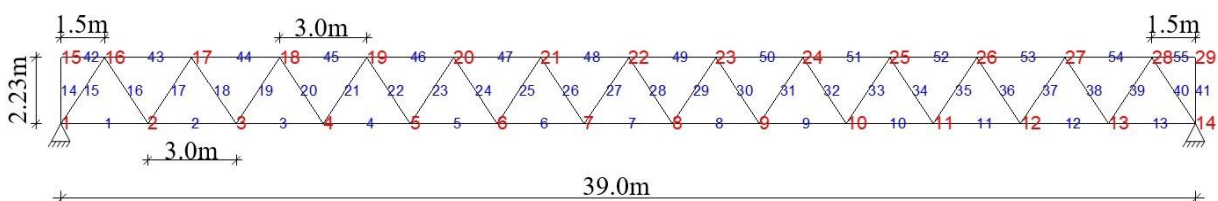


Figure 2 – Warren truss footbridge proposed by Miguel *et al.* (2015)

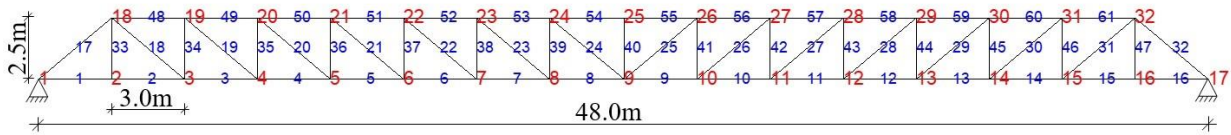


Figure 3 – Pratt truss footbridge proposed by Miguel *et al.* (2015)

Both structures were modeled in Ansys APDL and in Matlab, based on characteristics shown in Tab. 1.

Table 1 – Cross sectional areas of the members of the analyzed footbridges (Adapted from Miguel *et al.* (2015))

Group	Warren truss footbridge		Pratt truss footbridge	
	Member number	Area (m ²)	Member number	Area (m ²)
Inferior chord	1–13	0.0060	1–16	0.0090
Diagonals	14–41	0.0040	17–32	0.0065
Superior chord	42–55	0.0080	33–47	0.0055
Vertical members	–	–	48–61	0.0105

A comparative for the first five natural frequencies of Warren and Pratt truss footbridges obtained by modal analysis is shown in Tab. 2.

Table 2 – First five natural frequencies obtained by Ansys and Matlab

Frequency (Hz)	Warren truss footbridge			Pratt truss footbridge		
	Ansys APDL	Matlab	Difference %	Ansys APDL	Matlab	Difference %
1	5.9960	5.9969	0.0150	4.0016	4.0022	0.0150
2	16.036	16.0498	0.0860	11.385	11.3957	0.0939
3	33.916	33.9660	0.1472	24.370	24.4044	0.1410
4	41.245	41.3063	0.1484	29.755	29.8194	0.2160
5	55.977	56.0714	0.1684	41.043	41.0698	0.0652

Figure 4 shows the vertical (a) displacement and (b) acceleration for the central node 22 of the Warren footbridge, which has damping ratios for the first and second modes equal to 0.4%, considering 13 people. Maximum displacement and acceleration are 0.0067m and 9.09m/s², respectively.

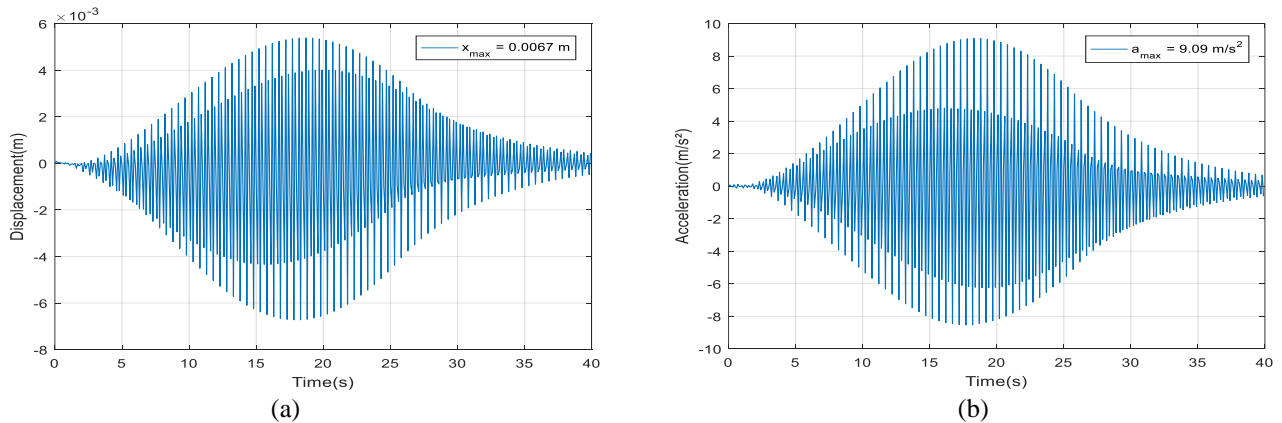


Figure 4 – Vertical (a) displacement and (b) acceleration in the central node 22 of the Warren footbridge under vibration by pedestrians walking.

In Fig. 5 are shown both vertical (a) displacement and (b) acceleration for the central node 25 of the Pratt footbridge, which has damping ratios for the first and second modes equal to 0.4%, considering 32 people. Maximum displacement and acceleration are 0.0094m and 7.19m/s², respectively.

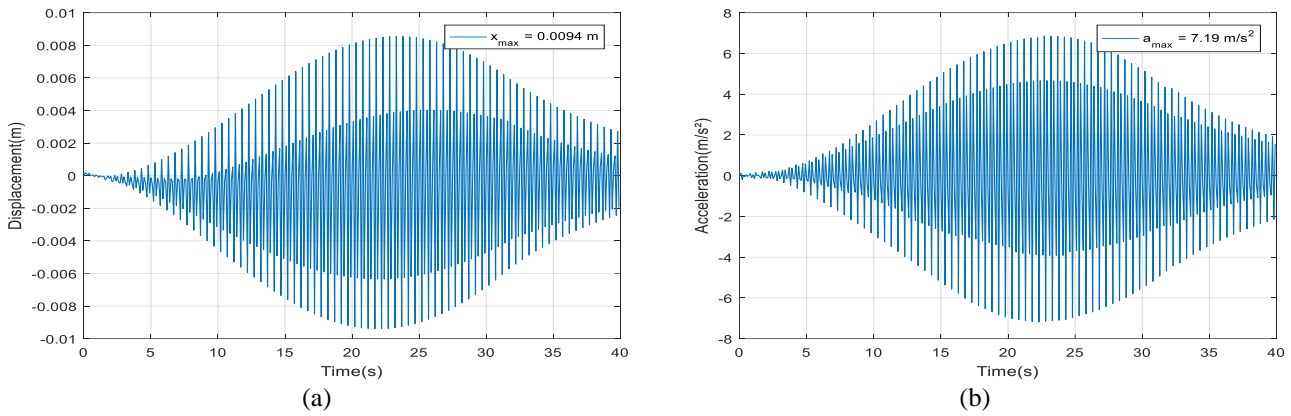


Figure 5 – Vertical (a) displacement and (b) acceleration in the central node 25 of the Pratt footbridge under vibration by pedestrians walking.

The behavior of both footbridges under harmonic load by running was also simulated. As explained by Schmidt (2018, p. 80), “since the applied load is harmonic and uniform, it represents a crowd marching in step, which is not realistic, but produces maximal response values”. The results were based on parameters for running given by Bachmann and Ammann (1987) and Bachmann *et al.* (1995).

Figure 6 shows the results for Warren footbridge, based on the following conditions: $f_s = 3.0 \text{ Hz}$, which is a harmonic frequency of its first natural frequency; $v_s = 5.0 \text{ m/s}$; $\Delta G_1 = 1.6G$, $\Delta G_2 = 0.7G$ and $\Delta G_3 = 0.2G$; $\phi_2 = \phi_3 = 0$; and $\lambda = 0.5 \text{ people/s}$, which results approximately in 4 people simultaneously on the footbridge. Maximum displacement and acceleration are 0.0140 m and 19.76 m/s^2 , respectively.

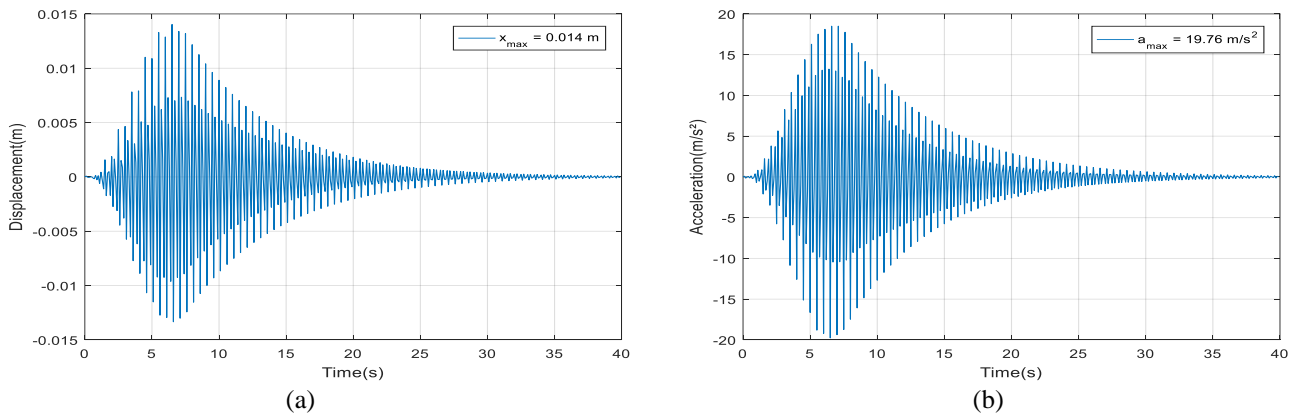


Figure 6 – Vertical (a) displacement and (b) acceleration in the central node 22 of the Warren footbridge under vibration by pedestrians running.

For Pratt footbridge were considered the following conditions: $f_s = 4.0 \text{ Hz}$, which is a harmonic frequency of its first natural frequency; $v_s = 7.0 \text{ m/s}$; $\Delta G_1 = 1.6G$, $\Delta G_2 = 0.7G$ and $\Delta G_3 = 0.2G$; $\phi_2 = \phi_3 = 0$; and $\lambda = 1.0 \text{ people/s}$, which results, approximately in 7 people simultaneously on the footbridge. Figure 7 shows the results for this case. Maximum displacement and acceleration are 0.0345 m and 21.94 m/s^2 , respectively.

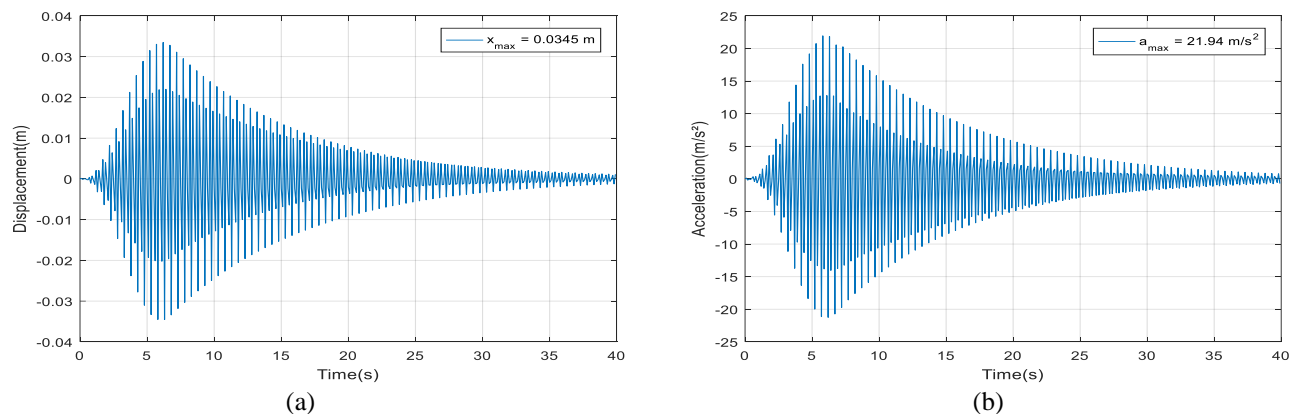


Figure 7 – Vertical (a) displacement and (b) acceleration in the central node 25 of the Pratt footbridge under vibration by pedestrians running.

Finally, it was performed static analysis based on finite element method, using plane truss elements. It was considered an equivalent load, corresponding to the number of people simultaneously on the footbridge multiplied by the weight of one person, applied at the central node. Table 3 shows the results for displacement on vertical direction obtained for the both footbridges, using Ansys APDL.

Table 3 – Vertical displacement under static load on central node

Node 22 of Warren footbridge			Node 25 of Pratt footbridge		
Number of people	Load [N]	Displacement [m]	Number of people	Load [N]	Displacement [m]
4	-3200	-0.763e-3	7	-5600	-1.625e-3
13	-10400	-2.480e-3	32	-25600	-7.431e-3

Results show a considerably less displacement for the structures under static loads. Dynamic loads subjected Warren and Pratt footbridges to displacements about 6 and 5 times larger than static loads, respectively.

CONCLUSIONS

In this paper an approach for human induced vibration on truss footbridges is followed based on Newmark's method. The method was used to obtain the dynamic response of the structures based on displacement and acceleration.

The results show a good approximation using Newmark's method. In terms of natural frequencies, modal analysis presented a difference of 0.216% between Ansys APDL and Matlab. On the same way, vertical displacement and acceleration presented a low difference between the values obtained by Miguel *et al.* (2015). On the extreme cases of static load, where all the vertical load was applied at the same node, both footbridges presented the expected behavior compared with dynamic cases.

Thus, the implemented method proved to be an accurate approach for future studies on optimization of footbridges and of characteristics of passive energy dissipation devices installed in this kind of structure in order to reduce the dynamic response.

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