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FRACTIONAL STATISTICS IN THE CHERN–SIMONS THEORY*

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A (2 + 1)-dimensional theory of charged scalar particles coupled to an abelian gauge field with a Chern–Simons term in the action is canonically quantized in a generalized linear non-covariant gauge. We find, in all gauges, charged excitations obeying bosonic and fractional statistics. On general grounds, the fields are seen to develop translational and rotational anomalies. The introduction in the action of the conventional $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ suppresses the appearance of anyons.

1. Introduction

For the (2 + 1)-dimensional Chern–Simons theory★

$$\mathscr{L} = (\overline{D_\mu \phi})(D^\mu \phi) + \frac{\theta}{4\pi^2}\epsilon_{\mu\nu\lambda}A^\mu \partial^\nu A^\lambda, \tag{1}$$

describing charged scalar particles minimally coupled to an abelian gauge field, it was recently shown [1] that the Coulomb–superaxial gauge transformation is singular. As a consequence, ϕ obeys a bosonic equal-time algebra in the Coulomb gauge and a graded equal-time algebra in the superaxial gauge. It was also demonstrated [1] that the absence of the term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ is at the root of this statistical transmutation [2–4].

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★ The fully antisymmetric tensor $\epsilon^{\mu\nu\lambda}$ is normalized such that $\epsilon^{012} = +1$, the metric is $g^{00} = -g^{11} = -g^{22} = +1$ and $\overline{\phi}$ denotes the complex conjugate of ϕ. Contrary to what was done in ref. [1], the coupling constant e ($D^\mu \equiv \partial^\mu - ieA^\mu$) is not assumed to be dimensionless and equal to one but rather $d[e] = \mathrm{cm}^{-1/2}$. Then, the engineering dimensions of the fields are $d[\phi] = d[\overline{\phi}] = d[A^\mu] = \mathrm{cm}^{-1/2}$, while for the Chern–Simons constant one finds $d[\theta] = \mathrm{cm}^{-1}$. The term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ ($F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$) can now be added to the lagrangian (1) without altering the field dimensions.

In this paper we address the following questions:

(a) Are the anyons [2] physical excitations or gauge artefacts?

(b) Does the field ϕ develop rotational and/or translational anomalies in all gauges?

(c) Does the addition of the term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ to the lagrangian (1) suppress the appearance of anyons in all gauges?

To answer these questions, we start by introducing the real variables B_μ, η and φ through the following definitions:

$$\phi(x) \equiv \frac{1}{\sqrt{2}} \exp[i\varphi(x)]\, \eta(x)\,, \tag{2}$$

$$A^\mu(x) \equiv B^\mu(x) + \frac{1}{e}\partial^\mu\varphi(x)\,. \tag{3}$$

In terms of the new fields the lagrangian (1) is found to read

$$\mathscr{L} = \frac{\theta}{4\pi^2}\epsilon_{\mu\nu\lambda}B^\mu\partial^\nu B^\lambda + \frac{\theta}{4\pi^2 e}\epsilon_{\mu\nu\lambda}(\partial^\mu\varphi)\partial^\nu B^\lambda + \tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \tfrac{1}{2}e^2\eta^2 B^\mu B_\mu\,. \tag{4}$$

Notice that, unlike the case of scalar electrodynamics, the Chern–Simons lagrangian under analysis cannot be solely written in terms of the gauge invariant fields B^μ and η. The surface term in eq. (4), containing the gauge-dependent field φ, does not contribute to the Lagrange equations of motion but will prove essential for the appearance of anyons.

After this change of description, and only then, the theory becomes effectively quantizable in the generalized linear gauge

$$\chi \equiv \varphi(x^0, \boldsymbol{x}) + \int d^2y\, K_i(\boldsymbol{x}, \boldsymbol{y}) B^i(x^0, \boldsymbol{y}) = 0\,, \tag{5}$$

where K_1 and K_2 are real kernels not depending on time. In particular, the Dirac bracket quantization procedure (DBQP) [5,6] is implementable, thus providing the basic equal-time commutation relations (ETCRs). Furthermore, composite operators, such as the Poincaré generators and the electric charge, can be explicitly constructed. Within the present approach, the fields of interest, ϕ and $\phi^\dagger$ are also composite objects to be built from the basic fields, as indicated in eq. (2). Their statistical properties, in the generalized linear gauge (5), are easily found by exploring the basic ETCRs. The behavior of ϕ and $\phi^\dagger$ under Poincaré transformations and under rotations in charge space is similarly obtained. All these developments are presented in sect. 2.

The Coulomb, superaxial and unitary gauges are separately analyzed in sect. 3. These three gauges are, of course, particular cases of eq. (5). As far as the

Coulomb and superaxial gauges are concerned, we recover the results in ref. [1] and examine them further on. The formulation of the Chern–Simons theory in the unitary gauge is, as far as we know, new and starts from the observation that, in this gauge, the field ϕ becomes singular. We nevertheless show that two regular charged fields, describing bosons and anyons, can be constructed.

Sect. 4 is dedicated to studying the modifications induced by the term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ when added to the lagrangian (1), while sect. 5 contains the conclusions.

2. Statistical properties in the generalized linear gauge

Within the hamiltonian framework the system (4) is characterized by the canonical hamiltonian

$$H_0 = \int d^2y \left[\tfrac{1}{2}\pi_\eta\pi_\eta + \tfrac{1}{2}(\partial^j\eta)(\partial^j\eta) + \tfrac{1}{2}e^2\eta^2B^0B^0 + \tfrac{1}{2}e^2\eta^2B^jB^j\right], \tag{6}$$

the primary second-class constraints

$$\mathscr{P}_0 \equiv \pi_0^B \approx 0, \tag{7a}$$

$$\mathscr{P}_i \equiv \pi_i^B - \frac{\theta}{4\pi^2}\epsilon^{ij}\left(B^j + \frac{1}{e}\partial^j\varphi\right) \approx 0, \qquad i = 1,2, \tag{7b}$$

the primary first-class constraint

$$\mathscr{P}_3 \equiv \pi_\varphi - \frac{\theta}{4\pi^2 e}\epsilon^{ij}\partial^i B^j \approx 0, \tag{8}$$

and the secondary second-class constraint

$$\mathscr{S}_0 \equiv e^2\eta^2B^0 + \partial^i\pi_i^B + \frac{\theta}{4\pi^2}\epsilon^{ij}\partial^iB^j \approx 0. \tag{9}$$

Here we have designated by π_μ^B, π_η and π_φ the momenta canonically conjugate to B^μ, η and φ, respectively. According to Dirac's conjecture [7], all first-class constraints act as independent generators of gauge transformations. Hence, it follows from (8) that η, π_η, B^μ, π_0^B and π_φ are gauge invariant phase-space variables, while φ and π_i^B change under gauge transformations.

The quantization of the model through the DBQP is straightforward but algebraically cumbersome. The enlarged set of constraints resulting from adding the gauge condition (5) to the original set $\{\mathscr{P}_0 \approx 0,\ \mathscr{P}_i \approx 0,\ i = 1,2,\ \mathscr{P}_3 \approx 0,\ \mathscr{S}_0 \approx 0\}$ is second-class, which enables us to introduce the Dirac bracket in the standard manner. The classical–quantum transition is then performed by abstracting the equal-time commutators (ETCs) from the corresponding Dirac brackets, the constraints and gauge conditions thereby translating into strong operator relations [5,6]. For the non-vanishing ETCs one, thus, obtains ($\hbar = 1$)*

$$[B^0(x), B^j(y)] = \frac{i}{e^2\eta^2(x)}\partial_x^j\delta(x-y), \tag{10a}$$

$$[B^0(x), \varphi(y)] = -\frac{i}{e^2\eta^2(x)}\partial_x^j K_j(y,x), \tag{10b}$$

$$[B^0(x), \pi_j^{\mathrm{B}}(y)] = -\frac{i\theta}{4\pi^2e^2\eta^2(x)}\epsilon^{ij}\left(\partial_x^i\delta(x-y) - \frac{1}{e}\partial_y^i\partial_x^k K_k(y,x)\right), \tag{10c}$$

$$[B^0(x), \pi_\eta(y)] = -2i\frac{B^0(x)}{\eta(x)}\delta(x-y), \tag{10d}$$

$$[B^i(x), B^j(y)] = \frac{2\pi^2 i}{\theta}\epsilon^{ij}\delta(x-y), \tag{10e}$$

$$[B^j(x), \varphi(y)] = -\frac{2\pi^2 i}{\theta}\epsilon^{ji}K_i(y,x), \tag{10f}$$

$$[B^i(x), \pi_j^{\mathrm{B}}(y)] = \frac{i}{2}\delta^{ij}\delta(x-y) + \frac{i}{2e}\epsilon^{il}\epsilon^{kj}\partial_y^k K_l(y,x), \tag{10g}$$

$$[B^j(x), \pi_\varphi(y)] = -\frac{i}{2e}\partial_x^j\delta(x-y), \tag{10h}$$

$$[\eta(x), \pi_\eta(y)] = i\delta(x-y), \tag{10i}$$

$$[\varphi(x), \varphi(y)] = \frac{2\pi^2 i}{\theta}\epsilon^{ij}\int \mathrm{d}^2z\, K_i(x,z)K_j(y,z), \tag{10j}$$

$$[\varphi(x), \pi_j^{\mathrm{B}}(y)] = -\frac{i}{2}K_j(x,y) + \frac{i}{2e}\epsilon^{ji}\epsilon^{lm}\int \mathrm{d}^2z\, K_l(x,z)\partial_y^i K_m(y,z), \tag{10k}$$

$$[\varphi(x), \pi_\varphi(y)] = -\frac{i}{2e}\partial_y^j K_j(x,y), \tag{10l}$$

$$[\pi_i^{\mathrm{B}}(x), \pi_j^{\mathrm{B}}(y)] = \frac{i\theta}{8\pi^2}\left(\epsilon^{ij}\delta(x-y) - \frac{1}{e}\epsilon^{ik}\partial_x^k K_j(x,y) - \frac{1}{e}\epsilon^{kj}\partial_y^k K_i(y,x) - \frac{1}{e^2}\epsilon^{ik}\epsilon^{lj}\epsilon^{rs}\int \mathrm{d}^2z\, \partial_x^k K_r(x,z)\partial_y^l K_s(y,z)\right), \tag{10m}$$

$$[\pi_j^{\mathrm{B}}(x), \pi_\varphi(y)] = -\frac{i\theta}{8\pi^2e^2}\epsilon^{ji}\partial_x^i\left(\delta(x-y) + \partial_y^k K_k(x,y)\right), \tag{10n}$$

* To simplify the notation we shall not distinguish between a quantum field operator and its classical counterpart. Whenever possible, the time label will be omitted in the field argument.

where the factors $1/\eta(x)$ and $1/\eta^2(x)$ are to be understood as regularized in the sense of Bardakci and Samuel [8]. This algebra carries, by construction, the constraints and gauge conditions as strong operator relations [5,6]. Moreover, the ETCs involving only gauge-independent variables do not depend on the K's, as must be the case.

We next assert that the hermitian density operators*

$$\Theta^{00} = \tfrac{1}{2}\pi_\eta \pi_\eta + \tfrac{1}{2}(\partial^j \eta)(\partial^j \eta) + \tfrac{1}{2} e^2 \eta^2 B^0 B^0 + \tfrac{1}{2} e^2 \eta^2 B^j B^j \,, \tag{11a}$$

$$\Theta^{0k} = \pi_\eta \cdot \partial^k \eta + e^2 \eta^2 B^0 \cdot B^j \,, \tag{11b}$$

verify, under eqs. (10), the Dirac–Schwinger equation [9,10]

$$\left[\Theta^{00}(x), \Theta^{00}(y)\right] = -i\left(\Theta^{0k}(x) + \Theta^{0k}(y)\right)\partial_k^x \delta(x-y) \,. \tag{12}$$

Then, the Poincaré generators can be written as follows:

(i) Momenta:

$$P^0 = \int \mathrm{d}^2 x \, \Theta^{00}(x) \equiv H \,, \tag{13}$$

$$P^k = \int \mathrm{d}^2 x \, \Theta^{0k}(x) \,, \tag{14}$$

(ii) Rotations:

$$J = \int \mathrm{d}^2 x \, \epsilon^{jk} x^j \Theta^{0k}(x) \,, \tag{15}$$

(iii) Boosts:

$$J^{0k} = x^0 P^k - K^k \,, \tag{16}$$

$$K^k = \int \mathrm{d}^2 x x^k \Theta^{00}(x) \,. \tag{17}$$

It is easy to check that the Lagrange equations of motion deriving from eq. (4) can be recovered from the Heisenberg equations of motion deriving from eqs. (13) and (10). Moreover, for the Poincaré algebra to hold, the kernels K_i, $i = 1,2$, are

* We secure the hermiticity of the composite operators calling for symmetrization in the products of Bose fields. As usual, $A \cdot B \equiv \frac{1}{2}(AB + BA)$.

required to satisfy the boundary conditions

$$\lim_{|x|\to\infty} K_{1,2}(y,x) \to 0, \tag{18a}$$

$$\lim_{|x|\to\infty} \partial_x^i K_{1,2}(y,x) \to 0, \tag{18b}$$

$$\lim_{|x|\to\infty} \partial_y^i K_{1,2}(y,x) \to 0, \tag{18c}$$

which compatibilize the ETCRs (10) with the vanishing of all gauge-independent fields at spatial infinity. We emphasize that translational invariance for the K's is not assumed.

On the other hand, the quasi-invariance of (4) under the global transformation $\varphi \to \varphi + \alpha$ (α = constant) leads to the existence of the conserved electric current

$$\xi^\mu = e^2\eta^2 B^\mu, \tag{19}$$

with the corresponding electric charge given by

$$Q \equiv \int d^2x\, \xi^0 = e^2 \int d^2x\, \eta^2 B^0. \tag{20}$$

From eqs. (19) and (10) one arrives at

$$\left[\xi^0(x), \xi^0(y)\right] = 0, \tag{21a}$$

$$\left[\xi^0(x), \xi^k(y)\right] = ie^2\eta^2(y)\partial_x^k \delta(x-y), \tag{21b}$$

where the hermitian character of η secures $\langle 0|\eta^2(y)|0\rangle > 0$, as required [11].

Having completed the quantization of (4) we turn our attention to the fields of interest ϕ, $\phi^\dagger$ and their respective conjugate momenta p, $p^\dagger$. The structure of ϕ and $\phi^\dagger$ in terms of the basic fields is already specified at (2), while from (1) and (3) one obtains

$$p(x) = \frac{1}{\sqrt{2}}\left[ieB^0(x)\eta(x) + \pi_\eta(x)\right]\exp[-i\varphi(x)]. \tag{22}$$

Due to the lack of commutativity of B^0 and φ [see eq. (10b)], the order of factors in eq. (22) is fixed. By using eqs. (2), (22) and (10) one finds the equal-time algebra

$$\phi(x)\phi(y) - \exp\left[\frac{i}{\theta}\Delta(x,y)\right]\phi(y)\phi(x) = 0, \tag{23a}$$

$$\phi(x)\phi^{\dagger}(y) - \exp\left[-\frac{i}{\theta}\Delta(x,y)\right]\phi(y)^{\dagger}\phi(x) = 0, \tag{23b}$$

$$\begin{aligned}\phi(x)p(y) &- \exp\left[-\frac{i}{\theta}\Delta(x,y)\right]p(y)\phi(x)\\ &= \frac{i}{2}\delta(x-y) - \frac{i}{2e}\phi(x)(\phi(y))^{-1}\partial_y^j K_j(x,y),\end{aligned} \tag{23c}$$

$$\begin{aligned}\phi(x)p^{\dagger}(y) &- \exp\left[\frac{i}{\theta}\Delta(x,y)\right]p^{\dagger}(y)\phi(x)\\ &= \frac{i}{2}\delta(x-y)\phi(x)(\phi^{\dagger}(y))^{-1}\\ &\quad + \frac{i}{2e}\exp\left[\frac{i}{\theta}\Delta(x,y)\right]\phi(x)(\phi^{\dagger}(y))^{-1}\partial_y^j K_j(x,y),\end{aligned} \tag{23d}$$

$$\begin{aligned}p(x)p(y) &- \exp\left[\frac{i}{\theta}\Delta(x,y)\right]p(y)p(x)\\ &= -\frac{i}{2e}\exp\left[\left(\frac{i}{\theta}\right)\Delta(x,y)\right]p(y)(\phi(x))^{-1}\partial_x^j K_j(y,x)\\ &\quad + \frac{i}{2e}p(x)(\phi(y))^{-1}\partial_y^j K_j(x,y),\end{aligned} \tag{23e}$$

$$\begin{aligned}p(x)p^{\dagger}(y) &- \exp\left[-\frac{i}{\theta}\Delta(x,y)\right]p^{\dagger}(y)p(x)\\ &= \frac{i}{2}\left[\delta(x-y) + \frac{1}{e}\partial_x^j K_j(y,x)\right](\phi(x))^{-1}p^{\dagger}(y)\\ &\quad - \frac{i}{2}\left[\delta(x-y) + \frac{1}{e}\partial_y^j K_j(x,y)\right]p(x)(\phi^{\dagger}(y))^{-1}\\ &\quad - \frac{1}{4}\Big\{\delta(0)\delta(x-y) + \frac{2}{e}\delta(x-y)\partial_x^i K_i(y,x)\\ &\qquad + \frac{1}{e^2}\left[\partial_y^i K_i(x,y)\right]\left[\partial_x^j K_j(y,x)\right]\Big\}(\phi(x))^{-1}(\phi^{\dagger}(y))^{-1},\end{aligned} \tag{23f}$$

where★

$$\Delta(x, y) \equiv i\theta[\varphi(x), \varphi(y)] = -2\pi^2\epsilon^{ij}\int d^2z\, K_i(x, z) K_j(y, z)\,. \tag{24}$$

Meanwhile, under rotations in charge space ϕ transforms according to [see eqs. (2), (20) and (10)]

$$[\phi(x), Q] = -\phi(x)\int d^2y\, \partial_y^j K_j(x, y)\,. \tag{25}$$

By combining eqs. (23) and (25) one arrives to some interesting conclusions. We observe that for any gauge in the set

$$\partial_y^j K_j(x, y) = -e\delta(x-y)\,, \tag{26}$$

the corresponding field ϕ describes charged excitations with charge e, while, at the same time, the algebra (23) collapses into

$$\phi(x)\phi(y) - \exp\left[\frac{i}{\theta}\Delta(x, y)\right]\phi(y)\phi(x) = 0\,, \tag{27a}$$

$$\phi(x)\phi^{\dagger}(y) - \exp\left[-\frac{i}{\theta}\Delta(x, y)\right]\phi^{\dagger}(y)\phi(x) = 0\,, \tag{27b}$$

$$\phi(x)p(y) - \exp\left[-\frac{i}{\theta}\Delta(x, y)\right]p(y)\phi(x) = i\delta(x-y)\,, \tag{27c}$$

$$\phi(x)p^{\dagger}(y) - \exp\left[\frac{i}{\theta}\Delta(x, y)\right]p^{\dagger}(y)\phi(x) = 0\,, \tag{27d}$$

$$p(x)p(y) - \exp\left[\frac{i}{\theta}\Delta(x, y)\right]p(y)p(x) = 0\,, \tag{27e}$$

$$p(x)p^{\dagger}(y) - \exp\left[-\frac{i}{\theta}\Delta(x, y)\right]p^{\dagger}(y)p(x) = 0\,, \tag{27f}$$

where $\delta(0)$ in eq. (23f) has cancelled against the other two terms in the curly bracket. Hence, for all those gauges in set (26), the charged excitations described by the field ϕ will obey fractional statistics if $\Delta(x, y) \neq 0$. Needless to say, when

★ Notice that $\Delta(x, y) = -\Delta(y, x)$ and, therefore, $\Delta(x, x) = 0$, securing the consistency of the algebra (23) under the operation of hermitian conjugation.

$\Delta(x, y) = 0$ the field ϕ describes charged bosons. We stress the relevant role played by the commutator $[\varphi(x), \varphi(y)]$ in the appearance of anyons [see eq. (24)].

On the other hand, for the gauges not in the set (26), the algebra (23) becomes singular. This calls for the introduction of new composite fields verifying a regular equal-time algebra. This situation will be illustrated in sect. 3 in connection with the unitary gauge.

We close this section by investigating the behavior of ϕ under spatial rotations and translations. For obvious reasons of regularity, we shall limit our analysis to those gauges in the set (26). The application of the generators (14) and (15) to the field ϕ is found to yield

$$[\phi(x), P^k] = i\partial^k\phi(x) + \phi(x)\int d^2y\left\{e\delta(x-y)\left[B^k(y) + \frac{1}{e}\partial_y^k\varphi(y)\right]\right.$$
$$\left. - \frac{2\pi^2}{\theta}\epsilon^{kj}K_j(x,y)\xi^0(y)\right\} + \frac{2\pi^2 e}{\theta}\epsilon^{kj}\phi(x)K_j(x,x) \quad (28)$$

and

$$[\phi(x), J] = i\epsilon^{jk}x^j\partial^k\phi(x) + \frac{2\pi^2}{\theta}\phi(x)\int d^2y\, y^j K_j(x,y)\xi^0(y)$$
$$+ e\epsilon^{jk}\phi(x)x^j\left[B^k(x) + \frac{1}{e}\partial_x^k\varphi(x)\right] - e\frac{2\pi^2}{\theta}\phi(x)x^jK_j(x,x). \quad (29)$$

Had we kept $\hbar \neq 1$, the last terms of (28) and (29) would have been of order $\hbar^2$.

Hence, for the gauges in the set (26) the field ϕ develops, in general, translational and rotational anomalies [12]. As we shall see in sect. 3, the absence of anyons does not necessarily imply the absence of anomalies [1].

3. The Coulomb, superaxial and unitary gauges*

3.1. THE COULOMB GAUGE

By combining (3) and the Coulomb condition, $\partial^j A^{C,j} = 0$, one arrives at

$$\varphi^C(x) + e\int d^2y\, \partial_x^j G(x-y)B^j(y) = 0, \quad (30)$$

* Gauge-dependent field variables belonging to Coulomb, superaxial and unitary gauges will be denoted with the superscripts "C", "S" and "U", respectively.

where $\nabla_x^2 G(\boldsymbol{x}-\boldsymbol{y}) = \delta(\boldsymbol{x}-\boldsymbol{y})$, i.e.

$$G(\boldsymbol{x}-\boldsymbol{y}) = (4\pi)^{-1} \ln(|\boldsymbol{x}-\boldsymbol{y}|^2) + \text{const}. \tag{31}$$

From (5) and (30) one obtains the Coulomb gauge kernel

$$K_j^{\mathrm{C}}(\boldsymbol{x}, \boldsymbol{y}) = e\partial_z^j G(\boldsymbol{x}-\boldsymbol{y}), \tag{32}$$

which together with (24) leads to

$$\Delta^{\mathrm{C}}(\boldsymbol{x}, \boldsymbol{y}) = 0. \tag{33}$$

Since $K_j^{\mathrm{C}}(\boldsymbol{x}-\boldsymbol{y})$ obeys (26), one concludes that, in the Coulomb gauge, the graded algebra (27) collapses into a bosonic algebra. Correspondingly, ϕ^{C} describes charged bosons [1].

As for the anomalies, we start by noticing that (7b), (9), (19) and $\partial^j A^{\mathrm{C}j} = 0$ lead to

$$B^k + \frac{1}{e}\partial^k \varphi^{\mathrm{C}} = \frac{2\pi^2}{\theta}\epsilon^{kj} \int \mathrm{d}^2 y\, \partial_x^j G(\boldsymbol{x}-\boldsymbol{y}) \xi^0(\boldsymbol{y}), \tag{34}$$

which in turn allows one to cast (28) and (29) respectively, as follows:

$$\left[\phi^{\mathrm{C}}(\boldsymbol{x}), P^k\right] = i\partial^k \phi(\boldsymbol{x}), \tag{35}$$

$$\left[\phi^{\mathrm{C}}(\boldsymbol{x}), J\right] = i\epsilon^{jk} x^j \partial^k \phi + \frac{\pi e}{\theta}\phi(\boldsymbol{x}) Q - \frac{\pi e^2}{\theta}\phi(\boldsymbol{x}), \tag{36}$$

where Cauchy principal value regularization has been used. Therefore, ϕ^{C} develops rotational but not translational anomalies [1, 12].

3.2. THE SUPERAXIAL GAUGE

The superaxial gauge is specified by the conditions [1, 13]

$$A^{\mathrm{S},2}(x^0, x^1, x^2) = 0, \tag{37a}$$

$$A^{\mathrm{S},1}\left(x^0, x^1, x_{(0)}^2\right) = 0, \tag{37b}$$

$$A^{\mathrm{S},0}\left(x^0, x_{(0)}^1, x_{(0)}^2\right) = A^{\mathrm{C},0}\left(x^0, x_{(0)}^1, x_{(0)}^2\right). \tag{37c}$$

Here, $\boldsymbol{x}_{(0)} \equiv (x_{(0)}^1, x_0^2)$ denotes some arbitrary fixed point. A little thought reveals that this time

$$K_1^S(\boldsymbol{x},\boldsymbol{y}) = -e\Omega\left(x^1, x_{(0)}^1; y^1\right)\delta\left(y^2 - x_{(0)}^2\right) - e\partial_y^1 G(\boldsymbol{x}_{(0)} - \boldsymbol{y}), \tag{38a}$$

$$K_2^S(\boldsymbol{x},\boldsymbol{y}) = -e\Omega\left(x^2, x_{(0)}^2; y^2\right)\delta\left(y^1 - x^1\right) - e\partial_y^2 G(\boldsymbol{x}_{(0)} - \boldsymbol{y}), \tag{38b}$$

where

$$\Omega(x,y;z) \equiv \int_y^x \mathrm{d}u\, \delta(u-z) = \tilde{\theta}(z-y) - \tilde{\theta}(z-x) \tag{39}$$

and $\tilde{\theta}(x)$ is the Heaviside step function.

It is straightforward to check that $K_j^S(\boldsymbol{x},\boldsymbol{y})$ verifies (26) and that, in this case, (24) yields

$$\begin{aligned}\Delta^S(\boldsymbol{x},\boldsymbol{y}) = &-2\pi^2 e^2\Big[\Omega\left(x^1, x_{(0)}^1; y^1\right)\Omega\left(y^2, x_{(0)}^2; x_{(0)}^2\right) \\ &\qquad -\Omega\left(y^1, x_{(0)}^1; x^1\right)\Omega\left(x^2, x_{(0)}^2; x_{(0)}^2\right)\Big] \\ &+\pi e^2\left[\varepsilon\left(x_{(0)}^1 - y^1\right)\arctan\left(\frac{x_{(0)}^2 - y^2}{|x_{(0)}^1 - y^1|}\right)\right. \\ &\qquad \left. -\varepsilon\left(x_{(0)}^1 - x^1\right)\arctan\left(\frac{x_{(0)}^2 - x^2}{|x_{(0)}^1 - x^1|}\right)\right],\end{aligned} \tag{40}$$

where $\varepsilon(x)$ is the sign function. The ambiguities in $\Omega(x,y;y)$ are circumvented by adopting the regularization $\tilde{\theta}(0) = 1/2$. Then, after some rearrangements, eq. (40) can be cast as

$$\begin{aligned}\Delta^S(\boldsymbol{x},\boldsymbol{y}) = \pi e^2\Bigg\{&\varepsilon\left(x^1 - y^1\right)\left[\arctan\left(\frac{x_{(0)}^2 - y^2}{|x^1 - y^1|}\right) + \arctan\left(\frac{x_{(0)}^2 - x^2}{|x^1 - y^1|}\right)\right] \\ &-\varepsilon\left(x_{(0)}^2 - y^2\right)\left[\arctan\left(\frac{x_{(0)}^1 - y^1}{|x_{(0)}^2 - y^2|}\right) - \arctan\left(\frac{x^1 - y^1}{|x_{(0)}^2 - y^2|}\right)\right] \\ &+\varepsilon\left(x_{(0)}^2 - x^2\right)\left[\arctan\left(\frac{x^1 - y^1}{|x_{(0)}^2 - x^2|}\right) + \arctan\left(\frac{x_{(0)}^1 - x^1}{|x_{(0)}^2 - x^2|}\right)\right]\Bigg\} \neq 0,\end{aligned} \tag{41}$$

in agreement with previous results [1].

Therefore, it follows from eqs. (26) and (27) that ϕ^{S} describes charged excitations obeying fractional statistics. The existence of a gauge transformation linking the Coulomb and superaxial gauges [1] enables one to think of ϕ^{S} as of a composite Coulomb gauge operator. The same is, of course, true for ϕ^{C} with respect to the superaxial gauge. Hence, charged anyons and bosons are present in both gauges.

The specialization of eqs. (28) and (29) to the case of the superaxial gauge is not very meaningful because of the peculiar structure of the gauge conditions (37).

3.3. THE UNITARY GAUGE

The unitary gauge is defined by the condition [14, 15]

$$\varphi^{U}(x) = 0, \tag{42}$$

implying that [see eq. (5)]

$$K_i^{U}(x, y) = 0. \tag{43}$$

Hence, $K_i^{U}(x, y)$ does not satisfy eq. (26). One may erroneously conclude from (42) that ϕ^{U} is a neutral field, when what really happens is that ϕ^{U} and p^{U} are not well-defined quantum mechanical operators [see eq. (23f)]. This calls for the introduction of new fields. One may choose

$$\tilde{\phi}^{U}(x) \equiv \exp\left[-i\int d^2y K_j^{C}(x, y) B^j(y)\right] \frac{\eta(x)}{\sqrt{2}}, \tag{44a}$$

$$\tilde{p}^{U}(x) \equiv \frac{1}{\sqrt{2}}\left[ieB^0(x)\eta(x) + \pi_\eta(x)\right]\exp\left[-i\int d^2y K_j^{C}(x, y) B^j(y)\right], \tag{44b}$$

which are, respectively, nothing but the Coulomb fields $\phi^{C}(x)$, $p^{C}(x)$ now considered as composite objects in the unitary gauge. Therefore, $\tilde{\phi}^{U}(x)$ and $\tilde{p}^{U}(x)$ are regular field operators describing charged bosonic excitations. Needless to say, the fields $\phi^{S}(x)$ and $p^{S}(x)$ can also be thought as regular composite objects in the unitary gauge. One can, namely, define

$$\hat{\phi}^{U}(x) \equiv \exp\left[-i\int d^2y K_j^{S}(x, y) B^j(y)\right] \frac{\eta(x)}{\sqrt{2}}, \tag{45a}$$

$$\hat{p}^{U}(x) \equiv \frac{1}{\sqrt{2}}\left[ieB^0(x)\eta(x) + \pi_\eta(x)\right]\exp\left[-i\int d^2y K_j^{S}(x, y) B^j(y)\right], \tag{45b}$$

Then, charged bosons and anyons also emerge in the unitary gauge.

We shall next study the modifications induced by the term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ when added to the lagrangian (1).

4. The suppression of fractional statistics

We start by adding to eq. (1) the conventional term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$. The lagrangian thus obtained, when rewritten in terms of the variables B^{μ}, η and φ, is found to read

$$\mathscr{L}' = -\tfrac{1}{4}B^{\mu\nu}B_{\mu\nu} + \tfrac{1}{2}(\partial_{\mu}\eta)(\partial^{\mu}\eta) + \tfrac{1}{2}e^2\eta^2 B_{\mu}B^{\mu} + \frac{\theta}{4\pi^2}\epsilon_{\mu\nu\lambda}B^{\mu}(\partial^{\nu}B^{\lambda})$$
$$+ \frac{\theta}{4\pi^2 e}\epsilon_{\mu\nu\lambda}(\partial^{\mu}\varphi)(\partial^{\nu}B^{\lambda}), \tag{46}$$

where $B^{\mu\nu} \equiv \partial^{\mu}B^{\nu} - \partial^{\nu}B^{\mu}$. Within the hamiltonian framework, the new system is characterized by the canonical hamiltonian

$$H_0' = \int d^2y \Big[\tfrac{1}{2}\pi_i^{\mathrm{B}}\pi_i^{\mathrm{B}} + \tfrac{1}{2}\pi_{\eta}\pi_{\eta} + \tfrac{1}{4}B^{ij}B^{ij} + \tfrac{1}{2}(\partial^j\eta)(\partial^j\eta)$$
$$+ \tfrac{1}{2}e^2\eta^2 B^0 B^0 + \tfrac{1}{2}e^2\eta^2 B^j B^j - \frac{\theta}{4\pi^2}\epsilon^{ij}\pi_i^{\mathrm{B}}\Big(B^j + \frac{1}{e}\partial^j\varphi\Big)\Big], \tag{47}$$

the primary second-class constraint

$$\mathscr{P}_0' \equiv \pi_0^{\mathrm{B}} \approx 0, \tag{48}$$

the primary first-class constraint

$$\mathscr{P}_3' \equiv \pi_{\varphi} - \frac{\theta}{4\pi^2 e}\epsilon^{ij}\partial^i B^j \approx 0, \tag{49}$$

and the secondary second-class constraint

$$\mathscr{S}_0' \equiv e^2\eta^2 B^0 + \partial^i\pi_i^{\mathrm{B}} + \frac{\theta}{4\pi^2}\epsilon^{ij}\partial^i B^j \approx 0. \tag{50}$$

The second-class constraints (7b) are no longer present and, as a consequence, the direct connection between the π^{B}'s and B's disappears. This is the main effect provoked by the addition of the term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$.

As before, the system is quantized in the generalized linear gauge (5). The DBQP provides the corresponding ETCs. We shall not pause here to quote them, but we only mention that the algebra (23) is replaced by

$$[\phi(x),\phi(y)]=0, \tag{51a}$$

$$[\phi(x),\phi^{\dagger}(y)]=0, \tag{51b}$$

$$[\phi(x),p(y)]=\frac{i}{2}\delta(x-y)-\frac{i}{2e}\phi(x)(\phi(y))^{-1}\partial_y^j K_j(x,y), \tag{51c}$$

$$[\phi(x),p^{\dagger}(y)]=\frac{i}{2}\delta(x-y)\phi(x)(\phi^{\dagger}(y))^{-1}+\frac{i}{2e}\phi(x)(\phi^{\dagger}(y))^{-1}\partial_y^j K_j(x,y), \tag{51d}$$

$$[p(x),p(y)]=-\frac{i}{2e}p(y)(\phi(x))^{-1}\partial_x^j K_j(y,x)+\frac{i}{2e}p(x)(\phi(y))^{-1}\partial_y^j K_j(x,y), \tag{51e}$$

$$\begin{aligned}[p(x),p^{\dagger}(y)]=&\frac{i}{2}\left[\delta(x-y)+\frac{1}{e}\partial_x^j K_j(y,x)\right](\phi(x))^{-1}p^{\dagger}(y)\\&-\frac{i}{2}\left[\delta(x-y)+\frac{1}{e}\partial_y^j K_j(x,y)\right]p(x)(\phi^{\dagger}(y))^{-1}\\&-\frac{1}{4}\left\{\delta(0)\delta(x-y)+\frac{2}{e}\delta(x-y)\partial_x^i K_i(y,x)\right.\\&\left.+\frac{1}{e^2}\left[\partial_y^i K_i(x,y)\right]\left[\partial_x^j K_j(y,x)\right]\right\},\end{aligned} \tag{51f}$$

where the factor $\exp[\pm(i/\theta)\Delta(x,y)]$ is missing. Indeed, a careful analysis of the DB's structure reveals that, due to the absence of the second-class constraints (7b), the ETC (10j) has been replaced by

$$[\varphi(x),\varphi(y)]=0. \tag{52}$$

This, together with (24), implies that $\Delta(x,y)=0$. The additional term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ implements, then, an anyon suppression mechanism in all gauges.

5. Conclusions

We have solved in this work the problem of quantizing the Chern–Simons theory (1) in the generalized linear non-covariant gauge (5). The lack of flexibility of the original description was eliminated by introducing the new variables η, φ and B^{μ}, as indicated in eqs. (2) and (3). As far as the lagrangian is concerned, the just mentioned change of variables provokes the appearance of a surface term containing the gauge-dependent phase φ. Although this term does not affect the dynamics, we demonstrate that it is responsible for the emergence of excitations obeying fractional statistics. Our study of the Coulomb, superaxial and unitary gauges indicates that these excitations are not gauge artefacts.

Furthermore, we show that, on general grounds, the field ϕ develops translational and rotational anomalies. In particular, the absence of anyons does not imply the absence of anomalies.

We prove at last that when the standard term $-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}$ is present in the action no anyons arise, this being true in all gauges.

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