Reply to "Comment on 'Vortex distribution in a confining potential' "

Matheus Girotto,[1,*] Alexandre P. dos Santos,[2,3,†] Renato Pakter,[1,‡] and Yan Levin[1,§]

[1] *Instituto de Física, Universidade Federal do Rio Grande do Sul, Caixa Postal 15051, CEP 91501-970, Porto Alegre, RS, Brazil*

[2] *Departamento de Educação e Informação em Saúde, Universidade Federal de Ciências da Saúde de Porto Alegre, 90050-170, Porto Alegre, RS, Brazil.*

[3] *Departamento de Física, Universidade Federal de Santa Catarina, 88040-900, Florianópolis, Santa Catarina, Brazil*

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We argue that contrary to recent suggestions, nonextensive statistical mechanics has no relevance for inhomogeneous systems of particles interacting by short-range potentials. We show that these systems are perfectly well described by the usual Boltzmann-Gibbs statistical mechanics.

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In a recent Physical Review Letters [1], Andrade *et al.* studied a system of particles (vortices) interacting by the potential

$$V(r) = q^2 K_0\left(\frac{|\mathbf{x}_1 - \mathbf{x}_2|}{\lambda}\right), \tag{1}$$

where K_0 is a modified Bessel function, $r = |\mathbf{x}_1 - \mathbf{x}_2|$ is the distance between particle 1 and particle 2, q is the potential strength, and λ is the screening length. The particles were confined to a potential well

$$W(x) = \alpha \frac{x^2}{2} \,. \tag{2}$$

The principal conclusion of the paper by Andrade *et al.* was that a system of such particles, in contact with a reservoir at $T = 0$, "obeys Tsallis statistics". The authors argued that at finite temperatures, the system will maximize a mixture of Tsallis and Boltzmann entropy. In our Comment [2] on Andrade *et al.*'s paper, we pointed out that at $T = 0$, statistics is irrelevant and a system in contact with a reservoir at $T = 0$ will lose all of its free energy and will collapse into the ground state. We then explicitly calculated the particle distribution in the ground state in the limit $N \to \infty$, $q \to 0$, and $Nq^2 = 1$, and showed that it is different from the one predicted by Tsallis entropy.

In the follow-up paper [3], we have extended our theory to finite temperatures and shown how the system of Andrade *et al.* can be studied using a mean-field theory. The Comment of Ribeiro *et al.* [4] criticizes our paper and insists that the equilibrium state of the system, described by Eqs. (1) and (2), should be described by the nonextensive statistical mechanics.

Below we address the issues raised by Ribeiro *et al.*:

(i) The asymptotic form of the interaction potential in Eq. (1) is $V(r) \approx q^2\sqrt{\frac{\pi}{2r}}e^{-r/\lambda}$. This potential is short ranged and has a form very similar to Yukawa potential. It is well known that a system of Yukawa particles confined by hard walls or periodic boundary conditions crystallizes [5–10]. The process is perfectly well described by the standard Boltzmann-Gibbs (BG) statistical mechanics. Ribeiro *et al.* do not provide any argument why the equilibrium state of the Yukawa-like system confined by a parabolic potential should be described by a nonextensive entropy. The only arguments are based on fitting the particle distributions calculated using overdamped molecular dynamics (MD) simulations to q Gaussians. Such curve fitting, however, must be taken with caution. For example, recently it has been argued that sufficiently strongly correlated random variables also obey nonextensive central limit theorem in which the usual Gaussian distribution for uncorrelated random variables is replaced by a q Gaussian. Again the only basis for this belief was curve fitting. However, in an important paper, Hilhorst and Schehr [11] calculated exactly the probability distributions for strongly correlated random variables and showed that these are analytically different from the q Gaussians. Curve fitting is a shaky ground on which to build a new theory, in particular one that attempts to replace the BG statistical mechanics.

(ii) In their Comment on our work the authors state that "besides the long-range forces, other attributes, like strong correlations" make systems fall out of the "scope of BG statistical mechanics". Indeed some years ago, it was hoped that the nonextensive statistics could be helpful to study systems with *long-range* interactions, such as magnetically confined plasmas or gravitational clusters. However, recent work [12,13] has shown that long-range interacting systems relax to quasistationary states, which have nothing to do with Tsallis entropy.

It is also incorrect to say that BG statistics fails for strongly correlated systems. If this would be true, the theory could not be used to study either liquids or solids, which are very strongly correlated. Yet, BG statistical mechanics is able to account perfectly for the structural and thermodynamic properties of liquid and solid phases, as well as for the phase transitions between the different phases.

(iii) In their Comment on our paper, Ribeiro *et al.* claim that we did not "realize how poor mean-field approximation" was in the strong coupling regime. The discrepancy between the mean-field and MD simulations at low temperatures was clearly pointed out by us. Furthermore, it is very well know that the mean-field theory fails when the correlations between the particles become strong; see, for example, discussion in Ref. [14]. The failure of the mean-field theory, however, is in no way indicative of the failure of the BG statistical mechanics. Indeed, it is possible to solve exactly for the particle distribution predicted by the BG statistical mechanics

*girotto.matheus@gmail.com

†alexandreps@ufcspa.edu.br

‡pakter@if.ufrgs.br

§levin@if.ufrgs.br

using Monte Carlo (MC). This is precisely what we have done in our paper [3] [Fig. 5(a) from our paper]. We compared the results of overdamped dynamics simulations of Andrade *et al.* with the predictions of BG statistical mechanics for the same number of particles, the same parameters, and the same temperatures as in their paper. As expected the results of Andrade *et al.* are in perfect agreement with the predictions of BG statistical mechanics. Ribeiro *et al.*, say that the reason for this good agreement is that the temperatures that we looked at were "too high". They argue that at lower temperatures, more relevant for superconductors, BG statistics will fail and the density distribution will correspond to the maximum of Tsallis entropy. To answer this criticism, in this Reply we calculated a coarse-grained density distribution at the lowest temperature, $T = 0$. Within usual thermodynamics and BG statistical mechanics a system at $T = 0$ will be in the ground state, which can be calculated by minimizing the potential energy of the system. The coarse-grained density distribution is then constructed by binning the particles along the x axis. In Fig. 1 we show that this density distribution is in excellent agreement with the overdamped dynamics data of Andrade *et al.* Once again we conclude that there is absolutely no reason to introduce a nonextensive entropy for the system of particles interacting by a short-range potential.

(iv) Ribeiro *et al.* claim that their W-Lambert solution describes perfectly the MD data of Andrade *et al.* However, they fail to point out that the good agreement shown in the Fig. 1 of their Comment is obtained with the help of a fitting parameter a. Clearly if one has to decide between two very distinct theories that account equally well for the data, but one of which has a fitting parameter and the other one does not, there is no question which theory is preferable.

We showed that all of the overdamped dynamics data of Andrade *et al.*, including $T = 0$, is perfectly well described by the BG statistical mechanics. At high temperatures, the particle distribution can be accurately calculated using the mean-field theory. At intermediate temperatures the correlations can be included using a density functional theory in conjunction with the hypernetted chain (HNC) equation [15]. Therefore, there is absolutely no reason to introduce a nonextensive entropy for this problem.

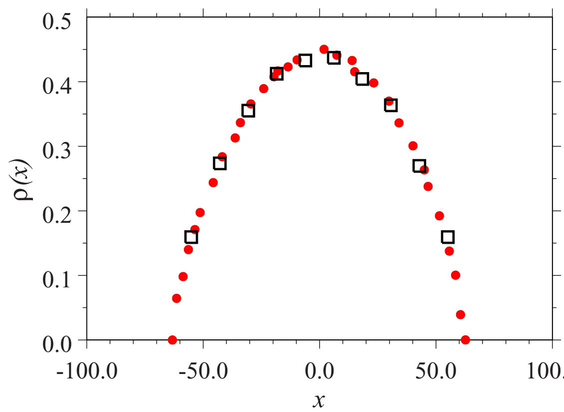

FIG. 1. (Color online) Comparison between the overdamped dynamics data of Andrade *et al.* [1] (circles) with the predictions of BG statistics (squares) for $T = 0$. The perfect agreement between the two clearly shows that the equilibrium state of the system of Andrade *et al.* is described by the standard BG statistical mechanics down to $T = 0$.

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