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A critical appraisal on the evolution of stock returns theory

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"It takes considerable knowledge just to realize the extent of your own ignorance"

— Thomas Sowell

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ABSTRACT

This study aims to describe and understand the evolution of the asset pricing theory. It is helpful to understand the state we encounter ourselves to observe the path and tendencies from the literature. The methodology used in this paper is a review of the literature with the perspective of the theorist, that is, focusing on the ideas that remained on scrutiny today over the ones that were important in the past. The results encountered show the evolution and patterns presented mainly in four key research aspects of the theory: stock market volatility, asset pricing tests, model properties and the search for variables that explain stock returns.

Keywords: Stock market. Financial Theory. Stock returns. Asset pricing.

LIST OF ABBREVIATIONS AND ACRONYMS

NY   New York

UK   United Kingdom

RE   Rational Expectations

PD   Price Dividend

U.S.   United States

IA   Investment factor

NA   North America

ME   Market Equity

P/E   Price-to-Earnings ratio

D/P   Dividend-price ratio

C/P   Cash flow to price ratio

E/P   Earning to Price ratio

B/M   Book-to-Market ratio

SMM   Simulated Method of Moments

SMB   Small Minus Big

HML   High Minus Low

APT   Arbitrage Pricing Theory

Mkt   Market Factor

ROE   Return on Equity

UMD   Up Minus Down

IES   Intertemporal Elasticity Substitution

UMP   Uniformly Most Powerful

GLS   Generalized Least Squares

OLS   Ordinary Least Squares

CMA   Conservative Minus Agressive

GRS   Gibbons Ross Shanken

EPU   Economic Policy Uncertainty

CIV   Common Idiosyncratic Volatility

GPH   Geweke and Porter-Hudak estimator

i.i.d   Independent and identically distributed

CAPM   Capital Asset Pricing Model

NYSE   New York Stock Exchange

AMEX   American Stock Exchange

NVIX   News Implied Volatility

BE/ME   Book-to-Market Equity ratio

GARCH   Generalized Autoregressive Conditional Heteroskedasticity

ARIMA   Autoregressive Integrated Moving Average

ICAPM   Intertemporal Capital Asset Pricing Model

NASDAQ National Association of Securities Dealers Automated Quotations

CONTENTS

1 INTRODUCTION

Stock markets are a key aspect of the economy. They allow for companies to finance themselves and raise capital. These structures became more important throughout the twentieth century, with notable raises mainly in the nineties. The increases in globalization with the new millennium brought with them a huge boost for stock markets, as the technologies allow them to be broader. The recent years mark the time which the stock market have been more important for the economy and for companies, with the highest amount traded in history of 99.792 trillion dollars in 2015, according to The World Bank.

As stock markets grew to be of key importance in financing company's investments nowadays, to understand returns, an important aspect of them, becomes crucial. Since the early 20th century, some authors began discussing unique characteristics about stock markets, with Sharpe (1964) aggregating and improving previous ideas and being the first that tried to explain returns with a model: the CAPM. Afterwards, a flurry of papers have been published up to these days, as authors continue to perfect and test new models to better explain stock returns.

The problem we address here is to describe and understand the evolution of the theory of stock returns from the perspective of the theorist, that is, focusing on the ideas that remained on scrutiny today over the ones that were important in the past. In fact, most of the papers on stock returns present a very limited review of the subject and therefore have limited value as mean of contextualization of current research. As a result, it is easy to be lost on the topic and little is pointed out as a direction of future research besides some minor modifications on the paper.

The literature review start in the famous Sharpe (1964) paper, and pass through the most important studies such as Fama and French (1993), Ross (1976) and Fama and MacBeth (1973), and up to these days. This paper's goal is to show how the asset pricing literature has evolved, demonstrating the focus points of each time and the breakthroughs that have been reached throughout history. It is important to note that this study does not aim to evaluate or present the econometric and statistic techniques used in the asset pricing literature, focus being on presenting the theory and its evolution.

The paper is organized as follows. Section II describes the origin of the researches in stock returns, starting in Bachelier (1900) and passing primarily through key papers published before 1964. Section III describes the first steps of the modern asset pricing theory, considered here to cover from 1964 to 1979. Section IV presents the evolution

of the literature from 1979 to these days. Section V shows the results and discussions obtained. Section VI includes the conclusion of the paper.

2 THE ORIGINS OF THE PROBLEM OF STOCK MARKET RETURNS: THE LITERATURE BEFORE 1964

Financial theory have a short history in economics. The asset pricing literature as a subject just erupted after Sharpe (1964) gathered previous knowledge and introduced a model trying to both explain and predict stock returns. Prior to that, the study and analysis of the financial market was not a main point, being largely based on intuitions from practitioners in the first fifty years of the twentieth century.

Even though technical analysis have been used since the rice markets in feudal Japan, the foundations of the problems in the asset pricing literature were only introduced in Bachelier (1900). The importance of that work however was not recognized until sixty years later, as the use of mathematics to analyze financial markets were not seen with good eyes by the time. Bachelier acknowledges the impossibility of exact forecasts for stock market returns, as those are influenced by an infinite number of factors. However, he introduce the possibility to use mathematics for studying the probabilities of market fluctuations in a given time. These probabilities can than be used to analyze if prices have a higher chance of falling or raising its value.

His paper provides a lot of concepts for stochastic analysis, giving the bases for the posterior sixty five years. He believed the fluctuations in price seen over short times were independent of the current price. Another assumption is about the independence of prices and their past process, which combined with the Central Limit Theorem result that additions in the process are independent and normally distributed. This analysis performed by Bachelier concluded that stock prices fluctuated in what is now known as a random walk, with the price changes being independent and probabilistic distributed.

The five posterior decades to Bachelier's work mark a period of little advance in terms of the asset pricing literature and understanding stock returns. Economists and practitioners believed the stock market should not be viewed as a market, but as a "casino", therefore having no need for analysis. As the random walk thesis was considered by most of the scholars and mathematical work was not viewed to be promising in the area, most of the articles regarding the stock market had little to do with understanding its pricing behavior, with the focus instead being on understanding the importance of those markets to the economy. Important economists from the century such as John Maynard Keynes and John Hicks acknowledged the important role of uncertainty and risk, but never provided theories that would explain stock returns behavior.

It was only in the late thirties and early forties that the casino view of stock markets started to be challenged. Williams (1938) was among the pioneers in this challenge. Already in the sixth decade of the twentieth century Harry Markowitz realized he could take advantage of the expected utility theory developed by John Von Neumann and Oskar Morgenstern to introduce a theory to optimize portfolio selection. The Optimal Portfolio theory presented in Markowitz (1952) and Markovitz (1959) explained how to assemble a portfolio with the highest expected return for a given risk. The theory argues the overall portfolio's risk and return should be evaluated altogether, and not the characteristics of an stock alone. The concept of "Efficient Frontier" is introduced as well. This concept shows investors the best possible return for a portfolio with a given level of risk.

Bachelier's framework was just recognized as an important theory in the beginning of the sixties. Mandelbrot acknowledged the importance of the random walk theory and the process developed by Bachelier in Mandelbrot (1963). In that article, Mandelbrot aimed to expand Bacheliers previous work to account for the data accumulated throughout the posterior sixty years. The author applied the Bachelier process to the log of stock prices and replaced the Gaussian distribution with another family of distributions he called "stable Paretian". Mandelbrot continuation on the work of Bachelier ignited the flame around the random walk theory, which would continue to be investigated later by Eugene Fama and Paul Samuelson among others.

The investigation from the authors above led to the Efficient Market Hypothesis, another important theoretical framework in Finance Theory. Developed in the mid sixties by Paul Samuelson, Eugene Fama and Benoit Mandelbrot, the theory provided that, if markets are working properly, all information about an asset would be composing its price. This implies that the prices are random and not predictable only because investors have already exploited the opportunities that appeared. As this theory was further expanded by Fama and was mainly developed post-64, the next section gives more details about it.

3 THE LITERATURE BETWEEN 1964 AND 1979

The first model that properly tried to explain stock returns was introduced in Sharpe (1964) and Lintner (1965). The goal of the authors was to describe the relationship between stocks and systematic risk. Previous work did not account for that type of risk, thus not having good results explaining stock returns and the stock market behavior. In Sharpe's theory, he argued that investors were compensated for time value of the money and risk. The ultimate goal of the model formulated by him was to evaluate the value of a stock comparing its risk and the time value of money to its expected return. The equilibrium model was baptized as Capital Asset Pricing Model, better known as CAPM. It considers the stock market to have risk averse investors with equal preferences. These investors aim to maximize their utility by choosing a combination of risky assets and riskless assets, which yield a constant risk-free rate, that provides the highest returns with the lowest amount of risk. The model assumes no transaction costs and all investors can borrow or lend money at a given interest rate. Prices are revised by the market in the following manner, stocks with high (low) demand will suffer an upward (downward) revision in prices, which will imply lesser (higher) future returns and consequently they will become less (more) attractive. With the specifications set, the model is defined as: the return of the investment is equal to the risk-free rate plus the investment beta times the market risk premium. The risk-free rate accounts for the time value of the money and the market component for risk. The beta of the investment is measured on how much risk this investment will add to a portfolio that looks like the market.

The predictability of stock returns continued to be in question even after Sharpe's model was introduced. In an environment of search for non-predictability proof in stock returns, the efficient market hypothesis was presented. The hypothesis was first seen in Fama (1965a) and continued a matter of discussion even in the latest decades. Fama's paper analyzed the Financial theory and its long discussion of the possibility to predict stock returns. Chartists believed that past price behavior from the stock would help to explain it's future price, while the adepts of the random walk theory believed successive price changes were independent and there was no possibility to predict stock prices. With this discussion in mind, Fama aimed to empirically test the validity of a random walk model. A validation of the model would imply that stock prices were not predictable and the opposite should point out towards a possibility of forecasting returns. The evidence Fama found in his results could not reject the null hypothesis that the model explains

the behavior of stock returns. This implicated that the possibility of stock prices being a random walk variable, with no possibility of future price prediction could not be refused. Fama also found these results to be compatible with an "efficient market", meaning that the stock prices adjusted to the available information.

In the same year, Samuelson (1965) aimed to challenge the random walk model previously described in the literature. The author argued that in an environment where the demand curve was the result of various independent measures, the results of a test may behave like a random walk without being one. The solution offered by him was to replace the random walk by another stochastic process, the martingale. He assumed the estimation at time t of the discounted spot price in a number of periods was represented by a distribution law. Samuelson aimed to examine the relationship between this variable and the future price. This relation could be seen at a specific period, when the spot price should be equal to the future price, as otherwise an arbitrage opportunity would exist. In a nutshell, this means that if the sequence of prices follows a martingale, the best estimation of tomorrow's spot price is the actual price of the future contract. Even though their framework went by different paths, both Samuelson and Fama reached the result that if traders had the correct expectation, prices fluctuate randomly.

The discussion continued in Malkiel and Fama (1970), a theory review that better explained the efficient market hypothesis. The efficient market hypothesis state that in an ideal market, security prices fully reflect all available information. Previous works in the literature had presented this hypothesis and tested its validity in empirical works. This paper aimed to review the theoretical and empirical literature about the Efficient market hypothesis as well as test three types of information subsets. After performing the theoretical and empirical review, the authors considered three sets of information to examine the empirical results for different levels of market efficiency. The first test checked for "weak form" of market efficiency and looked at an information set that only contemplated historical prices. The second set contained information that were obviously publicly available, which if prices efficiently adjusted, "semi-strong form" could be considered. The last test verified for "strong form" and examined if any groups had monopolistic access to information.

The results for the weak form tests showed strong evidence supporting this level of market efficiency. Dependence in successive price changes was found, but not sufficient to declare the market inefficient or to reject the random walk theory. This test, however, demonstrated day-to-day correlation in price changes, which could be used for profitable

trading. Semi-strong perform tests also supported the efficient market hypothesis. Evidence from previous empirical work found that three main aspects were absorbed by stock prices: information concerning the firm's future dividend payments, annual earning announcements by firms, and new issues and large block secondary issues of common stock. Finally the strong form tests saw two deviations from a perfect efficient market. Specialists on major security exchanges and corporate insiders had monopolistic access to information, respectively on not executed limit orders and about their firms. Given that this level of market efficiency was highly unlikely to be achieved, as monopolistic information seemed to be difficult to get rid of, the authors considered it to be more of a benchmark for deviation considerations than a proof of the Efficient Market Hypothesis rejection.

As Sharpe's article only aimed to propose the model with no intention to test it, further literature focused on that and pointed out problems within the model. Eight years after the original paper, Black et al. (1972) worked on empirical tests for the CAPM as well as on providing some adjustments. This study differs from previous as it tested the performance of the model based on grouped portfolios and used time-series regressions. The tests were conducted in ten portfolios of grouped securities. The article used the previous period beta of the stocks in selecting the portfolios for the year after to avoid selection bias. After that, they performed a time-series regression of the portfolio's excess return on the market portfolio's average returns to indicate if results would corroborate the original theory. In addition, they attempted to make explicit estimates of the time series of returns on the beta factor. This procedure would provide estimates of the mean and variance, enabling them to test if the mean excess return on the beta factor was zero. They assumed equal residual variances for the beta estimator, as the optimal estimator would require knowledge of the unobservable residual variances of each of the portfolios. They also suggested a two-factor model variation of the CAPM that was not properly tested in this article. The two-factor model described the excess return on an asset as a sum of a "beta factor" and the asset's beta times the market excess returns. Data used for the tests were from NYSE covering the 1926-1966 period.

Results pointed towards rejecting the original model as some of the relations the model proposed were not found. The beta factor had an upward trend in the period researched and had superior values to the risk free rate at 1.0-1.3% per month. The expected returns for low-beta portfolios are higher than the predicted by the model, with the opposite results reached to high-beta portfolios. The evidence presented in the research does

not indicate proportionality of the assets excess returns with their respective betas as the original model predicted.

In the following year, Merton (1973) also introduced a modified CAPM. The research aimed to present an intertemporal extension of the CAPM. The previous CAPM introduced in Sharpe (1964) was designed to predict based on a single period. The idea behind modifying this model to be intertemporal was that investors considered future expectations, which means they had impact in current prices. Merton maintained the pillars of the CAPM, such as being a linear equilibrium model, utility maximization and limited liability of assets, but expanded to a lifetime utility maximization instead of the single period nature of the original model. The main addition to the model were state variables acknowledging that investors hedge against shortfalls in consumption or against changes in the future investment opportunity set.

Adjusted versions of the CAPM continued to be introduced in the decade, as Lucas (1978) presented the bases for a variation of the model focused on consumption risk. His goal was to analyze the relationship between productivity changes and movements in asset prices. The model assumes an economy with one-good, risk averse investors, pure exchange and identical consumers. The analysis conducted in this research considered the market efficiency hypothesis previously presented by Fama. One difference standing out from the CAPM is that Lucas' model, just like Merton (1973)'s model, was derived in a multi-period set up, opposing the static condition of the CAPM. Also, instead of the market risk beta, the expected returns were related to a consumption risk beta. This implies the expected risk premium is proportional to the covariance between its return and consumption in the period of the return. The equation used generalizes the Martingale property of stochastic price sequences, and if a price series failed to possess the property it could be viewed as evidence of irrational behavior.

The CAPM and its variations were not the only model introduced in the early asset pricing literature. Ross (1976) provided an alternative model to the CAPM. The model is also based on systematic risk, but instead of having only one measure of it like the CAPM, it can use multiple macroeconomic variables which impact an asset risk. The theory provided a model that was able to measure stock returns based on the linear relation of asset's expected returns and the macroeconomic factors that affect the asset's risk. The goal of this model was to be able to calculate the fair price of a stock, detecting incorrect prices. In Ross's theory, the market was not always perfectly efficient, occasionally having assets being mispriced. Market's actions eventually correct the mispricing, but there is

still a window for arbitrageurs to take advantage of the situation and profit.

As the researches in the area became more popular, more space was given to researches that did not aim directly at introducing models. An example of this is seen in Fama and MacBeth (1973), where the focus shifted towards proposing a two step regression to be used to estimate parameters for asset pricing models. The basis of their theory was to test the relationship between average returns and risk. The assumptions of the methodology applied in the regression were derived from the two parameter portfolio model introduced in previous years by Fama (1965b), Tobin (1958) and Markovitz (1959). In this model, investors are price takers, risk averse and utility maximizers, there are no transaction costs and the one-period percentage returns for assets and portfolios are normal distributed. These assumptions led to the efficient set theorem, that says an optimal portfolio has to have no other portfolios with same or higher expected return and lower dispersion of return to be efficient. With these bases set, the method to estimate the parameters was derived using mathematical properties and equations. The parameters are estimated in two steps. First they regressed each asset against the proposed risk factors to determine that asset's beta for that risk factor. In the second step, all asset returns were regressed for a fixed time period against the estimated betas to determine the risk premium for each factor. Data used was from NYSE stocks covering the 1926 to 1968 period.

Fama and Macbeth found evidence supporting an efficient stock market. The hypothesis that says the NYSE average returns reflect the attempts of risk-averse investors to hold efficient portfolios could not be rejected by the tests. They found evidence pointing the average returns to have a positive trade-off between risk from the portfolio viewpoint and return. One interesting result found in the tests is the incapability of rejecting the hypothesis presented in the two-parameter portfolio model that says investors should assume a linear relation between portfolio's risk and return. At last, the model were not able to reject that no measure of risk systematically affects average returns in addition to portfolio risk.

Later in the decade, asset pricing tests were again subject of an important paper. Roll (1977) criticized previously developed asset pricing tests. The author's goal was to point flaws and difficulties in previous tests, such as the introduced in Black et al. (1972), Black (1972) and Fama and MacBeth (1973). Roll demonstrated problems in more than one aspect of the tests, and a few of them are shown next. For the two-parameter theory to be tested, there is a necessity to know the exact composition of the true market portfolio, which was an obvious problem for the time. As a solution for the problem, market

portfolio proxies were used, but those were subject to difficulties. The main difficulties presented by Roll were the possibility of the proxy to be mean-variance efficient (even though the true market portfolio weren't) and the chance of the proxy to just be inefficient or bad. Another problem lied in the possibility of bias in the case of a misspecification in the measured 'market' portfolio. Mean variance efficiency tests of the proxies were also a difficulty at the time, and the alternative option of using the return/beta linearity relation was hard to obtain via econometric procedures.

4 THE LITERATURE FROM 1979 TO THESE DAYS

As the 1964 to 1979 period marked the first steps of the asset pricing literature, the post 79 starts a period where more authors became interested in studying the theme. The literature expanded and a flurry of ideas and different theories appeared. This diversification can be exemplified in three papers that start the 1980 decade discussing a previously noted pattern in volatility. Shiller (1980) and LeRoy and Porter (1981) study the stock price indexes' volatility and its compatibility with the efficient markets model. The efficient markets model predicts real stock prices to be equal to the present value of optimally forecasted future real dividends discounted by a real constant discount rate. Shiller defines the simple efficient markets model and a series of inequalities that should indicate the bounds for stock price variance. The main inequality obtained by him identifies the variance of a real index being lesser or equal to the variance of present discounted value of the actual subsequent real dividend. Using a restated in innovations version of the model, he tests if the model accounts for the volatility found in the data.

Even though they have similar objective, in LeRoy and Porter (1981) the author's methodology is a bit different. As in Shiller's article, they also define the present value relation, but afterwards they prove three theorems that relate variance of dependent variables and independent variables. The first theorem predicts the variance of the dependent variable, in this case the stock price, to be less or equal to the variance of the independent variable, in this case earnings. For the second theorem, two time series are generated by altering the amount of information about future innovation. This time-series provides an upper and lower bound to the variance of the stock price. Theorem three guarantees the terms in the variance decomposition of the equation that states the variance of the independent variable are equal to the sum of the variances of the dependent variable. The theorem also says forecast errors can be estimated, allowing a point-test of the null hypothesis defined in the present-value relation to be performed. The three theorems give solidity for the author's construction of present-value relation tests.

The tests from both researches give similar results. As per theorem one or main inequality, the model predicts that earnings should have a higher value of variance than the dependent variable, the stock price. Empirically the results show quite the opposite, with dispersion of stock prices being at the 0.452 value and at 0.172 for earnings in LeRoy and Porter (1981). Shiller (1980) finds a volatility of five to thirteen times higher than the one predicted by the model. On top of that, the bounds estimated are severely violated,

which shows some evidence to the rejection of the efficient markets model.

Flavin (1983) challenged those results. Her focus was primarily in investigating biases in variance-bound tests on small samples that express variances as deviations from the sample mean rather than the population mean. She stated that these biases might have been responsible for the rejection of market efficiency null hypothesis in previous articles from the asset pricing literature. For instance, she believed these biases were responsible for the results previously reported in Shiller (1980) and LeRoy and Porter (1981). The author tested this claim by considering an economy in which the short rate is generated by an ARI process with an 0.95 autoregressive parameter. This economy had risk neutral investors that formed expectations of future short rates rationally, results being generated exactly as the expectations. From an one hundred sample size, she calculated the values of the variances and their differences. The evidence from Flavin's research pointed out towards a strong bias in variance-bound tests, favoring the rejection of the null hypothesis. Considering the sample size used in most of previous papers, the magnitude of the bias was very considerable. Excess volatility was found to be much less severe when the tests were controlled for small sample properties of the variance-bound statistics.

From another perspective, French, Schwert and Stambaugh (1987) also studied stock market volatility. This research focused on studying the relation between the risk premium and stock market volatility. This idea was based on the theory that claimed increases in expected volatility would lead to higher risk, thus in a higher risk premium. This was the first article to test the relation between volatility and the risk premium, with previous literature doing only exploratory work in the area. They tested volatility as an independent variable that would explain the risk-premium, having in sight to test volatility as a risk measure with power to explain stock returns. They used daily returns to compute monthly volatility for both the two statistical tests made. The used tests were an univariate ARIMA model as well as Engle, Lilien and Robins (1987) GARCH model. The sample considered in this research covered NYSE stocks from the 1928-1984 period.

The tests found a positive relation between predictable level of volatility and expected risk premium. As for the unpredictable component of volatility, it had a strong negative relation with excess holding period returns. The authors attributed the negative relation between contemporaneous returns and changes in volatility as evidence of a positive relation between expected risk premiums and ex ante volatility.

Chen, Roll and Ross (1986) was also in the search for variables that could explain stock returns. The goal was to test the relation between equity return and macroeconomic

variables. They argued that only general economic state variables and variables that affect the economic pricing operator would influence stock pricing. Hence, they modeled equity premium as function of the following variables: the spread between long and short interest rates, expected and unexpected inflation, industrial production, the spread between high and low grade bonds, and oil prices. The model revolved around the stock returns being the dependent variable with the macro variables and their respective loadings being the independent variables. First, the macro variable factors were constructed, and after building the model the authors applied a version of Fama and MacBeth (1973) technique, aiming to determine which variables related to asset pricing.

Industrial production, changes in the risk premium and twists in the yield curve found a strong link with asset pricing. Measures of unanticipated inflation and changes in expected inflation during periods found weak significance in explaining stock returns. The economic state variables most impressively outperformed value-weighted New York Stock Exchange index, which explained a portion of the time-series variability of stock returns. Two variables that never found to be correlated with stock returns were innovation on per capita consumption and oil prices.

Starting in the middle of the decade, three papers can be considered pioneers in trying to understand market behavior without relying on fully rational investors as representative agents. In Bondt and Thaler (1985), the authors were inspired by psychological discoveries of news overreaction to test its predictability in stock market. They also tried to address some unexplained behavior from stock markets as overreaction effects, such as the January effect, the small firm effect and the dividend yield and P/E effects. The overreaction hypothesis as an ex post explanation for stock market behavior is a valuable asset for understanding stock prices. In this article, the authors tested if this hypothesis could also predict these prices. Previous findings discovered that dividends did not vary enough to rationally justify observed price movements. This, combined with evidence suggesting stock price movements are strongly correlated with the following year's earning, showed a clear path to overreaction in these movements. In stock markets, the overreaction hypothesis can explain the big importance given to short-run economic developments and the P/E anomaly. The authors based their tests in two facts: extreme stock price movements in one direction will be subsequently countered by movements in the opposite direction, and this adjustment will have greater magnitude depending on the first movement.

The results were consistent with the overreaction hypothesis. The effect was much larger for losers than for winners, which is predicted by psychological analysis. In the

longer term (3-5 years), loser portfolios outperformed winner portfolios, with January being the month with the highest earnings for the former. The hypothesis was able to predict extreme return experiences and their following reversals. The P/E ratio effect was explained using this theory as an overvaluation of high P/E stocks and undervaluation of low P/E stocks based on overreactions. For the January effect, even though the announcements at the beginning of the year might have been a source of overreaction, it did not seem to be totally explained by those.

Epstein and Zin (1989) aimed to develop recursive preferences over intertemporal consumption with non-expected utility. This idea tried to address the problems faced by representative agents models with expected utility. One of the issues the class of recursive preferences tried to solve was the tangling of risk aversion with intertemporal substitution. The authors' general class of preferences contained three more important subclasses: intertemporally additive expected utility preferences and homogeneous von Neumann-Morgenstern utility index; homogeneous versions of the Kreps and Porteus (1978) structure extended to infinite horizon; and a multiperiod framework generalization to the atemporal non-expected utility theory of Chew and Epstein (1989) and Dekel (1986). The second and third subclasses allowed the separation between elasticity of substitution and risk aversion. This separation gave intertemporal consistency to their recursive preferences. Another feature of these preferences was the nonindifference implied by them. The authors explained that it is perfectly rational to have this feature, it is possible to test the existence of nonindifference with data and they assume indifference to timing and intertemporal consistency of preference, which imply expected utility ordering.

The result reports for the tests done with that framework were obtained in Epstein and Zin (1991). In this research, the authors used Euler equation restrictions to estimate the utility function parameters using generalized method of moments (GMM). The authors aimed to test for the expected utility hypothesis, the fit of their general model and its stability. The results found by the GMM estimates indicate a rejection of the expected utility hypothesis. The model results showed sensitivities to the choice of consumption measure regarding non-expected utility. Elasticity of intertemporal substitution was found to be less than one and relative risk aversion close to one. Another finding was that consumers prefer the late resolution of uncertainty.

The third research to diverge from the usual rational representative agents is Long et al. (1990). This study aimed to prove that in a market with positive feedback traders, which buy when prices rise and sell when prices fall, rational investors may jump in the

bandwagon in hopes of making profit. This behavior indicates an opposite outcome to the usual literature, as rational investors are usually taken as agents who stabilize prices. This stabilization is done by them going in the opposite direction of the trend, bringing stock prices close to fundamentals. In this paper, the authors argued against this position. The theory presented in this research was based on the investment strategy presented in Soros (1987). The idea behind it was that a rational investor can foresee a rise in the price of a stock based on good news, and use this information to buy the stock and stimulate its price even more. In the day after, positive feedback traders will also buy it and increase its price even further. After positive feedback traders buy the stock and the price rise once again, rational investors know the price is above fundamentals and sell the stocks, stabilizing prices. Rational investors operate as double agents in this case, destabilizing prices at first when they stimulate the price for positive feedback traders to buy them, and afterwards as a stabilizing agent when they sell the overpriced stock.

The model generated a positive correlation of stock returns at short horizon and negative at long horizon. That implicated the model was able to explain how investors can see an asset price move towards one way in the short-run and the opposite way in the long-run. It also accounted for the overreacting triggered by news, which is a feature of the positive feedback traders. An interesting empirical use of the model is its usefulness for understanding bubbles.

Although Mehra and Prescott (1985) does not examine the rationality of investors, it is also a study that focuses on some pillars of models used in asset pricing literature. The authors aimed to study the equity premium and if models such as the Arrow-Debreu set up, which not account for transactions costs, liquidity constraints and other frictions, can account for the high values of the premium. They employed a variation of Lucas (1978) model with the growth rate of endowment following a Markov process instead of the endowment level. This change enables capturing non-stationarity in consumption series. The economy considered has a single household, a single productive unit and one equity share competitively traded. With that, they formulated the equilibrium price of equity and risk-free bill in time t and derived the formula for expected return. Stocks used were from Standard and Poor's 500 Composite Stock Index and the data from U.S. in the 1889-1978 period.

The evidence from the sample period for the average real risk-free assets return is 0.80%. This value is incredibly low if compared to the average real return on Standard and Poor's 500 Composite Stock Index of 6.98% per annum. That said, the model failed

to account for this huge 6.18% equity premium, reaching a maximum value of 0.35% per annum. Predictions for the average real risk-free rate in the model specifications reached 3.7%. These results may flip the usual equity premium puzzle question from why the equity premium is so high to why the risk-free rates are so low. The only way the model gets close to the empirical evidence on risk-free rates is by considering individuals to be risk-neutral. With the model created by them ultimately failing to account for the premium, they suggested the use of a model considering the frictions, would be most likely to yield better results.

The equity premium was also studied in a more specific manner in an article that test investment strategies, Jegadeesh and Titman (1993). The study of investment strategies is very important, as it usually aims to capture and explain specific patterns in the market. Jegadeesh and Titman (1993) provided insights about the strategy of buying stocks that have performed well recently and selling bad performers. The goal of the paper was to analyze how this strategy fare in a 3-12 month horizon and in a longer 3 year horizon. The authors also tried to find an explanation to the abnormal returns found in contrarian strategies. With those objectives in mind, they formed portfolios for winners and loser based on past 6 month performance. In addition, an analysis of stock returns around earnings announcement dates was also made. The exploring periods analyzed are a three to twelve month and a twelve month to three year horizons. The reason for those periods was that previous literature focused on examining the strategies in either a very short horizon, of a week to a month, or a very long, of three to five years. The sample was collected from AMEX and NYSE stocks in the 1965-1989 period.

The results pointed out abnormal returns for a buying winners and selling losers strategy in the first twelve month horizon. The average returns accumulated in the first twelve months were up to 9.5%, but in the following months those returns fall and the loser portfolios started outperforming the winners. Their earnings announcement analysis found similar results, suggesting that losers do outperform winners in the period after the first twelve months. The authors analysis pointed out the evidence to neither be compatible with systematic risk nor with lead-lag effect resulting from delayed stock price reactions to information. However, delayed price reaction to firm specific information was identified.

At the end of the 1980 decade and during the 1990, there is a series of notable researches published by the duo of Eugene Fama and Kenneth French. Fama and French (1988) is the first of those. In this paper they aimed to test the explanatory power of divi-

dend to price ratio (dividend yield) in short and long horizon. This goal was motivated by previous findings of the literature that suggested a weak explanation for short-horizon return variances but stronger for longer horizons. The short horizon considered is a holding period of a month to four months, the long horizon covered holding periods of two to four years. For testing the explanatory power, Fama and French used regressions of returns on yields, as they believed those provided reliable evidence. The portfolios used for the tests are value and equal weighted and the data involved the 1927-1986 period.

The authors found similar results to previous literature. Their tests resulted in the dividend yield explaining only 5% of the monthly returns variance and 25% of the variance of two to four year returns. There is a two part explanation given by the authors for the evidence. The first part stated that high autocorrelation cause the expected component of variance to grow faster than the unexpected component, implying that in a longer return horizon there is more predictability. Moreover, they pointed to a discount rate effect, which associates shocks to expected returns with opposite shocks to current prices.

Published in the same year as the Fama and French research, Campbell and Shiller (1988) also study dividend-price ratio. However, this article studied the relation between dividend-price ratio and two variables: dividend growth rates and dividend discount factor. The authors used a dynamic version of Gordon (1962) model, which relates the dividend-price ratio to expected future values of the one period rates of discount and one period growth rates of dividends. Log dividends and discount rates, together with other variables compose a vector which summarizes the state of economy. Market agents observe this vector and use its patterns to predict the future values of log dividends and discount rates. As measuring discount rates is not straightforward, four versions of the basic model are considered. The versions use the following measurements: a constant rate, the ex-ante real return on short debt plus a constant risk premium, the expected growth rate of real aggregate consumption multiplied by the coefficient of relative risk aversion and the sum of a constant riskless rate and a time-varying risk premium. The model is tested using vector autoregressive methods and data from NYSE covering 1871-1986 and 1926-1986 periods.

Campbell and Shiller found three main results from their tests. Evidence showed that expected future growth in dividends were able to explain in part the movements in log of D/P. Even though they used more than one measure, discount rates had no evidence of being helpful to explain movements. At last, there was still a large part of the log of D/P that was not explained, which showed the need for other studies of this variable.

In Fama and French (1992), Fama and French started their work in factor models, which would lead to their posterior three and five-factor models. The author's goal was to verify the joint explanatory power of variables studied alone previously in the literature. They tested four variables that had previously showed good results alone in explaining the cross-section of average returns. The variables are book-to-market equity (BE/ME), earnings-price ratio (E/P), size (ME) and leverage. Using these variables together with the market beta, they tested for the joint explanatory power in the 1963-1990 period with cross-section regressions from Fama and MacBeth (1973). The authors acknowledged the fact that these variables can all be proxies for risk, which may explain their previous success to explain the cross-section of average returns.

The product found showed that the combination of size and BE/ME makes leverage and E/P redundant by absorbing their roles in explaining average returns. Also, the combination of size and BE/ME proved to have powerful explanatory power. This result suggested size and BE/ME captured different dimensions of risk, and other unknown variables may capture other dimensions. As the second objective of the authors, the market beta was actually found to not explain the cross-section of average stock returns.

In the following year, Fama and French (1993) aimed to extend the research from Fama and French (1992) by trying to explain bond returns and changing their testing approach. The authors tested a three-factor model which includes excess market return as a market factor using Black et al. (1972) time-series regressions. The portfolio SMB (small minus big) is created to mimic the risk related to size. The portfolio is composed by the monthly difference for returns from the three small-stocks portfolios and the three big-stocks portfolios. The HML (high minus low) portfolio has a similar goal, but for the book-to-market equity risk. It is the monthly difference from the average returns of the two high BE/ME portfolios minus the two low BE/ME portfolios. They also expanded their approach by trying to identify a risk factor that may help explaining variation in bonds. The bond factors used are two, one related to maturity and the other to default risk. A market portfolio of stocks and mimicking portfolios for the two variables considering monthly returns for bonds and stocks were regressed into portfolios built with twenty five stocks.

The results implied size and book-to-market equity proxy for sensitivity to common risk. The base for this result came in the fact that, no matter what else is in the regressions, the mimicking portfolios captured common variation in returns. The three-factor model (that includes excess market returns, size and book-to-market equity) found

intercepts close to 0, which suggests the model does a good job explaining the cross-section of average returns. The market factor ends up being responsible for explaining the difference between the average returns on stocks and one-month bills, a job the proxies fail to achieve. For the bonds, the two term structure captured most variations, also explaining average returns for bonds. The stock returns were also found to be linked to bonds as they share variation in their term-structure.

With solid evidence showing size and book-to-market equity have explanatory power for stock returns, Fama and French (1995) focused on trying to find if that relationship is related to earnings. At the end of their previous work, they acknowledged that understanding why such factors can be used as proxies for different dimensions of risk was important. This paper aimed to find explanations by testing the relation of those factors to the behavior of earnings. Firstly, they checked whether size and BE/ME reflected differences in profitability. Then, they analyzed the behavior of earnings around portfolio formation and aimed to show how the performance of size and BE/ME related to earnings.

Both size and BE/ME found relationship with profitability, but the effects of size in earnings were due to low profits of small stocks. As for BE/ME, for five years after portfolios are formed, low-book-to-market equity firms remained more profitable when compared to high. Their results also found no evidence supporting irrational pricing, as they concluded the market makes unbiased forecasts of earnings growth. The possibility presented by the authors based on the the slopes and equity results for BE/ME was that this variable may be a proxy for relative distress. Although the results found links in the behavior of BE/ME and size factors in earnings and stock returns, the lack of evidence proving the book-to-market factor in earnings drives the book-to-market factor in returns leads to conclude this may not be a final explanation.

The three previously reported works from Fama and French used only a set of portfolios built on size and book-to-market equity to explain the issues that were not explained by the CAPM, the so called "anomalies". This approach was criticized as it could have some sort of bias in the results. A previous work from Lo and MacKinlay (1990) had already pointed out for that type of practice. Lo and MacKinlay affirmed that, as the selection of stocks that formed portfolios were not random, but organized based on characteristics, this type of practice may create data-snooping biases. In their tests, they also found evidence suggesting the biases motivated by grouping because of theoretical motivations are strong. To address this issue, Fama and French (1996) tested the model using 25 stocks portfolios sorted by size, earnings/price, cash flow/price, book-

to-market equity, past sales growth, long-term past return, and short-term past return. Another objective that drove the tests on these anomalies using the three-factor model was the belief of the authors that the anomalies were all related and could be explained by the size and BE/ME variables.

The outcome obtained showed that, besides short-term returns, all other anomalies were accounted for in the model. The three-factor model captures returns from portfolios sorted on E/P, C/P and Sales growth. Low E/P and C/P and high sales growth imply lower expected returns. The reason behind this result is that those ratios are typical from strong firms, which have negative slopes on HML, similar to low BE/ME stocks. The contrary is also true, firms with high C/P, high E/P and low sales growth are similar to high BE/ME stocks and have higher average returns. As for long-term past returns, the effect is also captured. Stocks that have a low long-term past performance tend to have positive slopes for SMB and HML and higher future returns, the opposite is true for high long-term performers. Those results suggest the model is a good description on average returns.

In another important research, Fama and French (1997) changed the subject from factor models to the costs of equity. They used both CAPM and the three-factor model to show how inaccurate calculating costs of equity can be. The first problem they reported was not knowing which asset pricing model to use. Since there is no consensus in the literature, even though the CAPM was the common choice for the time, the three-factor model proved to be an interesting option to consider. In the author's calculations, the differences between the models in cost of equity values reached 2% per year, showing how important is the model choice. The other main problems are related to imprecision in estimates of risk loadings and risk premiums. The literature showed that risk loadings vary through time, making it hard to calculate them in a given period. As for the risk premium, its estimates have too much of a wide range. In the 1963-1994 period, Fama and French found 5.16% average returns with 2.71% standard error, which makes it very hard to use the historical premium to predict the expected premium.

At the end of the century, Fama and French published yet another important article in Fama and French (1998). Previous asset pricing literature uncovered that "value stocks", which are defined as stocks with high book-to-market equity, earnings to price and cash flow to price, generated excess returns in comparison to their low ratio counterparts. Fama and French extended this research on value premium testing its existence in international markets. They used an one state variable version of Merton (1973) ICAPM

or a two-factor version of Ross (1976) APT that considers global market return and a risk factor for relative distress to capture the value premium in international markets. Relative distress was found in previous literature to have relation with the value premium. This happens because the market undervalues distressed stocks and overvalues growth stocks, generating pricing errors and higher returns. The two factor model captured the value premium in 13 markets with sorts for B/M, E/P, C/P and D/P. Opposed to that, Merton's ICAPM without the addition of relative distress did not capture the effect. The average difference for high and low global portfolios sorted for book-to-market was 7.68% per year. This evidence showed the value premium was not a regional effect and could be seen in international markets.

With different objective than the Fama and French study, Zhang (2005) also studied the value premium. The author found odd that, even though conventional wisdom pointed towards growth stocks being more risky than value stocks, value stocks still had higher expected returns than growth stocks. To explain this anomaly, this study aimed to explain the value premium with tastes and technology, as well as by making use of neoclassical framework. The model relied its explanation for the value premium in costly reversibility and countercyclical price of risk. He pointed out that, in bad times, costly reversibility made it harder for value firms to reduce capital stocks than growth firms. In good times, growth stocks would invest more in capital than value firms, as the latter had unproductive capital becoming productive now. This showed dividends and returns covariation of value stocks would be higher with economic downturns. There was also a high dispersion of risk between value and growth strategies in good and bad times. With the use of rational expectations, value premium matched the risk dispersion between value and growth times the price of risk. When this price is constant, the differences in betas had to account for everything.

High average value premium was a result of the time-varying price of risk. The price of risk propagated the asymmetry, explaining the value premium. The link between risk and expected return to economic primitives that the author creates in his model allows for some conclusions: Value is riskier than growth, high book-to-market signaled to persistently low profitability (whilst low book-to-market does the opposite), expected value premium was atypically high when the value spread is wide and value and growth's earnings growth spread predicted value-minus-growth return. The model also provided some new refutable hypotheses, most notably that value firms disinvest more than growth firms in bad times and invest less in good times and the value premium is countercyclical.

The value premium subject had also been focused on a notable previous study a few years before Fama and French in Lakonishok, Shleifer and Vishny (1994). Noting that previous literature found evidence showing value strategies generated abnormal returns but failed to explain the reason for those returns, the authors aimed to clarify the reasons why such strategies work. They formed portfolios based on past 5 year using NYSE and AMEX data from 1963-1990 and analyzed the performance and characteristics of those for the subsequent 5 years. There are two main ideas explored by the authors in their framework. In the first, they used multiple versions of the contrarian model to examine whether value strategies actually outperformed "glamour" stocks. To do that, information on past growth in sales, earnings, and cash flow, and expected performance were used to measure past performance. Value stocks were defined as the ones which have done poorly in the past and had expectations to continue doing poorly. In an opposite manner, glamour stocks were those which have done good and were expected to continue doing good. The second idea argued that value strategies earned higher returns because they were riskier. There were two main reasons that motivated their tests comparing riskiness: to check whether value stocks underperformed glamour stocks with frequency and their performance in economic recessions.

The results found by the authors provided no evidence suggesting value strategies were riskier. Value strategies actually outperformed glamour strategies in a variety of ways. Even though future growth rates were highly mean reverting, investors continued to base their prediction in past growth. Evidence showed a systematic pattern of errors by investors, suggesting they were not adapting to the situation. The most likely reason to explain this behavior was an overreaction both to good and bad past performers by investors. This behavior was also compatible with the reluctance in updating expectations based on new evidence reported in Edwards (1968). There is a variety of explanations suggested by the authors on why investors would prefer glamour over value strategies. One of them is the tendency to invest in "good firms", having the conception that you would not lose on some big company. Fund managers might invest in glamour firms as those provide a more "prudent" path. That also may be a reason on why funds underperform. Value strategies tend to need longer horizons, meaning investors looking for short term gains would not consider them.

This behavior components compatible with the results from Lakonishok, Shleifer and Vishny (1994) are incorporated in a model later in the decade by Barberis, Shleifer and Vishny (1998). The authors aimed to explain the empirical evidence that showed over

and under reactions to news. Their methodology consisted in proposing a parsimonious model considering psychological evidence. Some of this psychological evidence are the representativeness concept and conservatism concept respectively introduced by Tversky and Kahneman (1974) and Edwards (1968). The first one accounts for the tendency to not consider probability and base judgements in recent history. The second represents the reluctance to consider new evidence for updating the present concepts. It is easy to see representativeness is compatible with overreactions and conservatism is compatible with underreactions. The model has one investor and one asset, with the investor's beliefs affecting prices and returns. In the model, asset earnings follow a random walk, but investors do not know that. Instead, they believe those earnings move in mean-reverting or trend. Within every period, the investor's beliefs are updated using Bayesian statistics. The model is able to capture and predict price overreaction and underreaction. When exposed to a consistent pattern of good or bad news, stock prices overreact. Strength and weight of the information are important aspects that impact the reaction based on the beliefs.

In the consecutive year, Campbell and Cochrane (1999) was published. It is a very celebrated article that also tries to explain stock market behavior with changes in aspects from the representative agents. Campbell and Cochrane adapted the traditional consumption-based model by introducing habit formations. The main area they tried to explain is the fundamental sources of risk in expected returns. Their goal was to be able to capture features of the stock market and its fundamental risk using this adaptation. The authors used the standard representative-agent consumption-based asset pricing model with the addition of habit formations in the power utility function. They explained that, in economic downturns, consumption declines towards the habit, leading to a rise in the utility function curvature, which means a fall in asset prices and growth in expected returns. The model considers a constant risk-free rate. Consumption growth is modeled as an i.i.d lognormal process based on the same mean and standard deviation of actual post war measures. After the model is formed, they generated artificial data based on the model to compare patterns with reality. Adding actual consumption data and seeing if the model accounted for the fluctuations in stock prices is the last step in their methodology.

The addition of habit formations helped the model to explain some of the questions from the asset pricing literature. The model's artificial data captured some patterns from the stock market. It replicated some important information seen in stock markets, such as the volatility of stock price/dividend ratios or returns not being accounted by

changes in expectations of future dividend growth rates. When actual consumption data was added to the model, it accounted for fluctuation in stock prices and performed really well explaining behaviors in the stock market.

A different framework is also used in Bansal and Yaron (2004). They used investor preferences previously introduced in Epstein and Zin (1989) to develop a model that made use of a measure of economic uncertainty to explain stock returns, returns' volatility, dividend-yield and the risk-free rate. The pillar in the methodology was to model consumption and dividend growth rates to contain economic uncertainty and a predictable component. The model introduced involved this measure of consumption and growth rates as well as Epstein and Zin's preferences, which enables separation between Intertemporal Elasticity of Substitution (IES) and risk aversion. The assumption of the model is that when IES is above 1, investors require larger equity risk premia because of the economic uncertainty. The model also contains a consumption volatility channel, which is important to explain the volatility feedback effect.

The equilibrium model found that economic uncertainty explain questions in the asset market. News related to growth rates impacted price-dividend ratio and the equity return. The price-dividend volatility could be attributed to economic uncertainty and variances in the growth rates. The growth rates had a slightly persistent component, distinguishing from previous literature which were i.i.d. Even though it is hard to argue which of the processes is better, it is important to point out the results from this article with the persistent component helps justifying some asset market puzzles. Consumption volatility is found to be time-varying factor, and this variance ratio was found to be increasing. All these results are based in an IES of 1.5 and risk aversion of 10. The IES results may be downward biased, which is led by the presence of fluctuating consumption.

In the beginning of the millennium, Fama and French continued their path of successful articles with Fama and French (2002). They focused on studying the equity premium by estimating the unconditional expected stock returns from 1872 to 2000. For that purpose, they used two models: an earnings growth and a dividend growth model. The methodology used on this paper was based on the logic that average dividend yield plus the average rate of capital gain results in the average of stock returns. When earnings-price ratio and dividend-price ratio are stationary, it is possible to derive two models from this logic: a dividend growth model and an earnings growth model. In the model composition, the average returns were explained by the dividend yield plus the earnings or dividend growth ratio depending on the model. The period studied is from 1872 to 2000.

The 1872 to 2000 period results for the expected versus realized equity premium are underwhelming. The expected average returns results ended up to be far lower than the empirical values. The dividend growth model found a 3.54 percent per year return while the realized average returns were 5.57 percent. Taking a closer look in the period, the authors noticed the 1872 to 1950 estimate of 4.17 percent was very close to the realized value of 4.40. This indicated the posterior fifty years were the ones the model had a poor prediction. The period from 1951 to 2000 indeed had bad results with the two models, the expected equity premium being 2.55 with the dividend growth model and 4.32 with the earnings growth model, in comparison to the observed 7.43 percent. Authors pointed out three reasons that might suggest this difference was not so high. The first lies in the fact that estimate from fundamentals are more precise than estimates from average stock returns. The 1951-2000 period equity premium Sharpe ratio from average stock returns is double the value from the 1872-1950 period, while the equity premium Sharpe ratio from dividend growth is similar for the same period. And third, book-to-market ratio relation with returns on investment suggest the estimate average returns are higher than it should be.

In a paper comparable to the previous asset pricing literature on stock market volatility of Shiller (1980) and LeRoy and Porter (1981), Campbell et al. (2001) examined the historic path of market, industry and idiosyncratic volatility. The primary focus of their research was to capture trends in firm-level volatility, as it is a less studied area than aggregate volatility. As secondary objectives, they studied cyclical patterns of the three volatility measures and the stocks needed to have a well diversified portfolio. The methodology was to use daily data from individual stocks to construct monthly variances. With that data, they provided a simpler composition of volatility, which had no need for estimation of covariances or betas for industries or firms. Since the goal of the paper is not forecasting, there was no use for a parametric model to be proposed.

Market and industry monthly data found no evidence of a trend in volatility. For idiosyncratic volatility in the other hand, an upward tendency is found between the 1962 to 1997 period, with firm-level volatility practically doubling in the period. All three measures of volatility increased in periods leading to economic recession. Results for correlation among stocks indicated those diminish through time, a product of the increase in idiosyncratic volatility.

An important variable that inspire researches in the early two thousands is liquidity. This variable was studied in previous asset pricing literature, but differences in

measures of the variable encouraged authors to review the relation between liquidity and stock returns. Amihud (2002) made use of a illiquidity measure called ILLIQ to examine the over time relation of illiquidity and stock returns. This measure is the daily ratio of absolute stock return to its dollar volume, averaged over some period. He recognized this was not the best measure for illiquidity, explaining that he used it because other measures do not have the data necessary for an over time study.

In a similar study, Liu (2006) tested liquidity as a main factor to explain the cross-section of stock returns. Liu introduced a new liquidity measure, the standardized turnover-adjusted number of zero daily trading volumes over the prior 12 months, which focused on the trading speed dimension of liquidity. His main goal was to test the predictability power of the liquidity measure as well as its ability to explain documented anomalies. The new measure captured more than one dimension of liquidity, but focused on the trading speed. It appeared to be consistent with intuition as the less liquid assets gravitated towards small, value, low-turnover, high bid-ask spread, and high return-to-volume stocks. Afterwards, the author developed an augmented CAPM with two variables: a market factor and a liquidity factor. The latter was constructed around profits from the portfolio that buys a dollar of a low-liquidity portfolio and sells a dollar of the high-liquidity portfolio. At last, Liu tested the predictability of the model and the ability to explain anomalies such as size, long-term contrarian investment and fundamental to price-ratios.

In a third paper, Pástor and Stambaugh (2003) studied the relation between marketwide liquidity and stock returns. The authors aimed to check whether fluctuations in aggregate liquidity are behind cross-sectional differences in stock returns. Their main goal was to determine if liquidity is in fact a good candidate for priced state variable. The authors started from the principle that investors need higher expected returns to hold stocks that are more sensitive to liquidity fluctuations. They used a monthly aggregate liquidity measure in a model that accounts for four other factors: market, size and value from Fama and French (1993) 3-factor model, and a momentum factor. For the tests, ten portfolios were sorted according to predicted liquidity beta for each year.

The results showed liquidity was an important priced factor in stocks. The decile of the least liquid stocks have on average 0.682% per month higher returns in comparison with the most liquid decile for Liu (2006), which is similar to the 7.5% per year value difference found in Pástor and Stambaugh (2003) between the top and bottom decile of the liquidity betas. Amihud (2002) found the relation to be seen across stocks and over

time. Tests on the models showed the liquidity factor was highly negatively correlated with the market. The liquidity premium continued to be significant even when removing the decile of assets which were not traded for the most days. That difference represented the superior returns needed for holding an illiquid stock. A reason for these results is the representative investor being risk averse, therefore he would aim for less risk and more liquid assets if they foresee a recession. Another common finding from these researches is that small and high book-to-market stocks are less liquid, leading them to be the most affected by the liquidity effect.

Since the start of the millennium, there has been a big increase in the asset pricing literature branch that identifies problems within the methodology. Campbell and Yogo (2006) questioned the use of t-tests for testing stock return predictability. They believed not only variables are highly persistent but also their innovations are too correlated with stock return innovation, causing the t-test to have non-standard distributions and misleading results. Their research came up with some contributions to the literature. First they argued that, when the persistence of a variable is known, a Uniformly Most Powerful (UMP) test was a good way for evaluating the advantages of different test procedures. The second contribution was a new Bonferroni test based on a UMP test. The main characteristics of this test were that it could be implemented with standard regression methods, it was valid under more general assumptions and it was more efficient in terms of Pitman Efficiency. A third contribution came in a pre-test indicating if a t-test was possible or if its results would be misleading.

Welch and Goyal (2007) aimed to examine previously introduced variables to see if they really predicted stock returns. They focused on testing whether success claimed on predicting the equity premium was indeed possible and if the models would help an investor. The authors re-examined variables used to explain asset returns with the same methods, time-period and estimation frequencies. They analyzed the in-sample and out-of-sample performance of the variables as well as if the model was useful for an investor looking for market timing. The findings suggested no models performed well. The only exceptions were some models that deserved more investigation on very-long term frequencies. The results suggested most of the models outcomes seemed spurious or unstable. The models performed well during the period of the oil shock (1973-75), and by excluding it the performance would be even worse. No model had superior out-of-sample performance in the last thirty years, and very few had good performance in sample. At last, the out-of-sample performance of the models suggested none of them would help an

investor looking for market timing.

In Lewellen, Nagel and Shanken (2010) the author's goal was to show the usual tests for assessing asset pricing models might be misleading. The motivation for that came with the fact that models with completely different economic variables with no relation yield good results in explaining size and book-to-market effects. Four suggestions were made by the authors to improve asset pricing testing. The first was to test models in a bigger variety of portfolios. Second solution was to impose limits to risk premia, as the problems were aggravated when those were estimated as a free parameter. They argued the problems are less severe for GLS in comparison to OLS regression, so it was advised to use those. They made it clear this was an imperfect solution, but it suffered from problems in less occasions than OLS. Last solution they advised was to use confidence intervals in the place of p-value and point-estimates. The main benefits explained by the authors for using confidence intervals were that they revealed high sampling errors in a more clean and easier way, and also they did not need a decision on a null hypothesis.

More recently, Harvey, Liu and Zhu (2016) also questioned asset pricing tests. They believed the criteria for significance was too low as, since there was extensive data mining for so many years, the hurdle should be higher than the usual 2.0 t-statistic. The methodology utilized by the authors was to use a new framework that allowed for multiple hypothesis testing. They used it to test 316 factors presented in previous papers starting in 1973, and from those they determined thresholds to be considered for statistical significance up to 2032.

Empirical results found by the three researches corroborated their critique. Results from the empirical work of Campbell and Yogo (2006) showed that, according to their pre-test, using a t-test might be misleading for dividend-price ratio and smoothed earnings–price ratios. Using Lewellen, Nagel and Shanken (2010) methodology, none of the five models tested performed well. The main point of their critique lied in the fact that the models used high cross-sectional R^2 in explaining 25 size-B/M portfolios to claim success. That was a problem based on Fama and French factors already explaining 80-90% of the cross-sectional variation, meaning it would only take one factor to be weakly linked to Fama and French (1993) SMB or HML factors. Harvey, Liu and Zhu (2016) tests determined a threshold of 3.0 t-statistic for today's researches. The factor analysis lead to a conclusion that about half of the published significant factors ended up being false using their approach (158 out of 296 under Bonferonni).

Within that context, authors tried to search for new variables and methods hoping

to improve previous work. Barro (2006) uses Lucas (1978) tree-model with addition of a probability of rare disasters to explain puzzles such as the high equity premium, volatility and low risk-free rate. He argued that there are two main reasons for probability of rare disasters explaining stock returns. The first one is uncertainty makes people shift toward risk-free assets. The second is that people also tend to accumulate more assets with that uncertainty, meaning P/E ratios will fall or rise depending on which effect is stronger.

Gabaix (2012) and Wachter (2013) continued that line of work from Barro (2006) that relates rare disaster probability as a possible variable to explain stock market puzzles. Both researches based on Barro try to improve his framework. They used time-varying rare disaster probability instead of fixed ones and representative agents with different preferences then the used by Barro. Gabaix (2012) and Wachter (2013) used representative agents with recursive preferences (or Epstein-Zin preferences), which were introduced in Epstein and Zin (1989), instead of power utility preferences used in Barro (2006). Gabaix differentiated his framework by using linearity generating processes to keep expressions in closed form. He also used Barro and Ursúa (2008) disaster values for calibration.

The three authors found very similar results in their research as time-varying disaster probability was not only revealed to explain the volatility, but also to increase the equity premium. Gabaix found that risk premia decreased following good news and increased following eminent disasters. Another interesting find is that the model generated predictability, as a High P/D ratio was found to have low returns and a Low P/D ratio to have high returns.

Volatility comes back as a subject at three articles that rather than trying to explain the volatility found in stock returns, tested its power for explaining those. In Ang et al. (2006) the authors tested if volatility was a priced risk factor for stock markets and estimated its aggregated price. To do that, they examined the relation between idiosyncratic volatility and expected returns. They used the cross-section of stock returns as opposed to options on the market, because it opened the possibility to create portfolios with different sensitivity for market volatility innovations. They also constructed portfolios build on idiosyncratic volatility to test if they provided a set with different average returns, a result that would be contrary to Fama and French (1993) model.

The result of the estimated price for aggregated volatility indicated the behavior for stocks with higher sensitivity to innovations. A negative result of -1% per annum found pointed out that stocks with higher sensitivity have low returns on average. The authors found results contrary to the previous literature in the idiosyncratic volatility tests.

Firms with higher volatility risk had lower expected returns and the firms with the highest had remarkably low returns. The results for both aggregate volatility and idiosyncratic volatility were robust for size, book-to-market and momentum. The low average returns for idiosyncratic volatility remained unexplained.

Three years later, the authors extended their work in Ang et al. (2009). The focus on this second research was to test the relation found in the previous article for other regions. The authors used Fama and French (1993) three-factor model to test stock market returns sorted on idiosyncratic volatility for 23 developed markets. They examined the relation in the developed markets to see if the results co-move with previous ones found for U.S data. The research analyzes past idiosyncratic volatility as well as present idiosyncratic volatility and expected returns. Afterwards, the authors controlled the results mainly for information factors that could explain the relation found.

The results found in other markets mimic the ones in U.S, with lagged high idiosyncratic volatility firms having low average returns. This outcome eliminated the chance of the relation being a regional phenomenon or data snooping. A second outcome was a co-movement in the difference of high and low idiosyncratic volatility between U.S and other markets. This commonality suggested the effect behind it was broad and hard to diversify. The expected returns difference found between stocks in the highest and lowest quintile portfolios were -1.31% per month. The authors ruled out all explanations based on market frictions, information dissemination, and option pricing for the effects.

A third article with comparable findings to those two is Bollerslev, Tauchen and Zhou (2009). The authors aimed to use the difference between implied and realized variation to explain stock returns in the post-90 period. The study used a stylized self-contained general equilibrium model. The authors used the difference between the "model-free" implied and realized variances, which they called the variance risk premium, to explain a fraction of the variation in stock returns. Their model generates a two-factor structure with the factors related to volatility dynamics of consumption growth. They argued that different volatility concepts in the model cover for risk factors, therefore giving it good predictability. The predictability found was strong for quarterly return horizon. The biggest finding of the research was that the difference between the model-free implied and the actual realized variation explains a fraction of the variation in stock returns between 1990-2007. The combination used of this variance risk premium and other predictors was found to be even better at predicting. These results reached suggested that both volatility and consumption risk played an important part in explaining stock returns.

Cooper, Gulen and Schill (2008) also look for significance in a new measure of a previously studied variable. The research used a year-on-year percentage change in total assets to represent asset growth. The motivation behind this choice lies in the fact that previous studies used components of firm investment and not a variable that captured the total. Value-weighted and equal-weighted portfolios are sorted by previous year asset growth. Authors compared t-statistic results for the variable with previous variables adopted by asset pricing literature. They also decomposed total asset growth, aiming to find reasons behind the effect observed. Data is from U.S stock market over the 1968-2003 period.

The results obtained are the following. In a value-weighted portfolio, firms in the lowest decile for asset-growth had average returns of 18% in comparison to 5% from firms in the highest decile. When adjusted for standard risk, the spread fell but was still high at 8% per year. Adjusted equal-weighted portfolio spread was even higher at the 20% per year mark. Low asset-growth stock returns surpassed high-asset growth stock returns at 71% and 91% of the years for value-weighted and equal-weighted portfolios respectively. The asset growth effect was also encountered to be persistent, with assets being affected by it even 5 years after the sorting year. Comparisons with book-to-market, lagged returns, firm capitalization, accruals and other variables, found asset growth obtained a t-statistic more than twice the value of those. The decomposition of total asset growth found a lot of components that predicted stock returns and had importance for different firms. With that said, total-asset growth is a good predictor by benefiting from the sum of those components.

The field is still broad, having multiple lines of research in the last decade. There are no consensus on a specific model. Even though this last decade has encountered good progress toward some of the anomalies and questions we've had, they are still very present. An example is the factor models, those still trying to find the best proxies to be considered for the variables. Barillas and Shanken (2018) is a recent study with the goal of comparing existing models and factor built models. The authors tested the models using a Bayesian procedure to compute probabilities for models built using a set of factors. Afterwards, they decomposed the models to see which of the conflicting factors from different models had the best results, and also tested which composition of factors would yield the best results. The model that resulted the highest posterior probability is a six factor model combining (Mkt, IA, ROE, SMB, HMLm and UMD). This model resulted in no redundant factors, which contrast to other models who had UMD being redundant,

for example.

The more sophisticated factor models however, are still not the most used by investors according to Berk and Binsbergen (2016). In this research, the authors used mutual fund data to derive the preferences of investors, and used that to test asset pricing models. They observed which opportunities were being recognized as net positive investment, and utilized that to infer investors preferences. Subsequently, the authors ran an OLS regression using t-statistic to assess significance and observing which model was closest to what the actual investors were using. The test results indicated CAPM was the model that explained more flows, thereby it was the nearest model to what was empirically used. That is surprising if you consider all the criticism over the years and how the model fails to explain cross-sectional variation in stock returns. This result also suggests the risks considered in augmented models are simply not contemplated by most of the investors.

One of the biggest critiques regarding the CAPM is its testability. Authors throughout the years questioned the possibility of empirically testing the most famous model in asset pricing theory. This issue continues to caught some attention, as Guermat (2014) proposed a method showing the CAPM is in fact testable. The main problems the literature suggested in testing the model are the fact that the variables are non-observable and the possibility of omitted variables. Guermat used both OLS and GLS to eliminate usual problems in testing Sharpe (1964) model. He argued that the only way of obtaining maximum R^2 for both was to use an efficient index. The desired value also would only be reached when there are no omitted variables. The logic behind the methodology, therefore, was the possibility to know when the index is right based on the value of the R^2 from both tests, thus one could proceed to formally testing the model afterwards.

The index found by the combination of GLS and OLS regressions would be efficient when the R^2 was at its maximum value for both, that eliminates the problem of non-observable variables and using the wrong index for them. As the maximum value was only obtained when there were no omitted variables, both main problems of testing the CAPM were solved by this approach. After the index was found to be efficient, he found that it was possible to proceed with the tests. It is important to point out that the results of the paper assumed no sampling error.

Another liability of the CAPM lies in its use for testing markets with selling constraints, a characteristic the model doesn't account for. Bai, Li and Qin (2017) tried to addres this issue by augmenting the existing model considering the constraints, giving

the model a good predictability not just for short-selling free markets, but also the ones with restrictions to it. The authors enhanced not only the CAPM but Fama and French's three-factor model as well.

The authors found a greatly improved R^2 (from 38.4 to 58.4%) for the markets with short-selling restrictions using the CAPM with the authors augments. The three-factor model encountered an improvement as well, not very big (from 61 to 65 per cent), but statistically significant. Those results proved the consolidated models had restrictions when applied to markets with short selling constraints. Also, it showed the considerations about the new risk factor made by the authors might have been right. They considered the usage of the new risk factor because non-shortable stocks have higher risk and higher expected excess returns than shortable stock, which comes from the "over-valuation risk", "the constraint-induced liquidity risk" and "constraint-induced information risk".

Along in the market characteristics theme, Chan and Kwok (2017) used narrow versions of CAPM used in Chari and Henry (2004) to test empirical evidence from the Chinese market liberalization. The data consisted in 856 stocks from the Shanghai market during the 8 month period between the reform's announcement and the actual implementation. The research found that risk-sharing mechanism explained 1/4 of the price revaluation, and also that the efficiency of price revaluation via the risk-sharing mechanism depends on the stock's turnover, analyst coverage and institutional ownership. Firms with global exposure were, in majority, less affected by risk-sharing than domestic ones.

An endless research proposition is the search for anomalies explanation and the significance of them. In that tendency, Hou, Xue and Zhang (2015) proposed a model inspired in the neoclassical q-theory of investment, which contemplates 4 factors: the market excess return (MKT), return differences between small and big size stocks (ME), and the same for low and high investment (I/A) and profitability (ROE). The authors used the model to test for 80 different documented anomalies, finding about half of them were not really significant at a 5% significance rate.

Frazzini, Israel and Moskowitz (2012) was another anomaly testing study. This research tested if strategies based on anomalies presented in the asset pricing literature would survive real world trade costs. They made use of a non-usual dataset, which contains nearly one trillion dollar of live trades from an institutional investor. The authors built the strategies with portfolios that accounted for real trading costs and tried to minimize it with some optimizations. Size, value and momentum strategies were found to be robust, sizeable and implementable, with just short-term reversals not surviving as a

strategy. Those results corroborated with the previous literature, showing the better documented anomalies are significant and do explain excess stock returns.

Some innovative studies test different roles for intermediaries in asset pricing. In He and Krishnamurthy (2013), the authors constructed a model in which intermediaries are considered to be the marginal investors. The explanation for such model was that most of the previous works did not consider those to be relevant, as they considered those to reflect their client preference. The model was developed considering intermediary sector to play an important role in asset market equilibrium. The authors used the dramatic increase of risk premia as a crisis factor. It was first established a normal level of risk premia and intermediary equity capital, and afterwards it was compared with crisis levels of both. Government policies efficacy was also tested using the model. The authors started with an abnormal 12% risk premium representing a crisis and they traced the recovery path after introducing 3 government policies: infusing equity capital to the intermediaries, lowering borrowing rates and direct purchase of the risky assets.

The results showed the model explained well the behavior during financial crisis. That happened in part because of the capability of the model in capturing the asymmetry during crisis. An average level of 3% risk premium was found when the intermediaries capital was within a good state. From the 3 policies, infusing equity capital was the one that yield better results. The fact this type of policy directly attacks the capital constraints of intermediaries is probably the reasoning for the result.

The purpose in considering intermediary financing was different in Adrian, Etula and Muir (2014). The authors utilized financial intermediaries as representative agents, as they considered that those fit assumptions made by modern financial theory better than households. Financial intermediaries participated in more markets than households, had a diminished transactions cost, utilized sophisticated models for trading and their investment strategies were continuously optimized based on forward-looking expectations. The goal of this study, thus, was to introduce a model centered in financial intermediaries to explain stock returns. To do that, the authors measured the Stochastic Discount Factor emphasizing intermediary behavior instead of households. The leverage of security broker-dealers was used as a proxy for the marginal value of wealth of financial intermediaries. The model performance was tested and compared to others from the literature. They used portfolios sorted on size, book-to-market and momentum. Afterwards, robustness tests were done.

The broker-dealer leverage as a single factor explained a variety of assets excess

stock returns. The factor was found to have high adjusted R^2 of 77% and low cross-sectional pricing errors around 1% per annum. With this criteria the model outperformed the Fama and French (1993) three-factor model as well as Carhart (1997) five-factor model. The impressive part lies in the fact that those models were created to explain excess returns in the portfolios sorted in this research. The authors also concluded that the leverage factor represented funding constraints.

In a more recent paper, the authors followed the steps of the previous articles and used intermediary asset pricing as the focus for understanding risk premia. He, Kelly and Manela (2017) offered a new perspective for understanding risk premia in seven asset classes using an intermediary asset pricing theory. Using intermediaries instead of households as a focal pricing kernel, brought the possibility of considering more sophisticated markets, such as derivatives and commodities, as households lack the expertise in those. The two key challenges considered within this approach were to identify financial intermediaries that were marginal investors in many markets and additionally measuring their wealth. To solve the aforementioned challenges, they used "primary dealers" that served as counterparts in NY Fed monetary policy implementation as the financial intermediaries and intermediary sector's net worth as the measure. Afterwards, the model was proposed with 2 factors, the excess returns on aggregate wealth and shocks to intermediaries equity capital ratio. The construction of the aggregate capital ratio was done by matching the primary dealer list with data on publicly traded holding companies from CRSP/Compustat and Datastream. The intermediary capital ratio was defined as the aggregated value of market equity divided by the same measure plus aggregated book debt of the primary dealers. Subsequently to the model construction, they performed cross-sectional asset pricing tests independently between classes as well as with them together. At the section prior to the conclusion, the authors executed robustness tests to their model. The data used is from the 1970-2012 period.

The exposure of assets to shocks in the intermediary capital ratio explained cross-sectional differences in average returns for all seven markets. Corroborating the model assumptions, the estimated returns are significantly positive for intermediary capital factor in all asset classes. There were two important economic implications that could be taken from their estimates. Assets that had low betas in the intermediary capital ratio shocks had lower expected returns in equilibrium, which means that primary dealers had high marginal values when capital ratio was low. Even though different asset classes involve different knowledge, those markets still produced estimated prices of intermediary capital

risk with similar degree.

Fama and French remained with high importance in the last decade. They continued their work on factor models and studied other key aspects of the stock market. In Fama and French (2010) they examined mutual funds who operate the U.S stock market with an equilibrium accounting perspective. Their approach bases on the belief that active and passive investing is a zero sum game. They used data history of funds to differentiate skill from luck and evaluate performance. The test to find which fund is a good performer requires long history to differentiate skill from luck, with skilled funds having a better history of winning. The performance results of mutual funds indicated very few managers have skill to produce enough expected returns to cover costs. The alpha for net returns to investors was negative for the majority of funds, and assuming a normal distribution for the true alpha they found 16% of funds had a net return greater than 1.25% per year and 2.3% had a greater than 2.5%. Those results implied mutual funds underperformed the CAPM, three-factor and four-factor models. The results being aggregated, there might be mutual funds with enough skill, but those were covered by the underperforming ones.

The performance of mutual funds had already been investigated in Berk and Green (2004). The authors used a rational model with mutual funds being a central figure of their research to explain some stock market behaviors. The lack of ability of mutual funds to beat passive strategies led to questions about the rationality of investors. They aimed to show that a rational model was compatible with empirical evidence. The authors created a model with three key features: Competitive provision of capital by investors to mutual funds, fund managers have differential ability to generate high returns and investors learn about ability from managers from past results. Although there is different ability between managers in their model, the returns are decreasing to scale in deploying these abilities. The model was used to try explaining behavior of funds and individual investors in the market.

The model reproduced many of the effects seen in financial markets, implying that these effects were consistent with rational and self-interested behaviors. The model captured the higher returns of passive strategies in comparison to active investing. Managers were able to increase the size and compensation of their funds to the point they were competitive going forward. These results demonstrated that was not a case of managers not having ability and relying on luck as previous literature suggested. What explained the variation in performance from funds was the chasing for high returns that makes the market competitive, with opportunities disappearing after a while. Opportunities, as well

as ability, are a scarce resource in the market. As all agents look for those, it results in no performance persistence of the funds.

The costs of active investing was already the focus of a previous research in French (2008). In this paper French investigated the amount spent in the stock market in active investing. The author compared gains and costs of active investing with a passive portfolio strategy. As an alternate goal, the study examined the structure of costs from today in comparison to 1980. The author used a representative investor with the market portfolio being the combination of all strategies. He assumed no net transfer between passive market portfolios and other investors. This assumption guaranteed a negative sum game in trying to beat the market. The primary reason was that since before considering costs, a gain in trading must be compensated with a loss, after costs that sum became negative. Another assumption he made was that active investors improved accuracy of financial prices in their search for trading gains. French also examined the composition of costs and tradings.

The results showed active investing was a negative sum game. Although some investors earned superior returns, those were rare and the majority of investors had worse performance than a passive strategy. The average cost difference in the period of 1980 to 2006 was 67 basis points of the total value of NYSE, AMEX and NASDAQ. A sub product of active investing for the society considered was price discovery. The amount spent by active investors converted in a positive externality in that sense, since their spending provided improvements in financial prices. The capitalized cost of price discovery was 10% considering expected U.S equity real return of 6.7%. That percentage was even higher considering U.S equity real return estimates in Fama and French (2002) resulted lower values. The composition analyses and comparison between 1980-2006 showed a shift towards mutual funds and funds of funds. Holdings from individuals decreased from 47.9% to 21.5%. In counterpart, open funds holdings rose from 4.6% to 32.4%.

Two years after their analysis on mutual funds, in Fama and French (2012) the authors aimed to test the role of size, value and momentum on 4 markets: North America, Europe, Asia Pacific and Japan. They did it using global and regional versions of the three-factor model introduced in Fama and French (1993) and the augmented four-factor model by Carhart (1997) that includes momentum. They tested if there was integrated asset pricing across the regions, how global models fared in regional markets and if the models were capturing international size and momentum patterns. Three regions found value premium that decreased with size as well as patterns in the momentum factor, Japan

being the exception. For global portfolios, the models passed the test and were useful for explaining stock returns, but in regional ones that was not the case. The models failed and the authors acknowledged the need for regional models to contemplate the specifics of each region. In respect to the regional models, the four-factor model with the momentum addition performed better than the three-factor in general.

A very key recent paper is Fama and French (2015), in which the authors expanded their iconic three factor model after observing the 20 years of accumulated critique. They augmented it with two new factors, proxies for profitability and investment, as they explained that their factor choice was based on the dividend discount model equation. The inclusion of two variables provided better explanation and performance for the model, which was believed to explain between 71-94% of the cross-section variance of expected stock returns. All factors were found to have patterns in average returns besides HML, which became redundant with the addition of the new factors. Two years before the five-factor model being published, Novy-Marx (2013) also studied profitability as an asset pricing factor. The author used a 4-factor model based on the following factors: a market factor, an industry-adjusted value, momentum and gross profitability. Portfolios were sorted on gross profitability, profitability and size, and profitability and value. Profitability ended up having almost the same power as book-to-market predicting cross section returns, with profitable firms generating higher average returns in comparison to unprofitable. The results found suggested profitability and size strategies were negatively correlated and the use of them together found great success in his research.

In the following years, Fama and French tested their new augmented model in comparison to the previous three-factor one. In Fama and French (2016), they tested the five-factor model focusing on the list of anomalies that was left when using it in comparison to the three-factor model. The results showed that the five-factor model accounted for anomaly average returns existent in three-factor model with the exception of two, one related to accruals and the other to momentum. The main reason for this improvement was that the returns explained that were not accounted, were actually part of the same phenomenon explained by included variables. Accruals were the one that seemed to represent the biggest issue, as the five-factor model did worse in the description of average returns, while other anomalies were at least improved in comparison to the three-factor model.

Fama and French (2017) went in the same line of testing the five-factor model, but this time with the goal being to check if the model improved from the previous one in a

global manner. The global results were similar to the ones from the previous papers, as they found very low average returns for small firms that had a high investment factor and low profitability in all regions. The tryouts for the global variation in all the international regions ended up failing, as the model was not in condition to explain asset pricing for all regions. All variables ended up being meaningful for NA returns, but that did not extend for all regions, as Europe and Japan had CMA and the investment factor being redundant. In Guo et al. (2017), the authors tested the three, four and five-factor model utilizing data from the Chinese stock market, the results being that the five-factor had a big improvement in comparison to the three-factor, but the differences between the GRS statistics for the four-factor and for the five-factor model were really low, in less than 0.05. In the same line as Fama and French, they also found CMA to be redundant, as it was in the Europe and Japan tests.

Another paper that found results that were robust for a regional stock market but did not replicate to a wider view was Baltzer, Koehl and Reitz (2019). This research aimed to determine the correlation between leverage and stock returns in the German and European stock market, but it only found relevance to the former, confirming the need for specific regional models.

Models with deviations from rational expectations have been a relevant topic as well. A few authors understand that full Rational Expectations (RE) hold too much the model, with some tweaks having the power to improve them in pursuit of better results. Barberis et al. (2015) is an example, as the authors used investor surveys that suggested investors had an extrapolative behavior, meaning they believed when the market was going well it would continue to go well and vice-versa, to build a non-rational agent representing 50% of the total agents (the other being rational). The interaction results ended up being quite interesting, as rational investors did not try to counter the 30% higher rise in stock prices based on the extrapolative behavior of their opposites. Adam, Marcet and Beutel (2017), which also used investor surveys, argued that RE, in terms of price beliefs, did not yield the right results, as it returns the opposite sign for PD ratios and Return Expectations. They adapted the consumption based asset pricing model from Lucas (1978) to deviate from full rational expectation and have subjective pricing beliefs adapted by the agents in a Bayesian manner (internal rationality). With that model, they found results far more explanatory than the ones using an RE model. Similar to Barberis et al. (2015), the authors acknowledged the usual model returns a negative relation between PD ratios and investors expectations. The problem was that investors expectation surveys pointed

quite the opposite as what was seen was the agents adapting their expectations based on previous gains, so if there was a surprise in comparison with their beliefs, they adapted and became more optimistic, which ended usually bringing an overoptimism.

One of the objectives of Adam, Marcet and Beutel (2017) was also to show that Rational Expectations (RE), in terms of price beliefs, did not yield the right results. To show that, they considered investors subjective expectations have a key role in understanding recent asset pricing flows by using a model with internal rationality, as the agents are rational but have subjective price beliefs. The model was tested using confidence intervals based on the Simulated Method of Moments (SMM), and results are compared to the ones in Lucas normal model with RE, the data usage are from the U.S stock market from the period of 1946 to 2012, with data for stock returns and investors expectation surveys.

The results found using the adapted model with internal rationality are far more explanatory than the ones using the usual RE model. The usual model returned a negative relation between PD ratios and investors expectations, which would imply that in booms the agents would be suspicious about the future and in busts they would be confident. The investors expectation survey points quite the opposite as what is seen is the agents adapt their expectations based on previous gains, so if there is a surprise in comparison with their beliefs, they adapt to that and become more optimistic, which ends usually bringing an overoptimism and overpessimism in the opposite case. That shows us how the beliefs can end up taking the asset prices out of their fundamental values temporarily.

Until this day, there is still a search for new factors to explain stock returns, even though a big sum of researches have tested and validated size and value factors and they have been found to be relevant in a lot of models. In Kuehn, Simutin and Wang (2017) for example, the model was built with the market tightness being the link between the labor market and asset pricing. The results were that market tightness negatively correlates with future stock returns. Park and Sohn (2016) used Campbell (1993) discrete time version of the ICAPM to discover a return innovation factor based on the state of the market variable. This factor was strongly and robustly priced across assets and had close relationship with momentum and liquidity.

Also in the search for variables that would explain stock returns, Brogaard and Detzel (2015) applied a measure for Economic Policy Uncertainty (EPU) introduced in Baker, Bloom and Davis (2016) as a proxy for observing the correlation between stock returns and EPU. The research found a negative contemporaneous correlation between changes in EPU and expected returns, but a positive relation at the two to three month

horizon. EPU is usually a cause of doubts in expected cash flows and discount rates, which could explain that correlation. The values found from testing are that a standard deviation in EPU measures translated into 1.5% increase in forecasted three-month abnormal returns. In Kelly and Ljungqvist (2012), the authors stipulated a proxy for information asymmetry based on brokerage closures. Results attained from the test were that increases in information asymmetry causes prices to fall and a reduction on demand of uninformed agents on risky assets.

Even though plenty of articles continued this search for new variables that could have a bigger explanatory power for stock returns, some of the more recent papers had the goal to improve the empirical research on previous ideas or to apply new techniques or variable measures for models. Studies on liquidity are an example of that. The previously studied variable became relevant again with authors discussing differences in measures of the variable. The authors in Butt Hilal and Suleman (2017) believed there was no consensual definition of illiquidity, and that might have an impact for risk analysis. They used different measures of illiquidity in usual asset pricing models to show the difference that it can cause in illiquidity premium using data from the Australian stock market. Their study showed the success of any asset pricing model in terms of magnitude of predicted risk premium was linked with measures of illiquidity chosen. That magnitude was subject to variation even within country analysis, meaning it was not necessary that a price impact measure consistently gave high premium for other markets.

In Marcato et al. (2018), the author aimed to look the premium for non-liquid assets. The vast majority of the asset pricing literature computes liquidity risk premium for liquid assets. This study helped explaining some puzzles about real estate, in particular, the two main facts. The first one was that some types of properties are preferred to others even if their risk/return profile would suggest the opposite. The other fact in view was that the overall allocation in a multi-state portfolio is lower than what an analysis would suggest. For the analysis of those issues, Marcato computed several liquidity proxies with the purpose of being used as measures to calculate the liquidity premium. Afterwards, he used three different models to assess the impact of liquidity premia for illiquid assets. At last, there was the usage of the models empirically to compute the liquidity premium to be embedded in the estimation of both ex-ante return and ex-ante risk for the UK real estate market. His research found that time on the market coupled with non-random returns could generate ex-ante risk, which is 30 to 40 per cent higher than the one observed in ex-post returns. The estimation of risk premia linked to market liquidity consistently showed

the significance of this risk factor. They found that, over time, premia were on average around 3.0 to 3.5 per cent, but they range between 1.5 per cent during rising markets and 10 during the most recent economic crisis.

Two recent papers focused on asset pricing volatility, but by different approaches. Manela and Moreira (2017) constructed a measure for uncertainty (which is called NVIX), using co-movement between news from the coverage of the Wall street Journal and volatility. A 3.3% excess returns increase for the next year was found at each standard deviation for the NVIX value. Nguyen, Prokopczuk and Sibbertsen (2019) tried to shed light in long memory volatility based on two estimators: the GPH and the Local Whittle. The authors found that the strategy of holding the investments with short memory in volatility and selling quickly the ones with long memory gave a per annum gain of 1.71%. Another finding was that anti-persistent volatility investments could earn up to 4.7% more than the ones with long memory in volatility.

Idiosyncratic volatility was also studied in a recent paper. Herskovic et al. (2016) showed that Common Idiosyncratic Volatility (CIV) could be used as a proxy for dispersion in consumption growth or household risk. The firm level cash flow volatility was correlated with the household risk, as the authors showed that the second can derive from the first because of the impact it has on its employees. In their methodology, the authors constructed a factor structure representing the idiosyncratic volatility, as they realized that volatility co-movement did not always arise from factors not represented in the model (residuals), and could be explained by a constructed variable. Afterwards, they created a heterodox agent model to be tested with the utilization of data from the Center for Research in Security Prices with more than twenty thousand stocks, in the period of 1926 to 2010.

The results showed that the CIV is highly related to returns, as firms with a higher CIV had an average 5.4% reduction in comparison to the low ones. Also, the cross-sectional differences in average returns for book-to-market, size, earning-to-price and corporate bond portfolios could be better explained introducing a CIV innovation factor into the model. Another result was that, based on the connection of the firm level volatility and household risks, it was plausible to say that the CIV could be used as a proxy to consumer growth dispersion as it was said before. Finally, the evidence brings us that shocks in the CIV are priced, which adds to the papers proposition.

More than thirty years after Chen, Roll and Ross (1986), Koijen, Lustig and Nieuwerburgh (2017) tried to connect macroeconomic state with asset pricing by mak-

ing use of firm cash flows and output risk. Comparing returns of value firm bonds and growth firm bonds in the business cycle, the authors found explanations about the superior returns that one has on the other depending on time. They created and tested an asset pricing model based on three factors: innovations to the CP factor introduced by Cochrane and Piazzesi (2005), differential exposure to shocks and exposure to the market return accounts for the aggregate equity premium. Those tests used U.S. stock market data from 1926 to 2012.

The authors encountered robust results and a 0.49% mean pricing error value per year. They were able to explain several portfolios, but were not able to do it in cross-section of momentum return to equity portfolios. There are three main contributions from the study, with the first being the aforementioned model. The second is that there was more covariance in value portfolios returns with innovations than in growth portfolios. This gives insight in the value premium puzzle (the difference between returns in value and growth firm bonds). That result also lean towards a corroboration of the risk based hypothesis, which explains that difference based on a higher risk existent in the value firm bonds. Again contemplating a risk-based explanation, the third contribution is acknowledging that value stocks experience cash flows shocks, as it has a lot of variance depending on macroeconomic state, which again can be explained by having a future low economic activity, therefore, risk.

Elshqirat (2019) tested the validity of the arbitrage pricing theory in the Jordanian stock market. The author used the Arbitrage Pricing Theory developed by Ross (1976), which contains a set of macroeconomic variables assumed to replace the systematic risk in the CAPM without specifying which factors to use. In the study, he tested different macroeconomic variables and examined their relationship with stocks returns. The author found only one variable with significance out of the four macroeconomic variables used, that being industrial producers' price index with a highly negative relationship. Even though both Exports and unemployment rate had positive relations with the returns, the value of that relation is insignificant, meaning that it does not explain changes in stock returns. Thus, the APT would not be the right tool to be used as a good prediction model of the stock returns (at least with the macroeconomic variables used).

Similar to those works considering macroeconomic variables, Bannigidadmath and Narayan (2016) used the time series predictive model proposed by Westerlund and Narayan (2012) to find which financial ratio variables is the most popular predictor. Also, they applied the tests not only to the market returns themselves but to sector returns, in-

tending to capture if sector evidence was stronger than market evidence and if predictors were able to explain all sectors. The authors used the time series predictive regression estimates as dependent variables and regressed them using the financial ratios shocks. Afterwards, these financial ratio shocks were used as the independent variables for regressing a sector profitability dependent variable, estimated using forecasted returns for a mean-variance investor with a mean variance utility function. The data used for those tests are daily, as the authors argued that it captures more information and there is a better fit for the lack of large enough time period from the Indian stock market, since the data for this market is only available starting in 1990.

The big result of the tests was that only the sectoral return predictability was strong. Dividend-payout ratio and dividend yield return were the two most popular predictors, as both predict returns in all sectors. There was difference in the sectoral returns, with some sectors being in excess of the market. Expected financial risks explained profitability in almost all sectors, while unexpected only in a few of them.

The use of recursive preferences was seen in Albuquerque et al. (2016), which constructed an asset pricing theory in which demand shocks played a central role instead of supply ones as per usual. These shocks played a key role in the equilibrium behavior of price-dividend ratio, equity returns and bond yields. They used two versions of the model: a benchmark one with sole purpose of observing time preference shocks, and an extended one in which consumption shocks together with dividend process are conditionally heteroskedastic. The model is estimated using SMM procedure with focus on the probability limit of the model-implied small-sample moments, using annual data from 1929 to 2011.

The demand shocks help explaining the equity premium if, and only if, risk aversion and elasticity of intertemporal substitution are greater or lesser than one, which was satisfied in both models. The extended model accounts for the correlation between stock returns and macroeconomic variables such as consumption, output and dividend growth. The authors found that the valuation risk (excess returns due to volatility in the shocks, as defined by them) was a more determinant factor on asset returns than conventional risk.

5 RESULTS AND DISCUSSION

The asset pricing literature evolution shows a lot of tendencies and discussions that continue throughout the years. In this review, some of this theory tendencies and the objectives the authors had were brought, as well as the critiques and counterpoints made by other authors to those theories. The goal of this session is to group similar topics that are recurrent over time and explain their importance and context within the theme.

An obvious pattern to note is the search for variables that are able to explain stock returns. The usual approach is to relate variables that have possibility to account for some type of risk, thus affecting the price of a stock. This approach is seen in the early literature, with the introduction of CAPM relating risk and returns. Papers which used the CAPM and deviations with other measures of risk such as Sharpe (1964), Lucas (1978) and Black et al. (1972) tested for different measures of risk.

As they highly affect the economy, macroeconomic variables are seen in many studies along the years as proxies for risk measures. The first paper to test these macroeconomic variables is Ross (1976), but the pattern to test those continues in Chen, Roll and Ross (1986), Koijen, Lustig and Nieuwerburgh (2017) and Elshqirat (2019). The logic behind these researches can be easily seen: as macroeconomic variables are measures of the economic state and this is mainly dictated by the production of companies and consumption, changes in those might be the best account for risk in stock market. This means the returns and prices in the stock market are a reflection of what is seen in the economy. However, a problem encountered is that, since there is an immense amount of factors that affect the economy and almost all of those can be taken as risk factors, it is impossible to present a model including all variables. This impossibility makes the focus of the authors throughout the years shift towards finding the single variables to capture the most risk and, by being more powerful, turn other variables redundant in explaining stock returns.

The firm characteristics are also tested as possible variables to explain stock returns. This contrasts with the macroeconomic variables method, as it considers characteristics for each individual stock instead of considering proxies for risk in the whole economy. Fama and French work in the nineties has big importance in this instance. Their three-factor model presented in Fama and French (1993) tests the explanation power of size and book-to-market equity, two firm-level characteristics, on stock returns. Similar researches can be seen in posterior years, as Amihud (2002), Pástor and Stambaugh (2003), Liu (2006), Marcato et al. (2018) and Butt Hilal and Suleman (2017) test for an-

other firm-characteristic: liquidity. Those papers believe there is a connection between the liquidity of a stock and its return, as problems with liquidity are linked with risk increase. They also show another side of the problem faced in the asset pricing literature, which is that there are many ways to measure a variable believed to explain stock returns. This variety of measures provide different results, making it hard to define the best one for the variables. This focus on firm characteristics continues in recent researches. Works such as Novy-Marx (2013) and Fama and French (2015) use profitability to explain stock returns and understand the behavior of portfolios of similar stocks. Moreover, all those works in firm-level characteristics are very important to understand patterns in the stock market, an example of that being the pattern seen in portfolios of small stocks with high investment and low profitability reported by Fama and French.

A third wheel that can be seen in the variable searching literature relates economic uncertainty and stock returns. This line of thought believes economic uncertainty and the probability of disasters provide a more direct proxy for the economic state than macroeconomic factors, thus, a better risk proxy to account for the risk premium. In recent papers such as Barro (2006) and its posterior augmentations Gabaix (2012) and Wachter (2013), the authors try to explain stock returns with consumption risk and a probability of rare disasters. Relating economic uncertainty and stock returns, Brogaard and Detzel (2015) and Bansal and Yaron (2004) also provide an interesting framework. Even though the results of those lines of research show promise, there are very few papers among this literature.

A second trend brought over time focused on the volatility of stock returns. Studies using volatility to try to explain stocks returns, discussing its historic and on idiosyncratic volatility, are among the ideas seen in the literature. Since Shiller (1980), LeRoy and Porter (1981) and Flavin (1983), there is a branch of the asset pricing literature who focuses on observing patterns in volatility. The importance of observing these patterns is to better understand if stocks carry this volatility through time and how long upward and downward tendencies usually last. An example of trying to understand these behaviors and what may lead to them is seen in the papers that study idiosyncratic volatility. The start of the millennium coincides with the first time idiosyncratic volatility is into the spotlight in studies such as Campbell et al. (2001), Ang et al. (2006), Ang et al. (2009) and Herskovic et al. (2016). This type of study aims to verify if firm-level volatility can be accounted as a firm characteristic that explains stock returns. Moreover, the historical of this firm-level characteristic may lead to a pattern in individual stocks, which would help to predict the future behavior of stocks with similar patterns.

Two of the most controversial tendencies were left for last: asset pricing tests and model properties. The latter is seen in multiple articles that challenge usual properties of models, such as rational expectations. The ignite of those challenges happened as some authors found that rational expectations did not yield the right results in the asset pricing literature. The stock market seem to respond to news and price changes in a non-rational matter all the times, with Bondt and Thaler (1985), Long et al. (1990) and Barberis et al. (2015) showing overreaction traces in market agents being an example of this behavior. With those traces being noticed, new models of investor's expectations started to be introduced, with the first and most famous one being formulated in Epstein and Zin (1989) and tested in Epstein and Zin (1991). Psychological evidence was also taken into account in some of these formulations, with Barberis, Shleifer and Vishny (1998) and Campbell and Cochrane (1999) incorporating some of this evidence to the representative agents. As the representative agents provide one of the foundations of models is obvious to see how important researches in this area are. They model some empirical facts and provide important knowledge about the behavior of investors, which is fundamental not just in understanding the stock market and how it operates, but also in opening the discussion for the validity of a rational expectations model in the stock market.

Rational expectations is not the only model property defied in the literature. In recent literature there are some papers that consider constraints and other interesting aspects. Kelly and Ljungqvist (2012) consider a measure of asymmetric information and check for its relation with stock returns. The idea behind the tests is that higher levels of asymmetric information will impact the riskiness of an asset, consequently affecting its price. The substitution of the classical representative individual investor (household) for financial intermediaries as a pricing kernel is seen in He, Kelly and Manela (2017) and He and Krishnamurthy (2013). Those researches believed that, since intermediaries operate in more sophisticated markets and use more sophisticated models, they might be better as a pricing kernel then households. At last, Bai, Li and Qin (2017) consider short-selling constraints, a mechanism that is seen in some markets. The inclusion of those constraints might help explaining the returns in such markets, as the strategy to buy and short-sell stocks is common, and the impossibility to practice it affects price flows. All these different frameworks help in understanding the puzzle that is the stock returns. Although the aforementioned papers each purpose one of these deviations from typical models, they also open up the possibility for future models to test a combination and reach the best

balance of these different properties.

The last topic presents itself as one of the most problematic in the literature. The asset pricing tests are sources of criticism and disbelief since the first steps. As the usual set of econometric and statistical techniques were not enough for testing the stock market data, authors saw the necessity of developing frameworks of their own for testing models, which is seen in Fama and MacBeth (1973) and Black (1972). However, their framework started to suffer critiques, with some examples being: claims of bias, of the significance being too low and the results of the tests being spurious. Already in the decade those papers were published, Roll (1977) pointed out flaws and problems with those tests. This assessment continued to identify problems, with more than a decade later Lo and MacKinlay (1990) indicating bias problems in the techniques used by that time. Low significance to accept the results was another problem shown in a recent paper Harvey, Liu and Zhu (2016). This doubt even evolved to a point that some authors were skeptic about the validity of some variables. Welch and Goyal (2007) verified if previous results found suggesting relation between some variables and the stock returns were not spurious.

The doubt and disbelief continued through these days, as an iconic paper in Lewellen, Nagel and Shanken (2010) tested a methodology developed in the paper in models from this millennium that yielded good results. The analysis done provides bad results for the models, showing how the methodology is still problematic. An important thing to point out is that there are continues tries to address the issues presented in asset pricing tests, for example the methodology developed in Campbell and Yogo (2006) that introduced a pre-test to be done before using a t-test. However, this topic continues to be a delicate aspect of the literature, with not many convictions and a lot of uncertainty presented.

The gathering of these trends shows us how complex is the stock market puzzle, with many variables to be tested in the chase of the best explanatory power for stock returns, but also the necessity of fixing the problems presented in those tests. Even a wider picture shows us the need to understand the market properties and its patterns in volatility to better figure out the behavior and tendencies of stock returns. Although the literature is going in the right path trying to understand all of those phenomenons and the increase of attention in the subject provides a good omen for the future, the focus should be more targeted in the asset pricing tests. A more complex and asserted theory on testing might be the safety net needed for the authors to be assured of their results and what path to pursue.

6 CONCLUDING REMARKS

This paper brought a review of the asset pricing literature focusing on the ideas that remained relevant to this day. The objective of this review was to present the most important articles throughout history as well as the evolution of the theory. Very recent papers are also included, as a showcase of which ideas are currently being discussed.

A review is important to present the contextualization on the evolution of the literature and which ideas remained important over the time. There aren't many papers that bring this methodology since most of the authors are interested in testing a theory of their own. This lack of reviews and the fact that the asset pricing literature completed fifty five years bring big importance to this work, as it can be used to navigate and understand the history of this topic.

The literature has evolved and came a long way since its start in Sharpe (1964). As the stock market grew to have more importance in the economy, more authors started to pay attention at it and try to understand market behavior. This increase was obviously beneficial as a flurry of new ideas and perspectives can be seen after the nineties, which marks the period in which the literature started to have more notice. This review concluded that, despite the intensification of researches, not many consensus were reached in the literature, with a lot of discussions dragging themselves through the years.

The results and discussion session shows us a part of the picture painted in the history of the literature, as well as the problems faced and needed to be addressed. A focus in understanding the stock market and its behavior is seen mainly through three focuses: a search for new variables or new measures, patterns in market and idiosyncratic volatility and investigation of market properties. The many studies involving different ideas and methodology in those areas shows us how the lack of consensus mentioned in the previous paragraph exists. However, these discussions altogether bring a lot of insights for posterior studies. The chance for mixing previous methodologies and testing different ideas jointly presents itself as a good path to pursue over time.

The results also shows us that an important part of researches that never reached agreement is the testing. Since the very first asset pricing tests, the criticism always happened, with claims of results being spurious or biased. Throughout the years some authors tried to give solutions and new methodologies for testing, but the topic is still fragile. That said, the main focus of the literature was never on top of this subject, and this difficulty to reach a more accepted testing technique should be a big priority in the

following researches. In that context, a possible path to pursue in the future would be to do a detailed review with empirical testing of all the testing literature. A comparison of the performance results of asset pricing tests would be beneficial for future researches, enhancing our understanding and knowledge in the segment.

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