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Stochastic Processes
Research Paper

Mixing conditions of conjugate processes

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Abstract

In this paper, we provide sufficient conditions ensuring that a ψ–mixing property holds for the sequence of empirical cumulative distribution functions associated with a conjugate process. Numerical examples are also provided to illustrate our results.

Keywords: Covariance operator · Functional time series · Random measure.

Mathematics Subject Classification: 60G57 · 60G10 · 62G99 · 62M99.

1. Introduction

Time series models where the dynamics is driven by a latent, unobservable *state* variable are ubiquitous in the literature – to name a few examples, we mention the ARCH and GARCH models (Engle, 1982; Bollerslev, 1986), the class of hidden Markov models (Baum and Petrie, 1966) and, more recently, the GAS model of Creal et al. (2012). Such models have found widespread use in quantitative finance, economics and other applied sciences, and it is then natural to consider extensions to a framework where the underlying state is infinite dimensional – especially when one takes into account the increasing availability of high dimensional data in the last 20 years. Contributions in that direction have been proposed, among others, by Hörmann et al. (2013) and Aue et al. (2017) who introduce functional versions of the ARCH and GARCH models, respectively. In fact, stochastic differential equations, Bayesian nonparametrics (Ghosal and Van der Vaart, 2017; Quintana, 2010) and many other probabilistic models can be interpreted as pertaining to the class of (infinite dimensional) latent variable models. Exploring such connections is beyond the scope of the present paper..

Also in the setting of an infinite dimensional state variable, Horta and Ziegelmann (2018) introduce the concept of a *conjugate process*, where the latent state is indeed the (random) conditional distribution of the observable continuous-time process. Consistency results are available, and as is common in the framework of Functional Time Series, they rely on imposing a strong mixing condition on the model. However, in this setting some additional difficulties arise because the mixing property is imposed on a functional of observable data, whereas the dynamics is specified in terms of the latent, infinite dimensional state variable. This means that it can be cumbersome to derive the required mixing condition directly from higher level model assumptions (see the discussion in Remark *1*). In this paper, we provide sufficient conditions which ensure that a ψ-mixing property is inherited by the functional of the data whenever the underlying state process is itself ψ-mixing.

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The remainder of the paper is organized as follows. In Section 2, we present the theoretical background, following Horta and Ziegelmann (2018). In Section 3, we state and prove our main results. Section 4 illustrates the theory through a computational example. Section 5 provides some concluding remarks.

2. Theoretical background

In Horta and Ziegelmann (2018) a *conjugate process* is defined to be a pair (ξ, X), where $X := (X_\tau : \tau \geq 0)$ is a real valued, continuous time stochastic process, and $\xi := (\xi_t : t = 0, 1, \dots)$ is a strictly stationary sequence of $M_1(\mathbb{R})$-valued (here $M_1(\mathbb{R})$ denotes the set of Borel probability measures on $\mathbb{R}$) random elements, for which the following condition holds:

$$\mathbb{P}(X_\tau \in B \,|\, \xi_0, \xi_1, \dots) = \xi_t(B), \qquad \tau \in [t, t+1), \tag{1}$$

for each $t = 0, 1, \dots$ and each Borel set B in the real line. From the statistical viewpoint, the sequence ξ is to be understood as a latent (i.e. unobservable) process, and thus all inference must be carried using information attainable from the continuous time, observable process X alone.

A crucial objective in this context is estimation of the operator $R^\mu \colon L^2(\mu) \to L^2(\mu)$ defined by

$$R^\mu f(x) := \int R_\mu(x, y) f(y)\, \mu(\mathrm{d}y), \qquad x \in \mathbb{R}$$

where the kernel R_μ is given by

$$R_\mu(x, y) := \int \operatorname{Cov}(F_0(x), F_1(z)) \operatorname{Cov}(F_0(y), F_1(z))\, \mu(\mathrm{d}z), \qquad x, y \in \mathbb{R},$$

and where μ is a fixed, arbitrary probability measure on $\mathbb{R}$ equivalent to Lebesgue measure. In the above, $F_t(x) := \xi_t(-\infty, x]$, $x \in \mathbb{R}$, is the (random) cumulative distribution function corresponding to ξ_t. One of the key results in Horta and Ziegelmann (2018) is Theorem 2.1 below, which provides sufficient conditions under which R^μ can be $\sqrt{n}$-consistently estimated. These conditions involve an *a priori* ψ-mixing assumption on the Data Generating Process, and therefore it is of crucial importance to provide tractable conditions which in turn ensure the required ψ-mixing property. Our Theorem 3.1 below provides one such sufficient condition.

Before stating the theorem, we shall shortly introduce the estimator $\widehat{R}^\mu$ which is (as one should expect) a sample analogue of R^μ. Consider, for each $t = 1, \dots, n$, a sample of observations $\{X_{i,t} : i = 1, \dots, q_t\}$ of size q_t from $(X_\tau : \tau \in [t, t+1))$. Typically one has $X_{i,t} = X_{t+(i-1)/q_t}$. Also let $\widehat{F}_t$ denote the empirical cumulative distribution function associated with the sample $X_{1,t}, \dots, X_{q_t,t}$,

$$\widehat{F}_t(x) := \frac{1}{q_t} \sum_{i=1}^{q_t} \mathbb{I}[X_{i,t} \leq x], \qquad x \in \mathbb{R}.$$

Notice that both F_t and $\widehat{F}_t$ are random elements with values in the Hilbert space $L^2(\mu)$, and thus we find ourselves in a framework similar to Horta and Ziegelmann (2016).

In this setting, $\widehat{R}^{\mu}$ is defined to be the operator acting on $L^2(\mu)$ with kernel

$$\widehat{R}_{\mu}(x,y) := \int \widehat{C}_1(x,z)\widehat{C}_1(y,z)\mu(\mathrm{d}z), \qquad x,y \in \mathbb{R},$$

where $\widehat{C}_1$ is the sample lag-1 covariance function

$$\widehat{C}_1(x,y) := \frac{1}{n-1}\sum_{t=1}^{n-1}\Big(\widehat{F}_t(x) - \bar{F}_0(x)\Big) \times \Big(\widehat{F}_{t+1}(y) - \bar{F}_0(y)\Big), \qquad x,y \in \mathbb{R},$$

with $\bar{F}_0 := (1/n)\sum_{t=1}^{n}\widehat{F}_t$.

Last but not least, let $X^{(t)}$ denote the stochastic process $(X_{t+\tau} : \tau \in [0,1))$, so that $X^{(0)}, X^{(1)}, \dots, X^{(t)}, \dots$ is a sequence of $\mathbb{R}^{[0,1)}$-valued random elements. We say that a conjugate process $(\xi,\, X)$ is *cyclic independent* if, conditional on ξ, we have that $\big(X^{(t)} : t = 0,1,\dots\big)$ is an independent sequence. This means that, for each n and each $(n+1)$-tuple $\mathcal{C}_0,\dots,\mathcal{C}_n$ of measurable subsets of $\mathbb{R}^{[0,1)}$, it holds that

$$\mathbb{P}(X^{(0)} \in \mathcal{C}_0, \dots, X^{(n)} \in \mathcal{C}_n \,|\, \xi) = \prod_{t=0}^{n}\mathbb{P}(X^{(t)} \in \mathcal{C}_t \,|\, \xi).$$

We are now ready to state the consistency theorem.

Theorem 2.1 (Horta and Ziegelmann, 2018) Let (ξ, X) be a cyclic–independent conjugate process, and let μ be a probability measure on $\mathbb{R}$ equivalent to Lebesgue measure. Assume that $\big(\widehat{F}_t\colon\ t = 1,2,\dots\big)$ is a ψ–mixing sequence, with the mixing coefficients $\Psi(k)$ satisfying $\sum_{k=1}^{\infty} k\,\Psi^{1/2}(k) < \infty$. Then, it holds that

(i) $\|\widehat{R}^{\mu} - R^{\mu}\|_{HS} = O_{\mathbb{P}}\Big(n^{-1/2}\Big)$;

(ii) $\sup_{j\in\mathbb{N}} |\widehat{\theta}_j - \theta_j| = O_{\mathbb{P}}\Big(n^{-1/2}\Big)$.

If moreover the nonzero eigenvalues of R^{μ} are all distinct, then

(iii) $\|\widehat{\psi}_j - \psi_j\|_{L^2(\mu)} = O_{\mathbb{P}}(n^{-1/2})$, for each j such that $\theta_j > 0$.

In the above, $\|\cdot\|_{HS}$ denotes the Hilbert-Schmidt norm of an (suitable) operator acting on $L^2(\mu)$, $(\theta_j\colon\ j \in \mathbb{N})$ $((\widehat{\theta}_j\colon\ j \in \mathbb{N}))$ denotes the non-increasing sequence of eigenvalues of R^{μ} $(\widehat{R}^{\mu})$, with repetitions if any and, for $j \in \mathbb{N}$, ψ_j $(\widehat{\psi}_j)$ denotes the unique eigenfunction associated with θ_j $(\widehat{\theta}_j)$. Notice that there is some ambiguity in defining things in this manner; to ensure that everything is well defined, we adopt the convention that the sequence (θ_j) contains zeros if and only if R^{μ} is of finite rank. Thus if the range of R^{μ} is infinite dimensional and 0 is one of its eigenvalues, it will not show up in the sequence (θ_j). On the other hand, $\widehat{R}^{\mu}$ is always of finite rank.

3. Main Result

In what follows it will be convenient to assume that the latent process is indexed for $t \in \mathbb{Z}$ and that the continuous time, observable process X is indexed for $\tau \in \mathbb{R}$. That is, we update our definitions so that $\xi := (\xi_t : t \in \mathbb{Z})$ and $X := (X_\tau : \tau \in \mathbb{R})$. Recall that a strictly stationary sequence $(Z_t : t \in \mathbb{Z})$ of random elements taking values in a measurable space $\mathcal{Z}$ is said to be ψ-*mixing* if the ψ-mixing coefficient Ψ_Z defined, for $k \in \mathbb{N}$, by

$$\Psi_Z(k) \coloneqq \sup\left|1 - \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(A)\mathbb{P}(B)}\right| \tag{2}$$

is such that $\Psi_Z(k) \to 0$ as $k \to \infty$, where the supremum in (2) ranges over all $A \in \sigma(Z_t\colon\ t \leq 0)$ and all $B \in \sigma(Z_t\colon\ t \geq k)$ for which $\mathbb{P}(A)\mathbb{P}(B) > 0$; see Doukhan (1994) and references therein for a thorough treatment of the topic, and also Bradley (2005) for basic properties of mixing conditions.

The ψ–mixing condition in Theorem 2.1 imposes restrictions on the sequence of empirical cumulative distribution functions $(\widehat{F}_t)$ and thus constrains (F_t) and (X_τ) jointly. One could argue that it is more natural to impose a ψ–mixing condition on the latent process (ξ_t) instead, the issue being that it may be the case that a mixing property of the latter sequence is not inherited by $(\widehat{F}_t)$. If a condition slightly stronger than cyclic–independence is imposed, however, then inheritance does hold. This is our main result.

Theorem 3.1 Let $(\xi,\, X)$ be a cyclic–independent conjugate process, and let μ be a probability measure on $\mathbb{R}$ equivalent to Lebesgue measure. Assume ξ is ψ–mixing with mixing coefficient sequence Ψ_ξ. If, for each t, the conditional distribution of $X^{(t)}$ given ξ depends only on ξ_t, in the sense that the equality

$$\mathbb{P}\big[X^{(t)} \in \mathcal{C} \,\big|\, \xi\big] = \mathbb{P}\big[X^{(t)} \in \mathcal{C} \,\big|\, \xi_t\big] \tag{3}$$

holds for each measurable subset $\mathcal{C}$ of $\mathbb{R}^{[0,1)}$ and each t, then $\big(X^{(t)}\big)$ is ψ–mixing with mixing coefficient sequence $\Psi_X \leq \Psi_\xi$.

Corollary 3.2 In the conditions of Theorem 3.1, if $\sum_{k=1}^{\infty} k\Psi_\xi(k)^{1/2} < \infty$, then the ψ–mixing assumption of Theorem 2.1 holds.

Remark 1 Theorem 3.1 and Corollary 3.2 are important as they provide the applied statistician a framework for introducing models in which the ψ-mixing condition of Theorem 2.1 is satisfied, ensuring the possibility of adequate estimation procedures and statistical analyses. A particular setup in which knowledge of ψ-mixing models for multivariate time series is sufficient for obtaining a ψ-mixing sequence of random measures (ξ_t) is the scenario in which the latter sequence is in fact driven by a finite dimensional process. This is the case whenever (ξ_t) satisfies

$$\xi_t(B) = \mathbb{E}\xi_0(B) + \sum_{j=1}^{d} Z_{tj}\lambda_j(B), \qquad t \in \mathbb{Z}, \qquad B \in \mathrm{Borel}(\mathbb{R}),$$

where d is a positive integer, the $Z_{t,j}$ are scalar random variables and the λ_j are signed measures of finite total variation. Indeed, in this setting the dynamic features of (ξ_t) are entirely captured by the multivariate time series $\boldsymbol{Z}_t = (Z_{t1}, \ldots, Z_{tj})$, and it is not difficult to see that if the mixing coefficient sequence $\Psi_{\boldsymbol{Z}}(1), \Psi_{\boldsymbol{Z}}(2), \ldots$ of $(\boldsymbol{Z}_t)$ satisfies the summability condition of Theorem 2.1, then so does Ψ_ξ.

Proof [Proof of Theorem 3.1] For $k \in \mathbb{N}$, let T_1 and T_2 be finite, nonempty subsets of $\{0, -1, -2, \ldots\}$ and $\{k, k+1, k+2, \ldots\}$ respectively, and set $T_0 \coloneqq T_1 \cup T_2$. Let $\{\mathcal{C}_t,\, t \in T_0\}$ be a collection of measurable subsets of $\mathbb{R}^{[0,1)}$. By definition, $\sigma(X^{(t)} : t \leq 0)$ coincides with the σ-field generated by the class of sets of the form $\bigcap_{t \in T_1}[X^{(t)} \in \mathcal{C}_t]$ over all finite, nonempty $T_1 \subset \{0, -1, -2, \ldots\}$ and all collections $\{\mathcal{C}_t : t \in T_1\}$ of measurable subsets of $\mathbb{R}^{[0,1)}$, and similarly for $\sigma(X^{(t)} : t \geq k)$.

Notice that by equation (3) and the Doob–Dynkin Lemma (see Kallenberg, 1997, Lemma 1.13) we have $\mathbb{P}\Big[X^{(t)} \in \mathcal{C}_t \,|\, \xi\Big] = g_t \circ \xi_t$, for some measurable function $g_t \colon M_1(\mathbb{R}) \to \mathbb{R}$. This fact, together with the cyclic–independence assumption, ensures that

$$\mathbb{P}\left\{\bigcap_{t\in T_j}\Big[X^{(t)} \in \mathcal{C}_t\Big]\right\} = \mathbb{E}\left\{\mathbb{P}\left\{\bigcap_{t\in T_j}\Big[X^{(t)} \in \mathcal{C}_t\Big] \,\Big|\, \xi\right\}\right\} = \mathbb{E}\left\{\prod_{t\in T_j} g_t \circ \xi_t\right\},$$

$j = 0, 1, 2$ (a similar computation yields strict stationarity of the process $(X^{(t)} : t \in \mathbb{Z})$). Thus, the quantity

$$\left|1 - \frac{\mathbb{P}\Big\{\bigcap_{t\in T_0}\Big[X^{(t)} \in \mathcal{C}_t\Big]\Big\}}{\mathbb{P}\{\bigcap_{t\in T_1}[X^{(t)} \in \mathcal{C}_t]\}\mathbb{P}\{\bigcap_{t\in T_2}[X^{(t)} \in \mathcal{C}_t]\}}\right| \tag{4}$$

is seen to be equal to

$$\left|1 - \frac{\mathbb{E}\{\prod_{t\in T_0} g_t \circ \xi_t\}}{\mathbb{E}\{\prod_{t\in T_1} g_t \circ \xi_t\}\mathbb{E}\{\prod_{t\in T_2} g_t \circ \xi_t\}}\right|. \tag{5}$$

Substituting each g_t in (5) by an arbitrary measurable, bounded and positive $g'_t \colon M_1(\mathbb{R}) \to \mathbb{R}$, and taking the supremum over all collections $\{g'_t : t \in T_0\}$ of such g'_t, and over all $T_0 = T_1 \cup T_2$ as above, gives an upper bound to (4). It is easily seen that this supremum yields precisely $\Psi_\xi(k)$. This establishes that $\Psi_X(k) \le \Psi_\xi(k)$ and completes the proof. (By definition $\Psi_\xi(k)$ is obtained by taking the supremum over all collections of g'_t which are indicator functions of measurable subsets of $M_1(\mathbb{R})$.) ■

Proof [Proof of Corollary 3.2] By definition (or using the Doob–Dynkin Lemma) we have that $\widehat{F}_t$ is of the form $\widehat{F}_t = g_t \circ X^{(t)}$ for some measurable $g_t \colon \mathbb{R}^{[0,1)} \to L^2(\mu)$. Since $\mathbb{P}(\widehat{F}_t \in B) = \mathbb{P}(X^{(t)} \in g_t^{-1}(B))$, it follows that the supremum in the LHS over all measurable subsets B of $L^2(\mu)$ is bounded above by $\sup \mathbb{P}(X^{(t)} \in \mathcal{C})$, with $\mathcal{C}$ ranging over all measurable subsets of $\mathbb{R}^{[0,1)}$. An easy adaptation of this argument shows that the mixing coefficient sequence $\Psi_{\widehat{F}}$ is bounded above by Ψ_X. ■

4. Examples

We refer the reader to Horta and Ziegelmann (2018) for an interesting application of the theory of conjugate processes to the problem of financial risk forecasting. Below we provide a simple example to illustrate the theory.

As discussed in Horta and Ziegelmann (2018), the case where (ξ_t) is an independent sequence is of no interest, since in this case R^μ is trivially the zero operator. Consider then an independent identically distributed sequence $(\vartheta_t : t \in \mathbb{Z})$, where ϑ_t is uniformly distributed on $[0, 1]$, and let η_t be the random probability measure defined by (abusing a little on notation) $\eta_t(0) = \vartheta_t$ and $\eta_t(1) = 1-\vartheta_t$. Setting $\xi_t := (\eta_t+\eta_{t-1})/2$, we clearly obtain a ψ-mixing sequence which satisfies the summability condition of Theorem 2.1. Indeed, (ξ_t) is 1-dependent. A straightforward computation shows that $\mathrm{Cov}(F_0(x), F_1(y)) = 1/48$ for $x, y \in [0, 1)$ and is identically zero otherwise, and therefore $R_\mu(x, y)$ is a positive constant for $x, y \in [0, 1)$ which only depends on the chosen measure μ.

Now, aside from the assumption that relation (1) holds, the nature of the process $(X_\tau : \tau \in \mathbb{R})$ is rather arbitrary. Below we simulate the case where, conditional on ξ, the process $(X_{t+\tau} : \tau \in [0, 1))$ is a continuous time Markov chain on the state space $\{0, 1\}$ with stationary distribution $(\xi_t(0), \xi_t(1))$. There is a free parameter in the construction, which is the mean holding time $1/q_0$ of state 0. We set $q_0 = 10$. Thus, conditional on $\xi_t = \lambda_t$, the process $(X_{t+\tau} : \tau \in [0, 1))$ is a Markov chain with initial distribution $(\lambda_t(0), \lambda_t(1))$ and generator

$$Q = \begin{pmatrix} -q_0 & q_0 \\ r_t & -r_t \end{pmatrix}$$

where $r_t \coloneqq q_0 \lambda_t(0)/\lambda_t(1)$.

The conjugate process (ξ, X) described above can be informally summarized as follows. At each day, the world finds itself in a (unobservable) *state* which is characterized by a number lying in $[0, 1]$. Within each day, given the state of the world, a system can find itself in two distinct (observable) *regimes* (say, regime $\bar{0}$ and regime $\bar{1}$). This system switches between $\bar{0}$ and $\bar{1}$ according to a stationary, continuous time Markov chain, where the state of the world in that day represents the probability of the system being on regime $\bar{0}$ at any given point in time within that day. Figure 1 displays a simulated sample path for the first 4 days of the process just described.

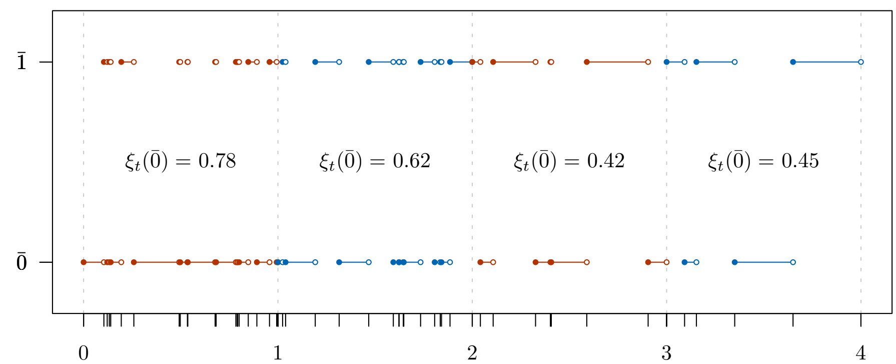

Figure 1. A simulated sample path. Even days are colored in red; odd days in blue.

We also illustrate the consistency result via a Monte Carlo simulation study. For each $t = 1, \ldots, n$, we sample the process $(X_{t+\tau} : \tau \in [0, 1))$ once per cycle (that is, we take $q_t = 1$ and $X_{1,t} = X_t$) and compute the corresponding value of $\widehat{C}_1(0, 0)$. Figure 2 displays the boxplot of the estimated values of $\widehat{C}_1(0, 0)$ across 10000 replications of the above procedure, with the sample size varying in $\{100, 1000, 10000\}$. The blue line indicates the true parameter value $C_1(0, 0) = 1/48$.

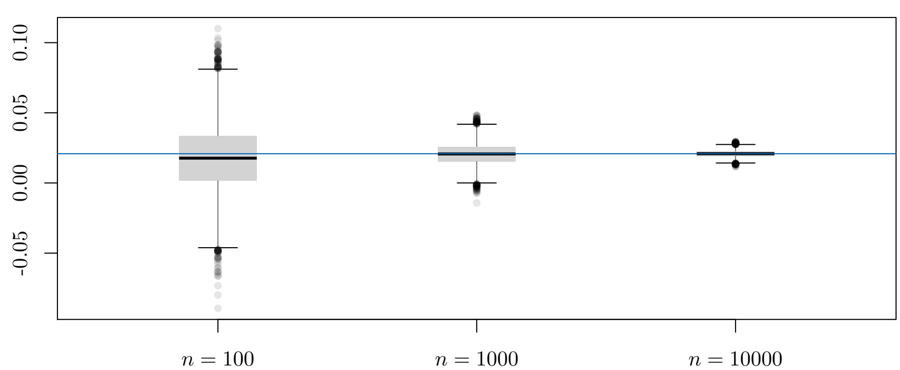

Figure 2. Boxplots of $\widehat{C}_1(0, 0)$ values across replications.

5. Concluding remarks

This paper investigated conditions under which a certain ψ-mixing condition is inherited by the empirical cumulative distribution functions associated with a conjugate process (Horta and Ziegelmann, 2018). Our theoretical results, presented in Theorem 3.1 and Corollary 3.2, ensured that whenever the underlying state sequence possesses the required ψ-mixing property, so does the corresponding sequence of empirical cumulative distribution functions. The results are of relevance in settings where the dynamics is governed by an infinite dimensional latent process, as they allow the applied statistician to propose conjugate process models whose parameters can be consistently estimated – thus ensuring the possibility of adequate statistical analyses.

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