MATHEMATICAL MODELLING FOR STRESS ANALYSIS OF COMPOSITE HEMISPHERES SUBJECTED TO UNIFORM PRESSURE

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The main objective of this study was to write a computer program in ADA to predi the collapse pressure of Fiber Reinforced Plastic (FRP) thin hemispherical pressure vesse under the action of uniform, static, internal or external pressure. The mathematical model the mechanical behavior of the structure consisted of a nonlinear second order differenti equation, which was solved in terms of hypergeometric functions. The latter were evaluat by taking their representation in series, based on the Gamma function (Kraus, 1967). Th FRP material was modeled as layered orthotropic, according to the Classical Lamina Theory (Gibson, 1994).

All the analytical results obtained so far were compared with numerical simulatio based on the finite element method (COMPSHELL, Levy Neto, 1991), and, in some case the calculations from the computer code developed in this work were also compared wi experimental results found in the literature (Levy Neto, 1991). The latter consisted of t failure pressures of externally pressurized carbon/epoxy and E-glass/epoxy hemispheric domes, reinforced with woven fabrics, and clamped at the edges, which were tested failure in a pressure chamber. It was assumed that the pressure in which the first-ply-failu (FPF) of composite laminate takes place, based on the criterion of Tsai-Wu, is the fin failure of the structure as a whole. All the stacking sequences of laminates simulated in th work were quasi isotropic, and the number of FRP layers varied from 4 to 12.

The agreement of the calculations based on the software developed in this study wi the numerical (in the range of 96 to 98%) and experimental results (average correlations fro 60 to 80%) can be regarded as good (Lin, 1991). The interactive factor of the Tsai-W Failure Theory ($-0.5 \leq F_{12}^{*} \leq 0.5$) changed the FPF pressures by about 20% (Gibson, 199 In addition, in some cases, the analytical results obtained were closer to the experimen results than the finite element simulations. The program developed in this work has also t advantage to run significantly faster than the finite element code, which was used to check results.

References from the literature.

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Sistemas de Parâmetros Concentrados no Estudo de Processos de Solidificação

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Neste trabalho formulações chamadas de "Sistemas de Parâmetros Concentrados" [1] são usadas na obtenção de uma solução em forma fechada para um problema de solidificação de ligas, em um meio cilíndrico composto [2], descrito pela equação

$$\frac{1}{\alpha_i}\frac{\partial}{\partial t}T_i(r,z,t) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}T_i(r,z,t)\right) + \frac{\partial^2}{\partial z^2}T_i(r,z,t) + \frac{Q_i(t)}{k_i}, \quad i = 1,2, \qquad (1)$$

$0 < r < a$, $0 < z < c$, onde $\alpha_i = k_i/\rho_i Cp_i$ é a difusividade térmica, T_i a temperatura em cada meio, ρ_i a densidade, Cp_i o calor específico e k_i a condutividade térmica, sendo $i = 1$ para o metal e $i = 2$ para o molde. Ainda, $Q_i(t)$ é o termo de fonte que descreve a taxa de variação do calor latente resultante da mudança de fase, que é nula no molde e é dado por

$$Q_1(t) = L\frac{\partial}{\partial t}f_s \qquad (2)$$

para o metal, sendo f_s a fração de sólido. Nesse caso, as condições de contorno consideradas foram tais que não há fluxo de calor na base do cilindro, a temperatura é máxima no centro do cilindro, temos contato térmico perfeito entre o metal e o molde e há convecção nas interfaces do molde e do metal com o meio ambiente.

Os sistemas de parâmetros concentrados são obtidos a partir do sistema diferencial original (1), resultando em modelos matemáticos mais simples que relacionam a temperatura no contorno do meio com uma nova temperatura média ($\overline{T}_i(r,t)$) gerada por um processo de integração . Ou seja,

$$\frac{\partial}{\partial t}\overline{T}_i(r,t) = \alpha_i\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\overline{T}_i(r,t)\right) - \frac{\alpha_i h_c}{ck_i}T_i(r,c,t) + \frac{\alpha_i}{k_i}Q_i(t), \quad i = 1,2, \qquad (3)$$

onde

$$\overline{T}_i(r,t) = \frac{1}{c}\int_0^c T_i(r,z,t)dz. \qquad (4)$$

A determinação de equações que relacionam a temperatura média e do contorno geram diferentes abordagens sendo a principal delas associada ao uso de fórmulas de integração numérica de Hermite, que propiciam a introdução de informações do contorno no modelo simplificado.

As equações unidimensionais simplificadas obtidas pela integração

$$\frac{\partial}{\partial t}\overline{T}_i(r,t) = \alpha_i\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\overline{T}_i(r,t)\right) - \frac{\alpha_i h_c \beta_i}{ck_i}\overline{T}_i(r,t) + \frac{\alpha_i}{k_i}Q_i(t), \qquad (5)$$

sendo β_i calculado pelas fórmulas de Hermite, são tratadas pela técnica das Transformadas Integrais Generalizadas (GITT) [1]. Resultados numéricos obtidos através do software matemático *Maple V* nos levam a comparações satisfatórias com a literatura.

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