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# Simulation of Herschel-Bulkley Fluids in Sudden Expansions

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This work presents the numerical solution of incompressible Herschel-Bulkley fluid flows through a two-dimensional channel with a 3:1 sudden expansion ratio using the Generalized Newtonian Liquid model. The governing equations are solved using the finite differences explicit Runge-Kutta three-stages scheme for second order time and space approximations.

Herschel-Bulkley fluids are a class of non-Newtonian fluids that require a finite stress, known as yield stress, in order to deform. Therefore, these materials behave as rigid solids when the local stress is below this value. Once the yield stress is exceeded, the material flows with a non-linear stress-strain relationship either as a shear-thickening fluid, or a shear-thinning one. Some examples of fluids behaving in this manner include paints, food products, plastics, pharmaceutical products and polymer solutions. The non-Newtonian flows analysis is very interesting in the case of sudden expansions.

Expansions have important applications in engineering processes like in refrigeration, extrusion and free jets. The study of Newtonian fluid flows through a sudden expansion of various ratios and conditions is a classical problem which has been analyzed by many workers. So, it is well known that these flows present a critical Reynolds, depending on the expansion ratio, in which the problem has three solutions, corresponding to a bifurcation in the Navier-Stokes equations: the symmetric one (unstable) and two asymmetric (stable). Increasing the Reynolds number, the solution becomes three-dimensional and time dependent.

For non-Newtonian flows, such investigation is recent and there is not much information about it. So, this study presents results comparing the Newtonian and Herschel-Bulkley flows through a sudden expansion. Simulations are performed for Reynolds numbers ( $Re$ ) ranging from 30 to 300 and Bingham numbers ( $Bi$ ) from 0 (Newtonian) to 5. Results show agreement with those found in the literature.

As an example, figure 1 shows streamlines for  $Re = 130$  and Bingham numbers from 0 to 5. These results show that the vortex size decrease when the Bingham number is increased. Therefore, the critical Reynolds for these fluids are higher when com-

pared to the Newtonian case.

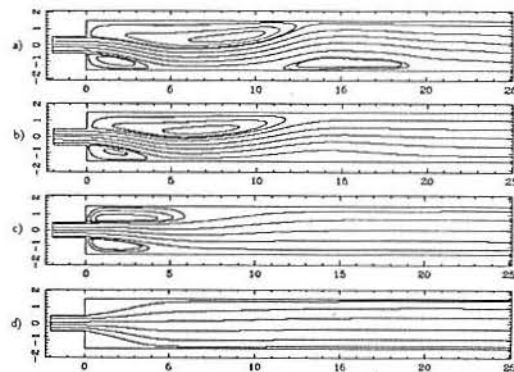


Figure 1: Streamlines for  $Re=130$  and a)  $Bi=0$ , b)  $Bi=0.2$ , c)  $Bi=1$  e d)  $Bi=5$

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