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AMANDA OLIVEIRA PUERARI

RANKINGS NO CONTEXTO DO DESENVOLVIMENTO HUMANO: UMA ANÁLISE DE ORDEM PARCIAL DO ÍNDICE DE DESENVOLVIMENTO HUMANO

Porto Alegre
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Trabalho de conclusão submetido ao Curso de Graduação em Ciências Econômicas da Faculdade de Ciências Econômicas da UFRGS, como requisito parcial para obtenção do título Bacharela em Economia.

Orientador: Prof. Dr. Flavio Vasconcellos Comim

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BANCA EXAMINADORA:

____

Prof. Dr. Flavio Vasconcellos Comim – Orientador
UFRGS

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Profa. Dra. Izete Pengo Bagolin
PUCRS

____

Prof. Dr. Sabino Pôrto Junior
UFRGS

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RESUMO

O estudo do desenvolvimento humano está no centro de diversos debates no campo da Ciência Econômica, e indicadores que visam a mensurar o desempenho de diferentes países em esferas críticas do desenvolvimento humano desempenham um papel fundamental na avaliação desse fenômeno. O Índice de Desenvolvimento Humano, por exemplo, agrega as dimensões educação, saúde e renda, oferecendo um ranking completo dos países. Rankings e ordenamentos são, dessa forma, de grande relevância em análises de desenvolvimento humano. Assim, o presente estudo apresenta uma aplicação empírica de análises de ordem parcial, desenvolvidas por Brüggemann e Patil (2011), com dados do Índice de Desenvolvimento Humano de 2015, buscando analisar incomparabilidades entre os países e o impacto dos diferentes indicadores no desempenho daqueles. Com base nesse exercício, o trabalho propõe uma discussão sobre rankings no âmbito das avaliações de desenvolvimento humano, defendendo que rankings parciais são uma metodologia mais flexível nesse contexto, em contraste a rankings completos. De fato, o estudo identificou novos e importantes aspectos acerca das estatísticas analisadas com base nessa nova abordagem. Ainda, os resultados das análises são discutidos buscando sugerir como tais aspectos poderiam potencialmente auxiliar na formulação de políticas públicas e como a abordagem dos meta-ranking poderia complementar a análise.

Palavras-chave: Desenvolvimento humano. Teoria da Escolha Social. Rankings. Análises de ordem parcial. Índice de Desenvolvimento Humano.

ABSTRACT

The study of human development is at the core of several debates in the field of economic science, and indicators that aim to measure the performance of different countries in critical areas of human development play a central role in the evaluation of this phenomenon. The Human Development Index, for instance, combines three different dimensions—education, health, and income—providing a complete ranking of all countries. Rankings and orderings are, thus, extremely important in human development analyses. Therefore, the present study presents an empirical application of the partial order analysis—developed by Brüggemann and Patil (2011)—with data from the 2015 Human Development Index, seeking to analyze 'incomparabilities' between countries and the impact of the different indicators on their performances. Based on this exercise, this work proposes a discussion on ranking in the context of human development evaluations, supporting that partial rankings are a more flexible methodology within this context, in contrast to complete rankings. Indeed, the study identified new and important aspects concerning the analyzed statistics by means of this new approach. Furthermore, the results were discussed seeking to suggest how those aspects could potentially help the formulation of public policies and how the meta-ranking approach could complement the analysis.

Keywords: Human Development. Social Choice Theory. Rankings. Partial order analysis. Human Development Index.

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1 INTRODUCTION

Human development is a complex and multidimensional phenomenon that has been the object of much debate in the field of economic science. As the United Nations Development Programme (UNDP) elaborates, “human development—or the human development approach—is about expanding the richness of human life,” focusing on people and their opportunities and choices. Consequently, assessing and measuring human development in different societies is fundamental for identifying deficiencies—and advancements—in human well-being achievement. The Human Development Index, for instance, calculates the performance of different countries in a few critical socioeconomic indicators, seeking, as the UNDP holds, to provide a basis for international comparisons and potentially stimulate discussions about government policy priorities.

Furthermore, social choice theory is a theoretical framework that analyzes the composition and ordering of individual opinions, preferences, interests or notions of well-being to arrive at a collective decision or social well-being in some sense. That is, it seeks to investigate how individual preferences are combined and translated into a collective decision. Rankings and orderings are, thus, central elements in the context of collective choice. However, as Amartya Sen (1977) claims, full rankings may be an oversimplification in this context as they fail to capture the variety of motivations involved in the process of choice. From this notion, he then develops the concept of meta-rankings—that is, rankings of rankings—as a more elaborate structure for social deliberations.

Within this context, Flavio Comim holds that “The use of rankings can be understood as ‘an approach’ in Sen’s analysis because it provides a methodological framework for helping with valuation exercises.” (COMIM, 2015, p.6) Indeed, rankings—and particularly partial rankings and the meta-ranking approach—can help enrich discussions in the field of human development once they allow comparisons between different alternatives in the social context while taking into account the complexity of the choice process and the variety of motivations that are involved in it.

Based on these ideas, the main purpose of the present study is to propose a debate on rankings in the context of human development evaluations, questioning whether complete rankings are the most adequate representation of multifaceted objects—in this case, different countries’ performances in selected indicators. The main hypothesis defended here is that, due to the intricate and multidimensional nature of human development, complete rankings may present limitations and a new—and more flexible—method could be more appropriate for examining human development statistics. In order to do so, a partial order theory and methodology, developed by Rainer Brüggemann and Ganapati P. Patil in their book *Ranking and Prioritization for Multi-indicator Systems* (2011), was applied to the 2015 Human Development Index results for 99 countries with online software PyHasse. Different analyses concerning 'incomparabilities' between countries and the individual impact of indicators on their relative positions were performed, and a more detailed diagrammatic representation of the HDI ranking—a Hasse diagram—was also considered. Finally, the study discussed how the results could potentially help orient public policy and how the meta-ranking approach could complement the analysis.

Therefore, this study is organized as follows: the first chapter is dedicated to an overview of the social choice theory as it lays the foundations for the discussion of rankings within the context of human development. Initially the chapter seeks to elucidate the purpose and historical background of social choice theory and to briefly describe the main concepts involved in the analysis—namely individual preferences and choices—as well as a few discussions derived from these notions, such as their relationship with rationality, freedom, and rights, based mostly on the works of Amartya Sen. Then, Kenneth Arrow's contribution was more deeply analyzed due to its great importance to the field as he worked on developing a more formal and axiomatic approach for the theory of social choice. Additionally, Sen's effort to expand the framework introduced by Arrow was also considered, as it substantially enriches the discussion. Lastly, the chapter ends with a debate on rankings and meta-rankings in the context of social choice, focusing on their relevance and applications in analyses of this type.

The second chapter of this study exposes the partial order analysis developed by Brüggemann and Patil (2011) and the different analyses it renders. First, the concepts of data matrix and Hasse diagrams are explained as they provide a basis for the all exercises—such as those related to incomparable objects, linear extensions, and levels—which are then scrutinized.

Finally, the last chapter further examines the concept of human development and how the Human Development Index contributes to the discussion, investigating its different dimensions and the indicators employed. Then, the partial order methodology is empirically applied to 2015 HDI statistics using the online version of the PyHasse software, evaluating countries' relative positions, potential 'incomparabilities' between them, and the individual impact of education, health, and income, as well as that of an additional indicator: environmental sustainability. Moreover, Brazil's performance is highlighted in all exercises.

The results are then discussed focusing on how public policy could potentially benefit from assessments of this type, within the scope of the meta-ranking approach.

2 SOCIAL CHOICE THEORY: AN OVERVIEW

Social choice theory is a theoretical framework that, in broad strokes, seeks to analyze how collective decisions and choices are made based on individual preferences and choices. As Amartya Sen (1999b) argues, its main purpose is to study the possibility of aggregating diverse preferences, values, and concerns of distinct individuals into a collective choice or judgment, when all the members of the society are free to engage in the decision process, directly or indirectly. An important issue that motivates social choice theory, and that has been extensively examined in the literature of the field, is the possibility of finding a rational basis upon which to consolidate aggregative judgments of this kind. Is it even possible to achieve a coherent social choice?

Some of the questions addressed by the theory concern, as John Craven (1992) claims, the design of electoral systems and the operation of committees, which use voting procedures as their strategy for decision-making and aggregating individual views. In the economic context, a market system can be seen as a structure for social choice, since, for given resources and technology, individual preferences are combined to generate a certain pattern of wages, prices, outputs etc., and thus an economic outcome. Furthermore, policymakers should—ideally—take into account peoples' concerns and interests when formulating economic policies, seeking fair and socially desirable outcomes. As Sen (1977) emphasizes, the analysis of moral concepts (e.g., liberty, rights, justice, equality), the application of social welfare measures, and the elaboration of statistics for economic evaluation (like national income, poverty or inequality) are other diverse fields of application for social choice theory.

It is important to note, however, that although all activities mentioned entail aggregation over different individuals and are, therefore, within the scope of social choice theory, they contrast with one another in various relevant ways. Sen points out that "one basis of distinction lies in the nature of the end product of aggregation." (SEN, 1977, p. 1539) While in some of them the aggregation results in judgments of 'social welfare', others simply attempt to achieve adequate and satisfactory decisions.

Even though the intricacy of combining conflicting concerns and preferences into acceptable collective decisions has been investigated since ancient times (by Aristotle, for instance, in his book *Politics*), social choice theory only emerged as a structured field of study in the late eighteenth century. French mathematicians like J.C. Borda and the Marquis de Condorcet, motivated by the attempt of avoiding instability and oppressive social choice arrangements, became "theorists of social coordination" (SEN, 1999b, p. 350), and in fact much added to the ideas behind the French Revolution. They mainly investigated democratic ways of achieving social decisions, such as voting mechanisms and majority rule—arriving, however, at rather pessimistic results.

Jeremy Bentham, also in the eighteenth century, developed and introduced a new way of aggregating individual interests: in terms of people's utilities. He is—for this reason—considered the founder of Utilitarianism. The main concern of this approach, however, was maximizing the sum-total of utilities of a community, with no particular interest on how that total is distributed among its members. Even so, the utilitarian theory much inspired the work of economists such as Francis T. Edgeworth (1881), Alfred Marshall (1920), and Arthur C. Pigou (1932), who helped develop the traditional welfare economics.

Nevertheless, in the 1930's, utilitarian welfare economics was intensely criticized by John Rawls, who—in his *A Theory of Justice*—denounced the narrowness of the former approach, especially regarding its neglect of distributional issues. Yet, the general critiques of the utilitarian theory in the 1930's tended to follow a different direction: that of the ideas presented by Lionel Robbins (1938), who argued that interpersonal comparisons of utility had no real scientific support, as they could not be verified by observation or introspection. He claimed that interpersonal comparisons of utility "are more like judgments of value than judgments of verifiable fact." (ROBBINS, 1938, p. 640)

In view of this informational limitation, a "new welfare economics" emerged in the 1940's. Its fundamental criterion of social development was the "Pareto comparison," which states that only by increasing the utility of everyone would an alternative situation be superior to the current one. Again, distributional issues were not at the core of the discussion here.

In 1951, Kenneth Arrow, following the demands for more elaborate criteria for making social welfare judgments, further investigated the difficulties involved in group decisions and the inconsistencies to which they may lead. Arrow developed a more structured—and axiomatic—framework for the discipline of social choice, introducing a "social welfare function" that links individual preferences to social preference (or choice). Nonetheless, from this basis, he arrived at what seemed to be a discouraging result: what became known as the "impossibility theorem" (or "general possibility" theorem, as he puts it). Indeed, it identifies a substantial susceptibility in this approach but, as Sen (1999b) argues, that does not undermine Arrow's contribution of establishing a technical framework for the social choice theory, rather the opposite. As he puts it,

> Arrow's "impossibility theorem" aroused immediate and intense interest (and generated a massive literature in response, including many other impossibility results). It also led to the diagnosis of a deep vulnerability in the subject that

> overshadowed Arrow's immensely important constructive program of developing a systematic social choice theory that could actually work. (SEN, 1999b, p. 351)

Indeed, Arrow's "impossibility theorem" laid the foundations for a great number of contributions in the field, such as that of Amartya Sen, who worked on developing ways of circumventing this obstacle and thus expanding the approach. Both Arrow's theorem and Sen's work will be more deeply analyzed in upcoming sections in this chapter. First, however, the fundamental notions involved in these discussions—i.e., individual preferences and choices—are presented and scrutinized, as they provide a basis for the theory of collective choice. Finally, after considering the contributions of Arrow and Sen, the chapter ends with a discussion on rankings and meta-rankings in the context of social choice, focusing on their relevance and applications in analyses of this type.

2.1 BASIC CONCEPTS

2.1.1 Individual preferences

John Craven (1992) describes that, in social choice problems, the primary scenario is a group of individuals being faced with a set of alternatives, which generally concern a judgment or choice to be made, commonly related to different possibilities of social states (i.e., the economic, political, and social circumstances of a particular society). Put simply, each individual would then have preferences regarding the available alternatives, which can be listed in a preference ordering. These preferences, in turn, inspire and guide that individual's choice. Preferences can be seen, thus, as "the basic 'input' of social choice." (CRAVEN, 1992, p. 13)

From these basic notions, however, it is possible to derive several complex—yet extremely relevant—discussions. One example is the relationship between individual preference, rationality, and freedom, acknowledged and explored by Amartya Sen in a number of his essays. He argues that "the concepts of rationality and freedom are among the basic ideas in economics, philosophy and the social sciences" (SEN, 2002, p. vii), and that the social choice approach can contribute greatly to a better understanding of these notions.

In this context, rationality is regarded—in general terms—as the "discipline of subjecting one's choices—of actions as well as of objectives, values and priorities—to reasoned scrutiny" (SEN, 2002, p. 4), that is, to the demands of reason. Similarly, John Rawls (1999) elaborates the notion of deliberative rationality as a principle of rational choice. The rational plan for a person to choose would be the one selected after a thorough examination of the consequences in the light of all the significant facts. Therefore, as Sen (2002) notes, rationality is fundamental to interpret complex ideas that require reasoning and reasoned choice, such as that of freedom.

This connection points to a complementarity between rationality and freedom. The latter is presented by Sen (2002) as a complex concept with two distinct, yet interdependent, aspects: the 'opportunity' and the 'process'. The first concerns the actual ability one has to attain what he or she values, and has reason to value. In fact, the available alternatives that were rejected are also important and must be regarded as part of the context. The opportunity aspect relates, therefore, to having diverse options and concrete opportunities of obtaining them—in particular, those that one wishes to seek, and has good reasons to.

The process aspect, in turn, regards the freedom in the very process of choice and achievement, since one may value not only the result of the choices, but also the way that leads to them. For instance, being free to choose and being forced to do so, even if at the end the same outcome is produced, are quite different scenarios, and the first is unquestionably preferable to the latter. Sen emphasizes that "The possibility of considering processes along with culmination outcomes as part of the description of states of affairs (that is, comprehensive outcomes) significantly expands the reach of choice analysis." (SEN, 2002, p. 658)

From this basis, Sen (1985) points out that one can draw a connection between the two aforementioned aspects of freedom and our preferences. For instance, a person should have the power and freedom to decide about his or her own preferences, to rethink and change them if he or she wishes to do so—since, in social choice exercises, not only should values be taken into account, but also one's values about his or her own values—and to not have others take these preferences as given, under any circumstances. These are different ways in which preferences are relevant in assessing the opportunity aspect of freedom.

Additionally, preferences also play a key role in evaluating the process aspect of freedom, since people may very well hold preferences over processes that happen in their own lives (what Sen calls "personal process concern"), but also concerning processes that impact the operation of the society on a larger scale (referred to as "systemic process concern"). Therefore, both aspects of freedom—opportunity and process—can be evaluated in terms of people's preferences.

Moreover, another important issue that should be highlighted is that, as Sen (2002) argues, a person's preference ordering may not always be complete. In many of the works in social choice, as Craven (1992) acknowledges, it is assumed that people have complete preferences, consistently put in order in the form of a full ranking. However, it could be that a person has multiple preferences, reflecting a multiplicity of valuations, or that his or her preferences intersect, consequently forming a partial ranking—referred to, in the latter case, as "intersection quasi-orderings" (SEN, 2002, p. 599). Choice, in this context, may be only maximal, not necessarily optimal.

In fact, Sen acknowledges that "Decisions for society can certainly be based on classes of information other than preferences, for example, historically established rules, customs or processes, or preference-independent formulations of procedural rights." (SEN, 1997, p. 16) Nevertheless—as a result of intellectual influences from the European Enlightenment, but mostly because of Arrow's formulations—individual preferences and values have been established as the adequate point of departure for collective decisions in this framework.

2.1.2 Multidimensional choices

Possibly the simplest way to try to guess a person's preference is by observing their behavior. Indeed, that's the basic assumption underlying the 'revealed preference' approach—that there is a correspondence between consistent choice and preference. In this conceptual framework, the existence of a 'counter-preferential' choice is impossible, as Sen (1982) asserts.

He also notes that "The public choice tradition has tended to rely a good deal on the presumption that people behave in a rather narrowly self-centered way—as *homo economicus* in particular." (SEN, 1995, p. 15) In fact, 'rational choice' is often defined as the pursuit of the maximization of one's own welfare. Edgeworth stated that "the first principle of Economics is that every agent is actuated only by self-interest." (EDGEWORTH, 1881, p. 16) Similarly, welfarist approaches make judgments about social states based exclusively on the personal welfares—or 'utilities', in the case of utilitarianism—generated by them, as Sen (1982) has claimed. Individuals would be, thus, concerned with maximizing their own personal welfare—or utility—and would make choices accordingly.

In the 'revealed preference' framework, that implies that, given that an individual chose, say, alternative *a* over alternative *b*, he must then prefer *a* because he considers that he is better off with *a* than *b*. The notion of 'revealed preference' was introduced by Paul Samuelson (1938), who analyzed consumers' behaviors and elaborated, based on the concept of utility, that their preferences may be revealed by what they purchase when faced with a particular set of prices and with a given income.

It may be argued nonetheless that the 'revealed preference' approach is somewhat limiting, for it fails to recognize the multiplicity of motivations and psychological issues involved in the process of choice, and that reason may have a broader reach. Indeed, it denies that individual behavior may be driven by collective values and ethical considerations—not solely by personal gain.

A great number of authors, such as Immanuel Kant (2002), Adam Smith (2006), and John Rawls (1999), have addressed this issue and argued that it is reasonable to assume that people may have more comprehensive values. Amartya Sen (1977), for instance, distinguishes between the notions of ‘sympathy’ and ‘commitment’. Sympathy—a combination of the Greek words *Syn*, “together”, and *Pathos*, “feeling”, meaning “fellow-feeling”—regards the case in which a person’s welfare is directly influenced by someone else’s welfare. That is, an individual may be psychologically affected by the suffering—or happiness—of others, and thus his or her own well-being depends on other people’s states.

Commitment, by contrast, relates to choosing an alternative that the person is aware will generate a lower welfare level yet does so anyway because he or she judges it is the right thing to do. It is thus associated with a sense of duty and responsibility, and is closely related to one’s morals—which are, in turn, affected by various influences, from cultural to religious and political ones.

Mark Peacock points out that “Amartya Sen’s notion of ‘commitment’ poses a challenge to understandings of rationality which conceive choices to ‘reveal’ or otherwise straightforwardly express the preferences of the chooser.” (PEACOCK, 2011, p. 35) Truly, ‘commitment’ breaks the rigid connection between behavior, preference, and personal welfare maximization assumed by the ‘revealed preference’ approach and introduces a new dichotomy. Furthermore, it supports that the concept of rationality and ‘rational choice’ must accommodate these broader judgments.

Therefore, in a social context, it is plausible to assume that people’s concerns can go beyond their own, and that their preferences may reflect that aspect. Indeed, the interests of groups, such as local communities, economic and social classes, and even families and friends, could also be taken into account. “The concepts of family responsibility, business ethics, class consciousness, and so on, relate to these intermediate areas of concern.” (SEN, 1977, p. 318)

That does not mean that egoistic behavior is ruled out—it simply means that individual welfare maximization shouldn’t be regarded as a rationality condition. Sen holds that

> Just as it is necessary to avoid the high-minded sentimentalism of assuming that all human beings try constantly to promote some selfless “social good,” it is also important to escape what may be called the “low-minded sentimentalism” of assuming that everyone is constantly motivated entirely by personal self-interest. (SEN, 1995, p. 15)

As Comim (2015) notes, this diversity of motivations gives rise to the notion of pluralism and different—and sometimes incommensurable—informational spaces. One can argue that welfarism fails to capture such pluralism, once it “reduces or eliminates diverging principles and conflicts into a common amorphous denominator” (COMIM, 2015, p. 5), the utility, and that informationally richer representations of state of affairs are more adequate within this context.

Based on these considerations, Sen further argues that the very idea of ‘preference’ should be revised, supporting a more elaborate and versatile conceptualization. While it has been used to invoke a multiplicity of notions such as values, desires, choices, and even mental satisfaction, “in much of standard economics, the differences between these distinct concepts are eschewed by making them all congruent.” (SEN, 1997, p. 17) Thus, it may serve to denote—even simultaneously—more than one definition, and yet it is possible to treat them as different and non-congruent aspects because it does nothing to obliterate the substantive differences in their contents.

Therefore, one can argue that a theory of social choice should accommodate a more elaborate structure. Sen asserts that this multidimensionality, or interpretational fluidity

> […] is a source of strength of the broad class of preference-based approaches to social choice. In different types of evaluative arguments about appropriate social decisions, diverse aspects of the individual’s will and agency are—explicitly or implicitly—considered, and the richness of the variety of interpretations permits the theory to invoke different features of the individual, depending on the context. (SEN, 1997, p. 18)

Even though social choice theory doesn’t normally engage in investigating how preferences are formed, tending to regard them as given (yet changeable), it is possible to underline the importance of social interactions, debates, and discussions in this process. Values are shaped and tested through discussion, and can be modified in the process of decision-making. However, Sen claims that this need for further examining the development of preferences “does not in any way reduce the importance in studying preference-based social choice theory.” (SEN, 1997, p. 23)

2.2 THE POSSIBILITY OF SOCIAL CHOICE: ARROW’S THEOREM

Based on the discussion presented in the previous section, it is possible to note that aggregating and combining diverse individual preferences into a collective choice is a notoriously complex task. Kenneth Arrow’s study entitled *Social Choice and Individual Values*, first published in 1951, sought to address this issue through the design of a social welfare function—an aggregation method that aims to translate a set of individual preference orderings into one social ordering of such states, for the purpose of social choice. As Arrow elaborates:

> By a social welfare function will be meant a process or rule which, for each set of individual orderings R1, …, Rn for alternative social states (one ordering for each individual), states a corresponding social ordering of alternative social states, R. (ARROW, 1963, p. 23)

Under the assumption that there exist at least three diverse social states and at least two—but not an infinite number of—individuals, and excluding the possibility of interpersonal comparisons of utility (for they are, in his opinion, meaningless), Arrow defines a set of intuitively suitable conditions that this aggregation procedure should obey:

a) unrestricted domain: the social welfare function must accommodate every possible individual preference profile and combine them into a social preference ordering;
b) independence of irrelevant alternatives: Arrow demands that the social ranking of two states, *x* and y, be subject only to the individual preferences over those states, regardless of how other alternatives are ranked;
c) nondictatorship: rules out the possibility of an individual being decisive—there must not be a person whose preferences weigh more than others, in a way that whenever he prefers a certain alternative, it results in this alternative being socially preferred;
d) Pareto principle: simply requires that, if all individuals prefer *x* to *y*, then *x* is socially preferred to *y*.

In this context, a group is considered decisive if it is so over any pair of alternatives, and any smaller group within that group is also decisive. In addition, the Pareto principle asserts that the collection of all individuals, considered as a group, is decisive. Once it can be repeatedly subdivided, we ultimately arrive at a decisive individual, or a ‘dictator’. It is, thus, impossible to satisfy all the conditions at once. This is Arrow’s “General Possibility Theorem,” or “impossibility theorem” as it is more commonly referred to.

Sen (1985) points out that this impossibility result can be regarded as a generalization of the old paradox of voting. Assuming there are three individuals (1, 2, and 3) and three alternative options (*a, b,* and *c*), Arrow (1963) holds that a natural method to reach a collective preference is by majority decision, that is, the preferred alternative is the one the larger part of the group prefers. However, suppose individual 1 prefers *a* over *b* over *c* (or $a > b > c$), while individual 2 prefers *b* over *c* over *a* ($b > c > a$), and individual 3 prefers *c* over *a* over *b* ($c > a$

> *b*). Then, a majority prefers *a* to *b* and *b* to *c* but, at the same time, *c* to *a*, which results in a paradox.

Thus, as Arrow defines it, once established the conditions aforementioned, "If there are at least three alternatives which the members of the society are free to order in any way, then every social welfare function […] must be either imposed or dictatorial." (ARROW, 1963, p. 59) Sen (1999b) further elaborates that, since only a 'dictator' would prevent such contradiction, this would entail a severe sacrifice of participatory decisions in the political domain, and a remarkable insensitiveness to the diverse interests of the populations in welfare economics.

Yet, Arrow's work has been of undeniable importance to the discipline of social choice. Besides putting the analysis of social aggregation in a more systematic and technical framework, the impossibility theorem paved the way for a variety of discussions and contributions by different authors, who were inspired by the attempt of developing 'escape routes' to this rather pessimistic result. Amartya Sen (1985), for instance, argues that by loosening some of Arrow's assumptions—in particular that of the inapplicability of interpersonal comparisons—the impossibility can be overcome. In fact, Arrow himself acknowledges the importance of this issue to the result:

> If we exclude the possibility of interpersonal comparisons of utility, then the only methods of passing from individual tastes to social preferences which will be satisfactory and which will be defined for a wide range of sets of individual orderings are either imposed or dictatorial. (ARROW, 1963, p. 59)

Therefore, the topic of interpersonal comparisons will be better analyzed in the following section. The discussion will focus on Sen's approach and his arguments for the relevance of this subject in the context of the social choice theory.

2.3 AN ALTERNATIVE WAY: INTERPERSONAL COMPARISONS

In making social decisions, Amartya Sen (1995) argues that both the appropriateness of the procedures that lead to them and the righteousness of the outcomes that derive from them should be carefully considered. Traditional social choice theory, however, seems to focus its attention more on the consequences rather than the processes, as he points out. In this approach, the states to be achieved are determined first, and then the procedures that would yield these "best" or "maximal" states. Processes are, in this view, a rather secondary concern, especially within the scope of the utilitarian theory.

By contrast, Sen (1995) supports the view that procedural considerations, or requirements, should be incorporated in consequential analysis—particularly in the ground of liberties and rights. He argues that there is a need to depict these scenarios in a more comprehensive way, taking into consideration the fundamental importance of rights and liberties, and hence enriching social, economic, and political arrangements.

These considerations make a strong argument for considering and questioning aggregation mechanisms and social decision procedures in the context of social choice. Arrow asserts that "in a collective context, voting provides the most obvious way by which individual preferences are aggregated into a social choice." (ARROW, 1951, p. 125, apud SEN, 1985, p. 1767) Sen (1999b), however, disagrees with that assumption and argues that voting-based methods are suitable for certain types of social choice issues (like committee decisions, elections, and referendums), yet inappropriate for many others. Indeed, context matters. As Sen points out:

> Through voting, each person can rank different alternatives. But there is no direct way of getting interpersonal comparisons of different persons' well-being from voting data. We must go beyond the class of voting rules to be able to address distributional issues. (SEN, 1999b, p. 355)

To address distributional issues, which are among the core concerns of welfare economics, majority rule and voting systems are therefore unsuitable. Sen (1995) illustrates this argument with the "cake division" example, which shows that majority rule is unable to differentiate between the following scenarios, both involving three individuals:

a) persons 1 and 2, who already have the largest portions of the cake, take even more cake from person 3, who has virtually nothing;
b) this time, persons 1 and 2 have very small portions of the cake and take part of person 3's share, the largest one.

In fact, Sen (1985) holds that this "neutrality" property derives from the set of conditions and axioms that Arrow established—when combined, unrestricted domain, Pareto efficiency, and independence of irrelevant alternatives result in this insensitiveness to distribution (and to basic rights and freedoms) simply focusing on the variations of the total welfare of the society. As he puts it:

> We must regard all these divisions—equal and extremely unequal ones—as exactly as good as each other from the social point of view. Once we have got to neutrality in this format, there is no real chance any more of making judgments concerning income distribution in a way that is relevant to welfare economics. (SEN, 1985, p. 1769)

There is, thus, a strong argument against majority rule as a form of aggregating preferences for judgments in the field of welfare economics. As a matter of fact, Sen (1985) claims that this is a problem that aggregation procedures used in the context of social choice can face once interpersonal comparisons are ruled out—a "common embarrassment" to a variety of them.

The solution offered by Sen is to expand the informational basis on which traditional social choice theory relies by admitting the use of interpersonal comparisons as a plausible way for measuring inequality. He argues that this would not only allow Arrow's impossibility result to be circumvented, but it would also incorporate normative and ethical considerations into the approach, which would contribute to enrich the discussions in the field.

One may then ask how it could be possible to analyze and contrast such a big number and diversity of individuals. Sen holds that interpersonal comparisons are not required to be complete—we may not need to juxtapose every individual with one another, and regarding every single dimension of their lives. There may be some degree of comparability between people, an intermediate stage between perfect comparability and no comparability whatsoever: a partial comparability. The lack of exactness does not, however, undermine the importance of this method; this flexible approach may be very useful in practical terms. Sen claims that "There may be no general need for terribly refined interpersonal comparisons for arriving at definite social decisions. Quite often, rather limited levels of partial comparability will be adequate for making social decisions." (SEN, 1999b, p. 356)

Indeed, it would not be reasonable to discard interpersonal comparisons altogether simply because the precision is not absolute. Moreover, Sen (1999b) argues that different kinds of interpersonal comparisons can be entirely axiomatized and integrated into social choice technical frameworks.

Another relevant issue in this discussion is the informational basis of interpersonal comparisons: in order to appropriately evaluate inequality, in terms of what should people be compared? The debate regarding the best method to assess individual advantage branches out into a number of different points of view. The utilitarian approach focuses its analysis, as the name suggests, on the utility (or well-being) derived by different individuals from diverse conditions and situations. John Rawls (1999), on the other hand, takes what he calls "primary goods" as a basis for interpersonal comparisons, i.e., goods that are both desirable and useful for any human being, regardless of what his or her objectives may be. The "natural primary goods" category includes health, intelligence, memory, creativity etc., whereas the "social primary goods" category includes rights, income and wealth, liberties and opportunities, and social bases of self-respect.

Nevertheless, Amartya Sen (1999b) claims that neither of the two approaches offers a complete and suitable metric for assessing individual advantage. Comparisons of mental states, which is the basis of the utilitarian approach, may be too subjective in this case, especially since people tend to adapt to long-lasting hardship and deprivation as a mechanism of survival. On the other hand, focusing on the ownership of "primary goods" fails to portray an important dimension of the relationship between a person and a good: the "functioning" aspect. As Sen (1982) elaborates, the chain consists in goods – characteristics – functioning – utility. While 'characteristics' are attributes of goods, 'functioning' describes what a person is actually able to do with those characteristics—that is, if he or she is able to convert the good into personal accomplishments. In fact, he argues that it is also important to take into account whether people have the substantive opportunity and capacity to function in certain ways, even if they do not choose to do so. This notion is central to the Capability Approach.

The Capability Approach was first formulated by Amartya Sen in the 1980's, but has since incorporated contributions from other authors, such as those of Martha Nussbaum (2011). Its core ideas are, as summarized by Ingrid Robeyns in the Stanford Encyclopedia of Philosophy,

> […] first, the claim that the freedom to achieve well-being is of primary moral importance, and second, that freedom to achieve well-being is to be understood in terms of people's capabilities, that is, their real opportunities to do and be what they have reason to value. (ROBEYNS, 2011)

Therefore, Sen (1982) argues that capabilities can be regarded as a comprehensive way of evaluating individual advantage, and can, consequently, be an adequate basis for interpersonal comparisons in the context of social choice.

However, issues concerning informational availability may pose challenges to this method, as Sen (1999b) acknowledges. It is thus important in this context to diversify the informational approach, relying on more than just one type. "In the recent literature in applied welfare economics, various ways of making sensible interpersonal comparisons of well-being have emerged." (SEN, 1999b, p. 359) For instance, it is possible to compare living standards and levels of quality of life of different individuals through the observation of relevant living conditions. Also, the use of questionnaires can be helpful in identifying consistencies and symmetries in people's answers concerning comparative well-being. Similarly, the investigation of expenditure patterns can provide a basis for inferences about their relative well-being. Furthermore, indicators of aggregate poverty and income inequality are also useful and important to consider when making interpersonal comparisons and social welfare judgments.

Although, as Sen (1999b) recognizes, all these methodologies have limitations, they have significantly enriched the empirical interpretations of individual advantages and interpersonal comparisons, thereby broadening the informational basis of welfare economics and social choice theory, and ultimately contributing to these approaches at the analytical and practical levels—especially in the last case. Indeed, "[…] what ultimately makes social choice theory a subject of importance is its far-reaching relevance to practical and serious problems." (SEN, 1985, p. 1774)

2.4 RANKINGS AND META-RANKINGS IN THE CONTEXT OF SOCIAL CHOICE

As Sen points out, "The outcomes of the social choice procedure take the form of ranking different states of affairs from a 'social point of view', in the light of the assessment of the people involved." (SEN, 2009, p. 95) Thus, rankings play a crucial role within the context of social choice theory and shall be further analyzed in this section.

Comim (2015) notes that one way rankings differ from one another is regarding how they combine different criteria, yielding a particular ordering. The criteria usually relate to four categories of information—capabilities, resources, rights, and subjective well-being. As argued in the last section, comparing people in terms of their individual advantage is fundamental to assessing inequality and thus to social choice.

Within this context, Comim points out that there are different categories of rankings: while complete rankings demand total comparability, i.e., that any pair of alternatives must be ordered *vis-à-vis* one another, determining which one is better or whether they are indifferent, partial rankings allow more flexibility in that sense, admitting partial comparability among options. Indeed, "[…] a partial ranking reflects a minimum standard, a lower limit, of what can be asserted without contradicting any of the other rankings." (COMIM, 2015, p. 10)

Moreover, one can try to search for a shared ranking among partial orderings—an 'intersection quasi-orderings', that has the advantage of not depending exclusively on any particular measure, as Sen (1997) affirms. It is important to note, however, that the resulting ranking can also be incomplete. Additionally, lexicographic rankings are organized based on a hierarchical arrangement of criteria—that is, the first criterion of the ranking must be fulfilled before the second criterion is considered, and so on. The name 'lexicographic' derives from the logic based on which dictionaries are built: words are ordered according to their first letters; within those beginning in 'a', words are ordered based on their second letters, and so on. This type of ranking became more widely known after Rawls (1999) employed it in his work "A Theory of Justice", establishing the principle of equal liberty as prior to all the other principles.

Furthermore, meta-rankings provide another system to order alternatives. As mentioned earlier in the discussion, given the multiplicity of preferences and motivations that inspire individuals to make certain choices, it may be an oversimplification to demand that a single ordering should be able to illustrate this variety of aspects. Indeed, as Sen holds:

> […] traditional theory has too little structure. A person is given one preference ordering, and as and when the need arises this is supposed to reflect his interests, represent his welfare, summarize his idea of what should be done, and describe his actual choices and behavior. […] To make room for the different concepts related to this behavior we need a more elaborate structure. (SEN, 1977, p. 335)

From this basis, Sen (1977) argues that, in order to broadly represent our moral judgments, we need to consider multiple preference and action rankings—and even rankings of such rankings. These 'meta-rankings' reflect the need to exercise morality and normative judgments in deliberating among distinct patterns of preferences and actions. They support a comparative analysis regarding the moral and social desirability of holding different preferences (or acting as if one did). As Robert J. van der Veen puts it, "[…] the meta-ranking approach may be seen as an attempt to exploit the psychology of mixed reasons." (VAN DER VEEN, 1981, p. 355) Therefore, being a more elaborate framework, meta-rankings take into account the complexity involved in the process of choice and admit a broader range of moral articulation, providing a suitable analytical structure for social evaluations.

Moreover, in this kind of analysis, some flexibility is allowed in the sense that meta-rankings are not required to be complete, that is, one does not need to fully order all set of rankings. "It can be a partial ordering, and I expect it often will be incomplete." (SEN, 1977, p. 338) That does not, however, diminish the importance of this method—these different considerations are still crucial in exercising morality and ethics.

When making social decisions, there needs to be a way of determining what to prioritize, that is, where to focus the attention, and the meta-ranking structure can be a useful tool in this sense. Even though comparative analysis may very often engender partial rankings, as it involves non-commensurabilities of many sorts, the meta-ranking framework still provides a helpful method, in spite of being incomplete. In fact, Comim claims, in regards to meta-rankings, that "[…] their incomplete and partial nature can be seen as a testimony to democracy and public reasoning." (COMIM, 2015, p. 11) Indeed, Sen (2009) maintains that social choice theory has much to offer to a theory of justice for it, among other contributions, allows different types of incompleteness—one that is temporarily accepted, while supplementary information or more thorough examination are expected, but also an 'assertive' incompleteness, reflecting the idea that sometimes two alternatives cannot be ranked in terms of justice.

Even so, Comim (2015) maintains that, in order to enrich this discussion, a more concrete and objective analysis should be carried out through the actual application of the meta-ranking approach to human development statistics, such as the Human Development Index, providing the basis for different human development valuations. Sen (1977) asserts that meta-rankings are a versatile technique which can be employed in different contexts in a variety of ways. Thus, Comim holds that

> The instrument of meta-ranking might be suitable for human development analysis once it provides a structure for normative discussions that don't try to hide the public morality of development interventions. Meta-rankings can be combined with different weighing schemes and articulation of different informational spaces. (COMIM, 2015, p. 19)

Therefore, given the importance of the meta-ranking structure for the field of human development, within the context of the social choice theory, a similar exercise was performed in this study using partial order analysis and the meta-ranking approach to investigate the Human Development Index. As described earlier, the methodology and its application, as well as the results and insights it provided, are detailed in the upcoming chapters.

3 METHODOLOGY OUTLINE: PARTIAL ORDER ANALYSIS AND THE META-RANKING APPROACH

Acknowledging that we are faced with ranking issues in a variety of situations, Brüggemann and Patil (2011), in the book *Ranking and Prioritization for Multi-Indicator Systems*, develop a systematic approach to partial order theory. This, as described by them, "is the theory by which objects, characterized by multiple indicators, can be compared and ordered" (BRÜGGEMANN & PATIL, 2011, p. ix), combining elements of discrete mathematics and graph theory.

One of the central questions in this context concerns, thus, how to rank multifaceted objects, described by a number of attributes, and the authors argue that composite indicators are a way forward to deal with this issue. The *Handbook of Constructing Composite Indicators*, published by the Organization for Economic Co-operation and Development (OECD) in 2008, elaborates that composite indicators are "formed when individual indicators are compiled into a single index on the basis of an underlying model. The composite indicator should ideally measure multidimensional concepts which cannot be captured by a single indicator […]" (OECD, 2008, p. 13) In order to construct a composite indicator, Brüggemann and Patil (2011) assert that one must identify its aim, that is, what is meant to be illustrated by it, and then select the indicators that best describe those properties. These indicators may be aggregated into pillars based on commonalities, and the values of the pillars may be expressed by weights. They claim that it is crucial that composite indicators provide not only rankings but also adequate metrics for comparing the objects in question.

From this basis, Brüggemann and Patil (2011) hold that partial order analysis offers important analytical tools to compare and order objects characterized by multiple indicators. Indeed, this theory yields useful insights into the relative rankings, helping, for instance, to discern the impact of individual indicators on the objects in question, to evaluate the relevance of the attributes selected, and to analyze the comparability between the objects and their relative position in the ranking.

Therefore, this chapter is organized as follows: sections 3.1 to 3.3 describe the basic elements in partial order analysis and the main evaluations it provides—such as incomparable objects, up sets, down sets, linear extensions, object heights, and levels. Those will be the evaluations illustrated in chapter 4, when an empirical application of the theory is presented. There are, however, several additional analyses that can be performed by means of partial order analysis, and a few of them are discussed in the Appendix of this study—e.g., analyses based on attributes and some procedures to simplify complex diagrams—which can contribute to enrich the approach. All the analyses were based on the previously mentioned book by Brüggemann and Patil, and most of the examples, tables, and figures were obtained from the book. Finally, the last section of this chapter introduces the discussion of how the meta-ranking approach can contribute to enrich these assessments.

3.1 BASIC ELEMENTS

3.1.1 Data matrix

Let us suppose that one would like to compare five objects, *a, b, c, d,* and *e.* Once the purpose of the ranking is established, one would then have to identify the properties of such objects that are relevant to the analysis—say, q_1, q_2, and q_3. All properties must have the same orientation, that is, they must contribute to the purpose of the ranking in the same way. If an increasing number of the attribute q_1 is desirable, then an increasing number of attributes q_2 and q_3 should also be regarded as so. This might not always be the case when we refer to positive attributes—such as percentage of children enrolled in primary school—vis-à-vis negative attributes, for instance percentage of contaminated rivers or CO_2 emissions. Moreover, the object set is denoted by X = {*a, b, c, d, e*}, representing the rows of the matrix, while the set of attributes, or information base, is expressed by IB = {q_1, q_2, q_3}, representing its columns.

Regarding comparability, the objects can be either comparable or incomparable between themselves. If there is no conflict between the attribute values of two objects, i.e., if all the attributes of one object are smaller or larger than those of another object, then they are considered comparable. In this case, a binary relation can be acknowledged, expressed by ≤ or ≥. Being $q(x)$ the data row for *x,* and $q(y)$ the data row for *y*, this can be formally denoted by:

$$x \leq y, \text{ if and only if } q(x) \leq q(y),$$
$$q(x) \leq q(y), \text{ if and only if } q_i(x) \leq q_i(y), \text{ for all } i$$

On the other hand, if a divergence is identified, the objects are regarded as incomparable, and the symbol || is used to express the relation. Also, if the objects have identical rows in the data matrix, they are considered as equivalent (while, however, still being regarded as different objects). This relation is denoted by ≈. The equivalent objects can be aggregated into an "equivalence class," and one object must be elected as a representative of the class. Table 1 illustrates a simple hypothetical and generic data matrix:

Table 1 - Illustrative data matrix

	q_1	q_2	q_3
a	1	2	1
b	2	3	5
c	4	5	6
d	3	6	7
e	6	8	9

Source: elaborated by the author based on Brüggemann and Patil (2011)

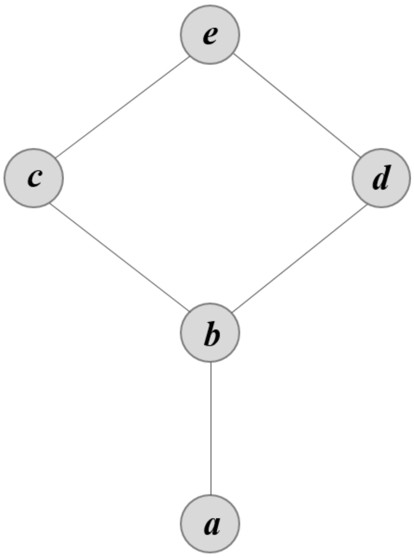
It is possible to see that $a < b$, $a < c$, $a < d$, $a < e$; $b < d$, $b < e$; $c < d$, and $c < e$. Therefore, these objects are comparable. However, objects *c* and *d* are incomparable, since $c > d$ concerning attribute q_1, but $d > c$ concerning attributes q_2 and q_3. Thus, $c \parallel d$. There are no equivalent objects in the data matrix.

Furthermore, an object set X containing a partial order, such as that described above, is called a "poset" (partially ordered set). (X, IB) is the partial order where $x \in X$ and $q_i \in IB$. By contrast, a complete, total, or linear order occurs when there are no incomparabilities among objects.

3.1.2 Hasse diagrams

A Hasse diagram is a graphical representation of a partially ordered set (poset), that helps visualize the relations between the objects. The objects are displayed vertically, organized in levels, with the object containing the lowest attribute values at the bottom and, similarly, the one containing the highest attribute values at the top. Moreover, the objects that are considered comparable are then connected by different lines. Thus, it is usually possible to identify incomparable objects based on Hasse diagrams. Figure 1 illustrates a Hasse diagram based on data of Table 1:

Figure 1 - Hasse diagram based on data of Table 1



Source: elaborated by the author based on Brüggemann and Patil (2011)

Object *a*, as seen before, is the object that possesses the lowest attribute values, and is therefore located at the bottom of the diagram, or first level. The second level is formed by object *b*; the third level, by objects *c* and *d* (incomparable); and the fourth, by object *e*.

Within this context, it is important to characterize some basic concepts concerning the structure of Hasse diagrams. Brüggemann and Patil enumerate several examples, and those considered more relevant shall be presented next:

a) chain: a subset, along with the partial order relation between its constituent objects. In the Hasse diagram above, $a < b < c$ is a chain;
b) maximal chain: the longest chain that can be found. $a < b < c < e$ is a maximal chain;
c) antichain: a subset consisting only of those objects which are incomparable. As well as maximal chains, there can be maximal antichains. $\{c, d\}$ is an antichain;
d) height of the poset: the number of elements in the longest chain. The height of the poset considered in the example is 4;
e) width of the poset: the number of elements of the maximum of antichains. Similarly, the width of that poset equals 2;
f) maximal element x: element for which no relation $x \leq y$ can be found. The set of maximal elements is denoted MAX. When there is only one maximal element, it is considered a greatest element. In the example, object *e* is the only maximal element—it is, thus, the greatest element;

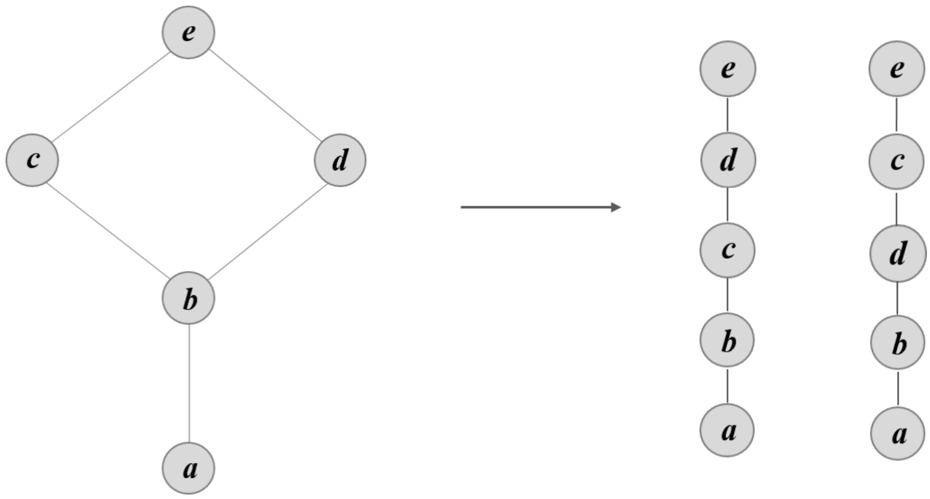
g) minimal element *x*: element for which no relation $y \leq x$ can be found. The set of minimal elements is denoted MIN. In similar fashion, when there is only one minimal element, it is regarded as the least element—which is the case of object *a* in the example. Maximal and minimal elements are usually of special concern, and can be considered priority elements;
h) isolated element: an element that presents no relation with other objects. They usually suggest singularities in the data matrix;
i) cover relation: *x* is covered by *y* if there is no other element between them;
j) set of incomparabilities: denoted by $U(x)$, it consists of all the incomparable objects, with respect to an object *x*, that can be found in the poset;
k) principal down set: considering a particular object *x*, its principal down set, or O(*x*), is the set of objects of the poset which have smaller values than *x* for all attributes;
l) principal up set: similarly, the principal up set, or F(*x*), is the set of objects that have higher values than *x* for all attributes;
m) interval of two objects *x* and *y*: denoted by $I(x, y)$, it is obtained by determining a third element *z* for which $x \leq z \leq y$.

Based on these notions, it is possible to derive several analyses regarding the objects in question. For instance, a poset with large antichains usually implies objects that present a considerable number of incomparable objects associated to them—which, in turn, suggests that these objects show conflicting data profiles. These aspects can help, for example, in the evaluation of the relationship between the attributes, providing a more comprehensive characterization of the dataset.

3.2 LINEAR EXTENSIONS AND OBJECT HEIGHTS

A linear extension consists of a linear order that conserves the order relations of a poset. It is therefore described as “order preserving.” A single Hasse diagram can generate multiple linear extensions; the only requirement is that they all preserve the order. *LT(X, IB)*, or LT, denotes the number of all linear extensions of a poset, whereas LE(X) is the set of all linear extensions. Figure 2 illustrates how the method operates using the Hasse diagram shown in Figure 1:

Figure 2 - Illustrative example of a set of linear extensions based on Figure 1



Source: elaborated by the author based on Brüggemann and Patil (2011)

It is possible to see that two linear extensions can be derived from the Hasse diagram, varying the relative positions of *c* and *d*, the incomparable objects. More generally, the height of an object *x* in the *i*th linear extension is expressed by $h_{le(i)}(x)$, and the probability for *x* to have height *h* can be calculated by the ratio between the number of linear extensions in which the element presents that height and the total number of linear extensions.

Furthermore, the average height, hav(*x*), is obtained considering the average of all heights of an object *x* over the entire set of linear extensions. Therefore, (X, {hav}) yields a canonical order and offers an alternative ordering to the poset in question, called O_{poset}. It is, thus, useful to derive linear orders from posets, and this can be done either with or without weights. The level method, which does not make use of weights, will be described in the following subsection as an illustration.

3.3 DEDUCING LINEAR ORDERS WITHOUT WEIGHTS: LEVELS

Levels are a simple way to obtain linear orders from posets. The aim here is to create equivalence classes among objects by forming level sets, and the method is simple: MAX being the set of maximal elements of a poset, and lg the number of cover relations in the maximum of all maximal chains, an object *x* that is part of MAX obtains the level number lev(*x*) = lg + 1. Then, X is partitioned by removing MAX and establishing a new MAX, which gets a level number reduced by one. The process continues until X is exhausted. Once this has been done, objects can be ordered according to their level number lev(*x*) in order to generate linear orders.

The benefit of this procedure is its simplicity; nonetheless, ordering objects by lev can lead to numerous ties, which can be seen as a disadvantage.

Furthermore, partial orders with many objects can generate messy Hasse diagrams, with too complex structures, making it an arduous task to perform evaluations and draw conclusions only by visual examination. Thus, simplifying the diagrams may be a useful tool to facilitate the analysis, and levels are an easy approach to make diagrams simpler, as they create equivalence classes among objects by forming level sets. Objects in the same level set have the same height, which facilitates the identification of antichains—a difficult process if one had to rely on the inspection of a messy Hasse diagram in its original form.

3.4 PARTIAL ORDER ANALYSIS AND THE META-RANKING APPROACH

Partial order analysis, in contrast with complete rankings, is a broader evaluation structure that allows the observation of 'incommensurabilities' and partialities, potentially stimulating insights about the idiosyncrasies of the objects considered. Moreover, it can provide a basis for an empirical application of the meta-ranking approach, enriching the discussion presented in the last chapter.

As argued before, the meta-ranking framework can be a useful tool for social decision-making, once it helps analyze what should be prioritized within a particular context, that is, where the attention should be focused. Meta-rankings, however, require taking a step further and incorporating normative considerations into the analysis, which the partial order methodology does not specifically perform. It can be argued, thus, that there exists a complementarity between them.

Therefore, it is possible to see that there exist different analytical tools that can help evaluate and interpret rankings of multidimensional objects. Aiming to investigate how they operate and to illustrate how they may complement one another, the next chapter is dedicated to an application of the partial order methodology to Human Development Index statistics, whose results are considered within the meta-ranking framework.

4 PARTIAL ORDER ANALYSIS AND THE HUMAN DEVELOPMENT INDEX

The idea of development has been the object of much discussion in the field of economics. As Edewor (2014) elaborates, in the 1940s, when the subject began, the debate was primarily focused on economic growth theory, understanding poor countries simply as low-income countries. However, as Amartya Sen (1999a) holds, traditional measures of economic development failed to portray the complex and multi-dimensional nature of this phenomenon, as they would concentrate on income and national product instead of people's entitlements and the capabilities they yield. Fomenting the productivity and wealth of the economy is but a part of what development is. Therefore, the discussions in the field began to focus on how to define human development and on the selection of alternative—and broader—measures of human welfare.

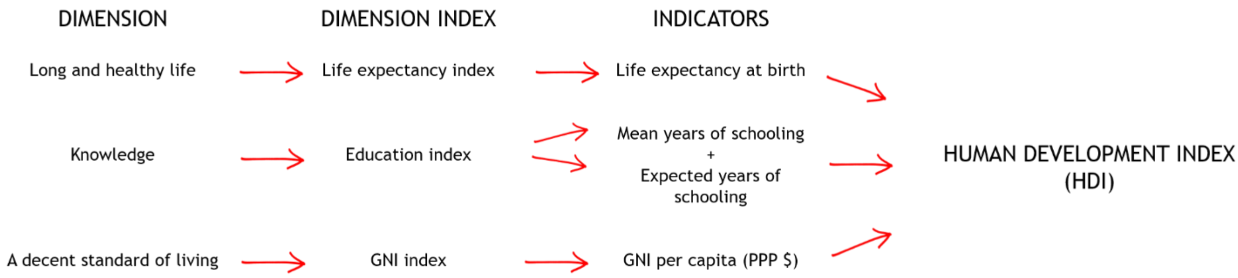
In line with these concerns, Pakistani economist Mahbub ul Haq, from the United Nations Development Programme (UNDP), created the annual Human Development Reports, first published in 1990, which aimed to shift the center of attention from national income to policies focused on people. Along with a group of development economists which included Amartya Sen, Frances Stewart, Paul Streeten, Keith Griffin, Meghnad Desai, Gustav Ranis, and Sudhir Anand, he developed the Human Development Index (HDI), first published that same year, seeking to assess the performance of different countries in three crucial dimensions of human development—living a long and healthy life, being knowledgeable, and having a decent standard of living.

Within this new approach, individuals' capabilities—or the real opportunities and freedom they have to lead the life they value—play a central role in evaluating human development. As argued earlier in this study, capabilities can be seen as a comprehensive way of assessing individual advantage and can, thus, provide an adequate foundation for interpersonal comparisons in the context of social choice.

This relates to a discussion proposed by Sen in his *Maximisation and the Act of Choice* (1997), where he distinguishes between 'comprehensive outcomes' and 'culmination outcomes'. The main element of difference is that comprehensive outcomes are inclusive of culmination outcomes plus the processes that generate them. By doing so, Sen establishes his Capability Approach within Rawls' procedural analysis, where processes are as valued as the final outcomes they generate.

From this basis, in its latest version—updated in 2010—the HDI considers four indicators: the health dimension is evaluated considering the life expectancy at birth; the education dimension considers years of schooling for adults aged 25 years or more and expected years of schooling for children of school entering age; and the standard of living dimension considers the gross national income per capita. Then, using geometric mean, the results for the three dimensions are combined into a composite index, based on which the countries are ranked. Figure 3 illustrates the structure of the HDI. The 2015 Human Development Report calculates HDI values based on estimates for 2014, covering 188 countries.

Figure 3 - Structural representation of the Human Development Index



Source: elaborated by the author based on UNDP (2016)

The Human Development Index allows, thus, international comparisons based on critical socioeconomic indicators, from which several analyses can be derived. As Indian economist Partha Dasgupta claims, “international comparisons of well-being tell us something about the nature of differing societies.” (DASGUPTA, 1992, p. 124) The HDI results are, however, traditionally represented in the form of a complete ranking, ordering all countries vis-à-vis one another. This approach, although useful for several different analyses, may present limitations. Indeed, there may exist incomparabilities between countries as a result of divergences in their data profiles, which are not easily captured by a full ranking.

Therefore, within this context, partial order analysis—described in the previous chapter—may yield interesting insights into the Human Development Index by allowing a more comprehensive evaluation of potential incomparabilities between countries and on the individual impact of indicators on their relative positions. In order to perform these exercises, the online version of the PyHasse software—the PyHasse-inet, available at https://pyhasse.org/calc—was used, and the results shall be presented in the upcoming sections.

As Brüggemann and Patil (2011) describe, there are various software packages available that allow an empirical application of partial order analyses, such as Prorank (which uses the Java programming language), Correlation and Whasse (which use the Delphi programming language), and Dart and Poset (which use the C++ programming language). The PyHasse software—which uses the Python programming language—was chosen as it is the one

Brüggemann and Patil utilize in their book, and because PyHasse-inet can be easily accessed online, offering a practical user interface. The online version provides a more limited number of modules than the software's "expert version"; nonetheless, it offers the basic tools for partial ordering studies based on data matrices of multi-indicator systems, allowing non-experts to investigate the different applications of ordinal analyses.

4.1 PYHASSE ANALYSES

4.1.1 Complete Human Development Index

Due to limitations in the PyHasse software, which supports a maximal number of 100 objects, the data matrix used in the analysis considers only the first 99 countries of the Human Development Index ranking. The first 49 countries within those considered here are in the "very high human development" category, while the remaining 50 are classified as "high human development" countries. Furthermore, 44 are European countries, 23 are Asian, 15 are North American, nine are South American, five are African, and three are Oceanian countries. In any case, the actual number of countries is only illustrative here given that the analysis is focused on the demonstration of the methodology. In fact, the more homogeneous the list the better because then it is possible to identify differences in lists that seem similar.

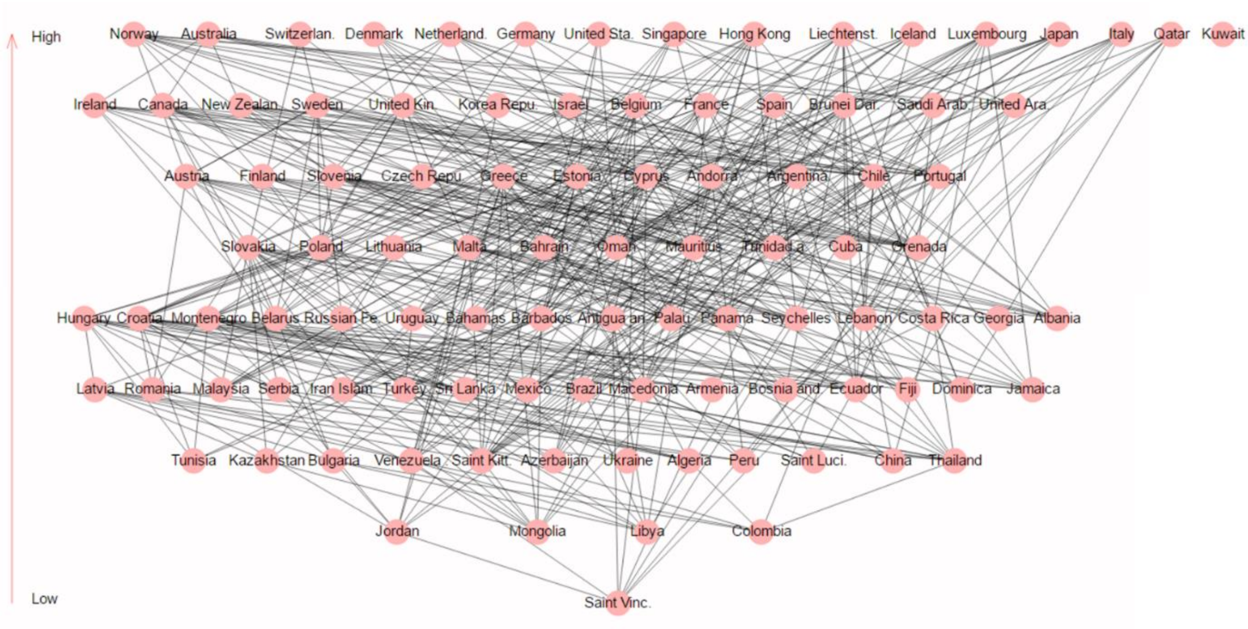
Table I—shown in Annex B—presents data regarding countries' scores in each of the four indicators, provided by the United Nations Development Programme (2016). In fact, the individual rankings that constitute the HDI inspire an interesting preliminary analysis, and the most relevant findings shall be presented next. The complete tables are also available in Annex B (Tables II to V).

Norway, Australia, and Switzerland are currently ranked as the top three countries in the HDI ranking, while Colombia, Saint Vincent and the Grenadines, and Jamaica are the bottom three among those examined here. Concerning the health dimension, Hong Kong, Japan, and Italy present the highest life expectancy values—84, 83.5, and 83.1 years, respectively. By contrast, Fiji, with a life expectancy equivalent to 70 years; Kazakhstan, to 69.4; and Mongolia, also to 69.4, are placed last. Considering, on the other hand, the education indicators, Australia, New Zealand, and Iceland rank the highest in terms of expected years of schooling (with 20.2, 19.2, and 19 years, respectively), whereas Armenia (12.3 years), Azerbaijan (11.9 years), and Albania (11.8 years) are at the bottom of the ranking. When it comes to mean years of schooling, Germany, the United Kingdom, and Australia are ranked as the top three countries, while

Colombia, Kuwait, and Tunisia are ranked as the bottom three. Their data profiles are 13.1, 13.1, and 13 years for the former, and 7.3, 7.2, and 6.8 for the latter. Finally, regarding income, Qatar, Kuwait, and Liechtenstein present the highest gross national income per capita values, while Jamaica, Cuba, and Georgia present the lowest. It is interesting to note the peculiar case of Kuwait, which ranks among the top three countries in the income ranking and among the bottom three for one of education's indicators. This as well as other distinct cases will be further examined in upcoming analyses.

Considering the PyHasse results for the complete HDI ranking, the Hasse diagram derived from the data matrix is illustrated by Figure 4 below.

Figure 4 - Hasse diagram based on the complete HDI ranking



Source: UNDP (2016). Elaborated by the author on PyHasse software.

Although not much can be concluded about the poset only through visual inspection once the resulting diagram is extremely fuzzy, the very fact that it is messy and shows large antichains indicates the presence of a significant number of incomparabilities between countries, which could not be seen only by analyzing the complete ranking. The diagram is organized in nine levels and, as expected, most countries in the category of "very high human development" are concentrated on higher levels. However, Slovakia, Poland, Lithuania, Malta, Bahrain—which are part of that class—share the same level as Oman, Mauritius, Trinidad and Tobago, Cuba, and Grenada, countries in the "high human development" category. In fact, Hungary, Croatia, and Montenegro, countries of a very high human development, are in an even lower level. Levels 4 and 5 present the highest number of objects—sixteen objects each, with most countries in the category of high human development. Each level and its respective full composition is illustrated in Table VI, in Annex B.

The objects with the largest sets of incomparabilities are Kuwait (which is, in fact, an isolated object, presenting zero comparabilities and therefore 98 incomparable objects), Saudi Arabia, with 91 incomparable objects, and the United Arab Emirates, with 90 incomparable objects. That is the result of their conflicting data profiles: considerably high levels of income but rather low education and health indicators. In fact, as mentioned earlier, Kuwait is among the top three countries in terms of income while also among the bottom three in terms of mean years of schooling. On the other hand, Australia, Norway, and New Zealand hold the smallest number of incomparable objects—23, 28, and 32, respectively, once their data profiles are more homogeneous. The objects' 'connectivities'—that is, the number of down sets, up sets, and incomparable objects associated with each of them—are presented in Table VII, in Annex B. Furthermore, there are no equivalence classes in the poset.

For a richer account of the attributes considered in the analysis, their maximal, mean, and minimal values are shown in Table 2. Uruguay presents the same value as the mean value of life expectancy; Croatia and Kazakhstan, the same number of expected years of schooling as this attribute's mean value; Finland, Greece, and Malta, 10.3 mean years of schooling, the average value; and Malta and Cyprus present GNI per capita values around the mean value.

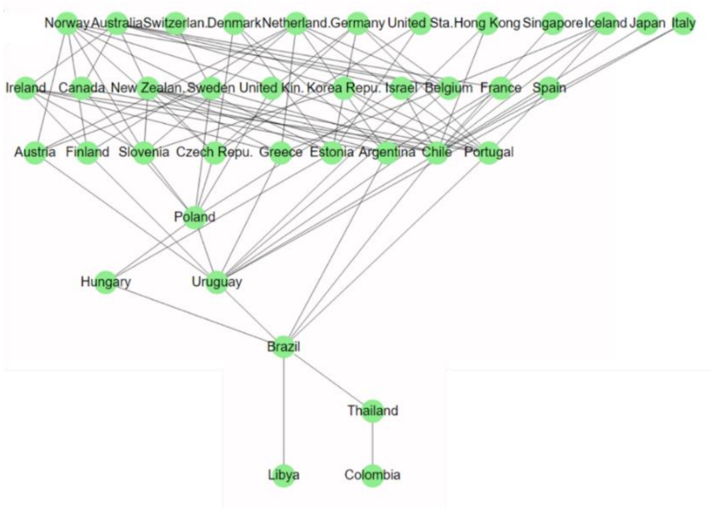
Table 2 - Attributes' statistical characteristics

	Life Expectancy	**Expected Years of Schooling**	**Mean Years of Schooling**	**Gross National Income Per Capita**
Max	84.0	20.2	13.1	123,124
Mean	77.2	14.9	10.3	27,998
Min	69.4	11.8	6.8	7,164

Source: UNDP (2016). Elaborated by the author on PyHasse software.

Moreover, it may be interesting to analyze Brazil's local Hasse diagram, which shows all the comparable objects associated with it, illustrated in Figure 5. As expected, most European countries are in higher levels than Brazil; however, South American countries such as Argentina, Chile, and Uruguay are also in better positions, indicating that they have higher values for all four attributes. Brazil is only relatively better than Libya, Thailand, and Colombia—which indeed rank lower than Brazil in the traditional HDI ranking.

Figure 5 - Brazil's local Hasse diagram based on the complete HDI



Source: UNDP (2016). Elaborated by the author on PyHasse software.

4.1.2 The individual impact of indicators

To assess the individual impact of attributes in the objects' positions, equivalent analyses were performed removing each dimension one after another and considering only the remaining ones. First, the results of removing both education indicators are presented; then, those of removing life expectancy; and finally, income. The resulting Hasse diagrams are presented in Annex A, and the discussion will focus on the most relevant findings derived from these exercises.

4.1.2.1 The impact of education

Once expected years of schooling and mean years of schooling are removed from the data matrix the resulting Hasse diagram becomes considerably taller, with 25 levels. That happens because the lower the number of attributes the more likely the objects are to be comparable between themselves. In this new scenario, Kuwait, Saudi Arabia, and Cuba have the largest sets of incomparabilities—74, 67, and 64 objects, respectively. On the other hand, those countries which present the lowest number of incomparable objects are Fiji, with 5 incomparabilities; Singapore, with 6; and Hong Kong and Ukraine, with 9 each. It is possible to note that these numbers are a lot inferior to those from the original poset, which revolved around 28 objects. The tables with the complete composition of each level and all the objects' 'connectivities' are shown in Annex B (Tables VIII and IX). As well as in the previous case, there are no equivalence classes here.

Regarding the impact on countries' average height, Table X, in Annex B, shows the variation for the complete set of countries, based on which it is possible to verify that most objects in higher positions tended to present a relatively low negative variation from their original average heights, but the impact becomes positive and larger as one moves towards the end of the table. Table 3 below illustrates the countries which present the most significant negative and positive variations as a result of the suppression of education, showing the original average height for the complete Human Development Index, the average height once education was removed, and the percentage variation.

Table 3 - Countries with the largest average height variations as a result of removing education

	Complete HDI	**Removing education**	**Variation (%)**
Georgia	5.88	1.44	-75.51
Fiji	3.57	1.05	-70.59
Belarus	23.80	7.69	-67.69
Dominica	2.50	22.00	780.00
Malaysia	2.77	35.82	1,193.14
China	1.88	25.71	1,267.55

Source: UNDP (2016). Elaborated by the author on PyHasse software.

The removal of education from the poset induced a similar effect on Georgia, Fiji, and Belarus—their average heights decreased around 70%, revealing that education impacts positively on those countries' performance in the Human Development Index ranking. Belarus and Fiji both present 15.7 expected years of schooling and rank, respectively, 31st and 32nd in that indicator's ranking—higher than countries like Singapore and Japan. Georgia, on its turn, occupy the 18th position in the mean years of schooling ranking (12.1 years), and Belarus is in 19th position (12 years)—ahead of the Netherlands and Japan, for instance. At the same time, these countries don't present such high levels of life expectancy or income, which explains the significant variation: Fiji is in fact among the bottom three countries when it comes to life expectancy, while Georgia is the last country in the income ranking. Thus, considering their relatively bad performance in health and income indicators, their positive performance is mostly due to education.

On the other hand, Dominica, Malaysia, and especially China are favored by the removal of education indicators from the analysis: their average heights increase in 780%, 1.193%, and 1.267%, respectively. China ranks relatively very low in the education rankings: 89th in the expected years of schooling ranking, and 94th in the mean years of schooling one. Malaysia and Dominica, in their turn, rank 91st and 92nd in the expected years of schooling ranking, and 55th and 89th in the mean years of schooling ranking, respectively. Neither of the countries presents a very good performance in the health or income dimensions, but their remarkably low scores in education indicators explain why the removal of such attributes causes a great positive impact on their average heights. This points to a relevant deficiency within these societies, and public policies targeting education could thus help foment their development processes.

Furthermore, the effect on Brazil's relative position was positive and significant, amounting to 60.1%. The country ranks 90th in the mean years of schooling ranking and 44th in the expected years of schooling ranking, as well as 70th in terms of income and 75th in terms of health. Furthermore, analyzing Brazil's local Hasse diagram (illustrated by Figure II in Annex A), one can verify that Brazil's down set increased when compared to its original local diagram, with the addition of Palau, Jordan, Grenada, Mongolia, Saint Vincent, Ukraine, and Fiji—meaning that Brazil is better than those countries in terms of health and income. Kuwait, nonetheless, no longer integrates Brazil's down set in this new scenario.

4.1.2.2 The impact of health

The suppression of life expectancy, in its turn, results in a Hasse diagram with 14 levels—five more than in the original diagram. The impact on the number of levels was not so significant in this case once only one attribute was removed from the data matrix, as opposed to two attributes in the previous exercise. Within this context, the objects with the highest number of incomparabilities are Qatar, the United Arab Emirates, and Kuwait—with 87 incomparable objects for the first two and 97 for the latter. By contrast, those counties presenting the smallest sets of incomparabilities are Norway (18), Australia (19), and Denmark (20). As in the previous section, the tables presenting the levels' compositions and the objects' 'connectivities' are shown in Annex B (Tables XI and XII). Again, there are no equivalence classes in the poset.

Regarding the changes in the objects' average heights, those countries in higher positions—as expected—did not show as considerable oscillations as those in lower positions. As in the last subsection, the complete table showing all countries' variations is presented in Annex B (Table XIII), and the most substantial variations are presented in Table 4 below.

Table 4 - Countries with the largest average height variations as a result of removing health

	Complete HDI	Removing health	Variation (%)
Dominica	2.50	1.31	-47.60
Albania	2.85	1.58	-44.56
Costa Rica	16.21	10.00	-38.31
Mongolia	2.27	15.68	590.75
Palau	6.66	51.72	676.58
Kazakhstan	7.40	56.89	668.78

Source: UNDP (2016). Elaborated by the author on PyHasse software.

Dominica, Albania, and Costa Rica showed the most significant negative variations, amounting to around 40%. Costa Rica ranks 34th in the life expectancy ranking, before countries like the United States, and Albania and Dominica rank, respectively, 40th and 41st. On the other hand, these countries' performances on other indicators are not as remarkable—Albania in fact ranks last in terms of expected years of schooling, while Dominica ranks 92nd and Costa Rica, 69th. Concerning mean years of schooling and income, the three countries also do not present very high scores. Thus, their position in the HDI ranking is positively influenced by their relatively good performance in the health indicator, explaining the tendency observed in the exercise.

By contrast, Mongolia, Palau, and Kazakhstan, show the highest positive oscillations in their average heights, around 600%. As expected, neither of the countries presents a high level of life expectancy—in fact, Mongolia and Kazakhstan are ranked last in the health ranking, and Palau ranks 90th. They also do not show very expressive scores in the remaining indicators, but their noticeably bad performance in the health indicator itself explains the great variation. Therefore, through this simulation one can identify that these countries present an inadequacy in the health dimension, which could inspire government policies to work to reverse this tendency.

Finally, the impact of eliminating life expectancy on Brazil's average height was positive—62.54%. This suggests that this indicator is contributing negatively to Brazil's relative position in the HDI ranking. Moreover, the country's local Hasse diagram is illustrated by Figure IV, in Annex A. Argentina, Uruguay, and Chile remain in higher levels than Brazil; however, in comparison to the original Hasse diagram, the country's down set increased from three to seven objects—namely Tunisia, Libya, Thailand, Colombia, Ecuador, Algeria, and China. This result means that Brazil has a better performance than those countries when considering income and education.

4.1.2.3 The impact of income

Once income is removed from the data matrix, the objects are redistributed into 11 levels—that is, two more than in the original Hasse diagram. Here, the countries with the highest number of incomparable objects are Palau, with 83 incomparabilities; Russian Federation, with 80; and Lithuania, with 79. On the other hand, the countries with the lowest number incomparabilities are Australia, New Zealand, and Norway, as in the original Hasse diagram, only this time presenting 9, 17, and 23 incomparabilities, respectively. Again, there are no equivalence classes in this scenario. As in previous sections, the complete tables are shown in Annex B (Tables XIV and XV).

Concerning the impact on countries' average heights, for most European countries on the top of the Hasse diagram the effect was not very significant once they present high levels of education and health that could compensate the suppression of the income from the analysis. Table 5 highlights those with the largest negative and positive height fluctuations as a consequence of withdrawing the income dimension, and the complete table is shown in Annex B (Table XVI).

Table 5 - Countries with the largest average height variations as a result of removing income

	Complete HDI	Removing income	Variation (%)
Kuwait	50.00	2.27	-95.46
United Arab Emirates	60.00	13.95	-76.75
Qatar	91.66	24.44	-73.34
Sri Lanka	3.12	21.42	586.54
Georgia	5.88	44.82	662.24
Cuba	5.00	58.69	1,073.80

Source: UNDP (2016). Elaborated by the author on PyHasse software.

It is possible to see that, for Middle East countries such as Kuwait, the United Arab Emirates, and Qatar, the impact was negative and relevant, since their high positions in the HDI ranking are mostly due to the high levels of GNI per capita they present. Indeed, Qatar and Kuwait rank first and second in the income ranking, while the United Arab Emirates ranks 7th. By contrast, these countries do not present such an ideal performance in the remaining indicators—in fact, Kuwait ranks among the bottom three countries in terms of mean years of schooling, as was mentioned earlier in the study.

On the other hand, countries such as Cuba, Georgia, and Sri Lanka present the opposite trend: their average heights increase significantly with the exclusion of GNI per capita from the analysis. Indeed, Cuba ranks 31st in terms of mean years of schooling, with the same value as, for instance, Japan, and highest value than countries such as Belgium and France, and 33rd in terms of life expectancy, with a higher life expectancy than countries like the United States.

These results arouse discussions on a possible conflict between efficiency—i.e., optimal production and allocation of resources—and equity—that is, how these resources are distributed throughout the society. Indeed, as Sen (1999b) argues, distributional issues are among the central concerns of welfare economics. Kuwait, the UAE, and Qatar illustrate a case when efficiency prevails, whereas the other countries can serve to exemplify the predominance of equity concerns. These considerations can be examined in light of the meta-ranking approach once, as Comim (2015) holds, it is a framework that allows a moral articulation between different sets of alternatives, aiming to identify what is most socially desirable for a particular context, while taking into account a variety of motivations.

Furthermore, the local impact on Brazil was positive, equivalent to a 16.1% increase, which means that income is making Brazil relatively worse off in the HDI ranking. The country's local Hasse diagram is illustrated by Figure VI in Annex A. South American countries such as Argentina, Chile, and Uruguay remain higher than Brazil in the local Hasse diagram, and Kuwait is now part of Brazil's down set.

4.1.2.4 The impact of an additional indicator: the Green HDI

As earlier mentioned, the HDI aims to measure the level of human development of different countries based on their performance in a few key socioeconomic indicators. However, as the Human Development Reports Office acknowledges, the HDI comprises only part of what human development encompasses. Indeed, it can be argued that the index fails to grasp important dimensions of this complex phenomenon, such as gender disparity, sustainability and environmental impact, political rights, among other aspects.

Indian economist Partha Dasgupta (1992), for instance, claimed that the political and civil spheres should also be taken into account in this analysis, advocating a more comprehensive assessment of human development, especially in poorer countries. Considering aspects such as freedom of the press and level of independence of the judiciary, he then observed that "improvements in per capita national income, life expectancy at birth, and infant mortality are positively correlated with the extent of political and civil liberties enjoyed by citizens." (DASGUPTA, 1992, p. 119)

Moreover, the Human Development Reports Office itself has worked on expanding the statistical analysis of human development by developing other composite indices, considering dimensions such as work, employment and vulnerability, poverty, inequality, human security, gender, demography, mobility and communication, trade and financial flows, and environmental sustainability. In 2011, the Human Development Report sought to address the relationship between equity and environmental sustainability, arguing that “promoting human development requires addressing sustainability, and this can and should be done in ways that are equitable and empowering.” (HDR, 2011, p.) Within this context, ‘greening’ human development, as stated in the report, can be seen as the pursuit of minimizing climate risks and environmental scarcities in the long run, ultimately aiming to raise human welfare and reduce inequalities.

Thus, in order to analyze how environmental sustainability may impact countries’ performance in the Human Development Index ranking, a similar exercise than that performed in previous sections will be carried out here considering an additional indicator: the level of carbon dioxide emission (tons per capita) of each country in 2013—data provided by the World Bank. However, once all attributes, as mentioned in the previous chapter, must impact the data matrix in the same direction—that is, the higher the better—the indicator was adjusted to consider the number of people necessary to emit 100 tons of carbon dioxide for each country.

As can be seen in the complete ranking of the indicator, shown in Table XVII in Annex B, the countries which pollute the most are Qatar, Trinidad and Tobago, and Kuwait. On the other hand, Costa Rica, Liechtenstein, and Sri Lanka are the least polluting countries. According to World Bank statistics, high income countries emit an annual average of 11.0 tons of CO_2 per capita, while middle income countries emit an average of 3.9, and low income countries, 0.3.

Once the sustainability indicator is introduced in the analysis, the Hasse diagram—originally organized in nine levels—now presents only four, as shown in Figure VII in Annex A. The first level is composed by four objects; the second, by 20 objects; the third, by 36; and the fourth, by 39. The consideration of an environmental sustainability indicator led to conflicting data profiles, once countries on a higher level of economic development tend to pollute more than those relatively less developed. Consequently, the incomparabilities within the poset increased significantly, as can be seen in Tables XVIII and XIX, in Annex B. Fiji, Grenada, Georgia, and Kuwait present the largest sets of incomparabilities: 98 objects each. By contrast, those countries with the lowest number of incomparabilities are Liechtenstein, with

57; Libya, with 63; and Switzerland, with 70. There are, again, no equivalence classes in the poset.

Furthermore, the maximal value the indicator achieves within the poset is 125 (that of Sri Lanka), while the minimal value is 2.5 (presented by Qatar). Portugal presents the same value as the average of the indicator—22.7. Table 6 below shows all attributes' statistical characteristics.

Table 6 - Attributes' statistical characteristics II

	Life Expectancy	Expected Years of Schooling	Mean Years of Schooling	Gross National Income Per Capita	People Per 100 Tons Of Carbon Dioxide Emitted
Max	84	20.2	13.1	123,124	125
Mean	77.2	14.9	10.3	27,998	22.6
Min	69.4	11.8	6.8	7,164	2.5

Source: UNDP (2016). Elaborated by the author on PyHasse software.

Regarding the impact of adding the environment dimension on the countries' average heights, it is possible to observe that for most countries with a very high level of human development the impact was negative, while the countries on the lower part of the ranking presented a positive and more expressive impact. The complete table is shown in Annex B (Table XX), and the countries with the most significant variations are presented in Table 7 below.

Table 7 - Countries with the largest average height variations as a result of adding the sustainability indicator

	HDI	Green HDI	Variation (%)
Bahrain	47.05	5.26	-88.82
Brunei Darussalam	82.35	25.00	-69.64
Estonia	86.79	28.57	-67.08
Colombia	1.56	25.00	1,502.56
Sri Lanka	3.12	50.00	1,502.56
Peru	1.96	33.33	1,600.51
Saint Vincent and the Grenadines	1.72	33.33	1,837.79

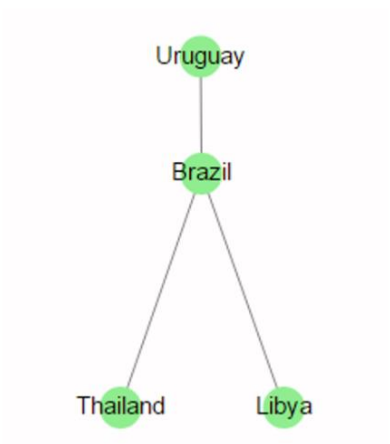
Source: UNDP (2016). Elaborated by the author on PyHasse software.

Bahrain, Brunei Darussalam, and Estonia presented the most considerable negative oscillations—a decrease of 88.82%, 69.64%, and 67.08%, respectively. Indeed, Bahrain and Brunei are the 4th and 5th most polluting countries, while Estonia ranks 13th. Moreover, all three countries occupy a relatively high position in the complete HDI ranking—30th (Estonia), 31st (Brunei), and 45th (Bahrain). Thus, even though these countries are all considered as "very high human development" nations, their performance on this selected environmental indicator is not desirable, indicating that there are still significant inadequacies that should be targeted in order to promote a more sustainable human development, even in countries that are considered to be in the highest category within the development classification.

On the other hand, Saint Vincent and the Grenadines, Peru, Sri Lanka, and Colombia presented the highest positive fluctuations caused by the addition of the new indicator. Indeed, Sri Lanka is the least polluting country in the set, and Peru and Saint Vincent and the Grenadines are among the 10 least polluting countries.

Brazil, considered by the World Bank as an upper middle income country, emitted an average of 2.5 tons of carbon per capita in 2013, below the category's average of 6.6 tons. Indeed, Brazil is among the 15 least polluting countries amidst those considered here. Concerning the impact on the country's average height, the addition of the environment indicator contributes significantly to raising Brazil's position in the ranking, in 485%. Brazil's local Hasse diagram is illustrated by Figure 6 below.

Figure 6 - Brazil's local Hasse diagram based on the complete HDI



Source: UNDP (2016). Elaborated by the author on PyHasse software.

One can note that the country's up set shrunk significantly—while there were 34 countries better than Brazil on the original Hasse diagram, the category was now reduced to Uruguay alone. That is because most high income countries, as shown by World Bank, tend to generate a higher carbon footprint, contributing negatively to their relative position in the ranking. Brazil's down set was also reduced, once Colombia is no longer part of it. Indeed, Colombia presents a better performance in that indicator than Brazil, being the 8th least polluting country.

Therefore, the partial order analyses performed in this chapter considering the individual impact of the HDI indicators and that of an additional one on countries, as well as how that translates into the Hasse diagrammatic representation, were but a germinal attempt to demonstrate that demands for full comparability may not be appropriate within the context of human development. By allowing a more detailed portrayal of countries' relative performance in selected human development indicators, partial order can help identify aspects which could not be verified considering only the HDI traditional ranking—such as 'incomparabilities' between them. For this reason, it could potentially help to better understand countries' vulnerabilities and critical deficiencies for achieving a sustainable human development. However, other analyses such as the examination of investment patterns in education or healthcare systems and so on could help enrich the discussion even more, and hopefully this methodology will be employed in other studies in the field.

5 CONCLUDING REMARKS

Human development, as Amartya Sen (2009) argues, should be viewed in terms of enhancing individuals' substantive freedoms and capabilities to live the life they have reason to value. He thus draws attention to the importance of reducing and ultimately extinguishing the most substantial obstacles to this process—as he puts it, "poverty as well as tyranny, poor economic opportunities as well as systematic social deprivation, neglect of public facilities as well as intolerance or overactivity of repressive states." (SEN, 2009, p.3)

Within this context, the Human Development Index is an attempt to empirically assess how this complex phenomenon unfolds in different countries. In spite of its structural limitations, it allows cross-country comparisons in terms of a few critical socioeconomic indicators, considerably contributing to debates in the field. However, complete rankings—as is the traditional HDI ranking—may be an oversimplified representation of countries' relative performance in the examined indicators, potentially concealing important aspects. It can be argued, thus, that there should be a more flexible structure for comparative analysis of different states and alternatives, and partial rankings can be a useful tool in that sense, as this study sought to illustrate through different empirical exercises using the PyHasse software. Indeed, several aspects regarding countries' relative positions, 'comparabilities', and 'incomparabilities' between them—which could not have been easily identified by means of the usual methodology—could however be more clearly detected through partial order analysis.

Furthermore, as a complement to this analysis, the meta-ranking approach could be a suitable analytical structure for social evaluations, as debated within the framework of the social choice theory. Through the incorporation of normative considerations, it could assist in the assessment of where to focus the attention and of what is more socially desirable for a particular society, based on a variety of motivations—even if the resulting meta-ranking is not complete. Indeed, as Comim holds, "The role of a meta-ranking is to allow a moral articulation between those different moral systems, for a given context, searching for the 'most moral' system in question." (COMIM, 2015, p.7)

Therefore, by analyzing the Human Development Index as an example, the study showed that, due to the intricate and multidimensional nature of human development, complete rankings may present limitations and that partial rankings—and particularly the partial order methodology—are a more flexible and comprehensive method, which could be more appropriate for examining human development statistics. Indeed, the striking contrasts identified in this study could provide useful insights on imbalances within human development processes and potentially stimulate debate about public policy priorities, based on the meta-ranking approach.

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APPENDIX A – Additional partial order analyses

The PyHasse software is only one instrument of application of the partial order methodology, and therefore does not provide a way to put in practice all possible exercises. A few additional analyses presented in Brüggemann and Patil's book are described here as an example of other examinations partial order renders, hopefully inspiring other studies to employ the methodology.

1 ANALYSES BASED ON ATTRIBUTES

In partial order analysis, it may be of interest to examine the role that attributes play in the poset, in order to identify the most relevant ones, in which the study should focus its attention. The concepts of sensitivity and ambiguity provide a way to perform this assessment, and shall be presented and explained in the next subsections.

1.1 Sensitivity

The attribute-related sensitivity analysis seeks to investigate the impact of a particular attribute on the position of objects in the Hasse diagram. This is done by determining a distance measure, which considers the ordinal change between a pair of objects before and after the attribute has been removed, or $(x, y) \in (\text{X, IB})$ and $(x, y) \in (\text{X, IB}(i))$, with $\text{IB}(i) = \text{IB} - \{q_i\}$.

This analysis can be conducted using down sets and up sets, and the starting point is the fact that any comparability in the initial poset, or (X, IB), must be reproduced in the modified one, (X, IB(i)). Consequently, the objects in the down set $O(x, \text{IB})$ are also in $O(x, \text{IB}(i))$, and the distance between the two posets can be calculated by $|O(x, \text{IB}(i)) \, \Delta \, O(x, \text{IB})|$, or $|O(x, \text{IB}(i))| - |O(x, \text{IB})| \geq 0$, in a simplified form. If one wishes to use up sets instead of down sets, the analysis is to be carried out in the same manner. This distance measure is denoted by $W(q_i)$, and can be interpreted as the sensitivity measure of the partial order to the attribute q_i excluded from the initial data matrix.

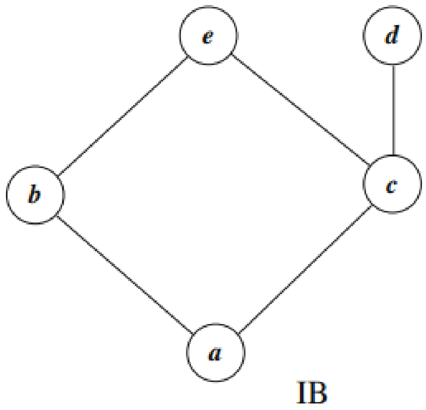
An illustrative example may contribute to a better understanding of the method. Table 1 shows a hypothetical data matrix, considering five different objects, *a, b, c, d,* and *e,* and three attributes, q_1, q_2, and q_3, and Figure 1 its respective Hasse diagram:

Table 1 – Illustrative data matrix II

	q_1	q_2	q_3
a	1	1	1
b	1	4	1
c	2	3	2
d	3	3	5
e	4	5	3

Source: Brüggemann and Patil (2011)

Figure 1 – Hasse diagram based on data of Table 1



Source: Brüggemann and Patil (2011)

W(X, IB, IB(1)) denotes the distance, or impact, related to attribute q_1; *W*(X, IB, IB(2)) to attribute q_2; and *W*(X, IB, IB(3)) to attribute q_3. Table 2 presents the measurement of *W*(X, IB, IB(2)).

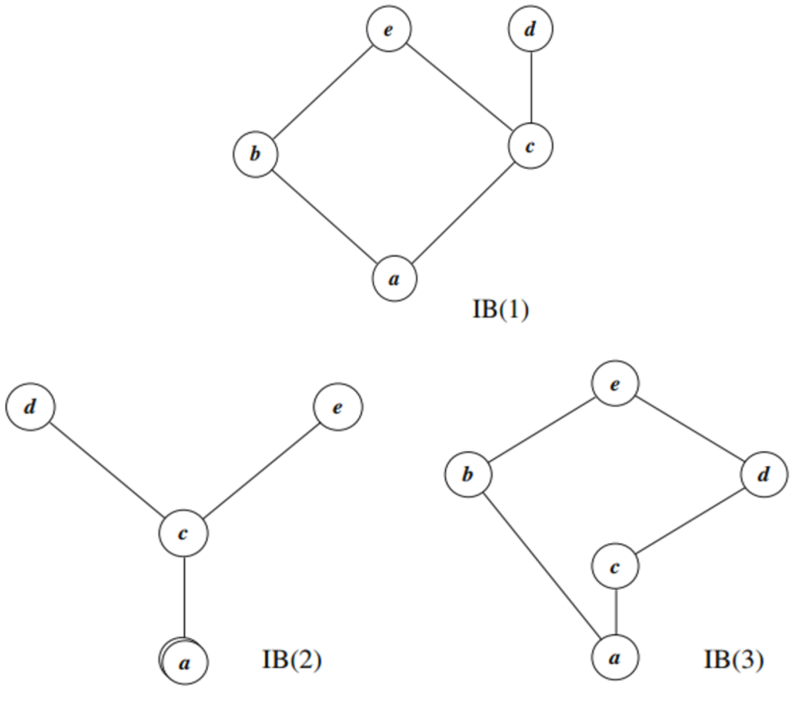
Table 2 – Measurement of the impact of attribute q_2

x	$A := O(x, \mathrm{IB})$	$B := O(x, \mathrm{IB}(2))$	$A \cup B$	$A \cap B$	$A \Delta B$
a	*a*	*a, b*	*a, b*	*a*	*b*
b	*a, b*	*a, b*	*a, b*	*a, b*	Ø
c	*a, c*	*a, b, c*	*a, b, c*	*a, c*	*b*
d	*a, c, d*	*a, b, c, d*	*a, b, c, d*	*a, c, d*	*b*
e	*a, b, c, e*	*a, b, c, e*	*a, b, c, e*	*a, b, c, e*	Ø
Σ					3

Source: Brüggemann and Patil (2011)

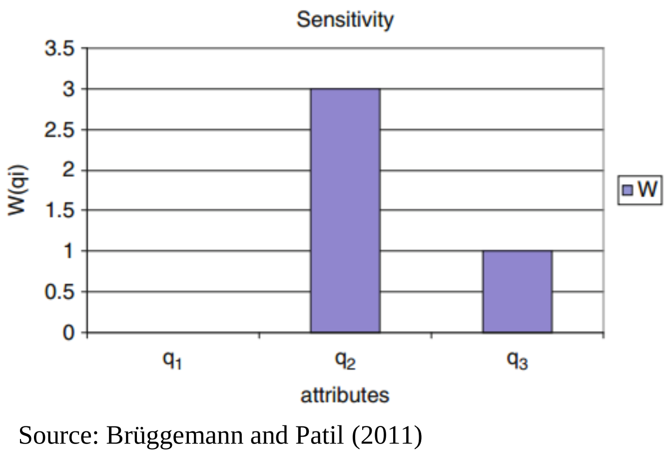
Columns A:=... and B:=... show the calculation of the contents of the down sets, produced by the objects in the first column; the remaining three columns demonstrate the steps to obtain the symmetric difference for every generating element. Finally, a sum of all generating elements results in 3, which expresses the overall impact of excluding attribute q_2 from the poset. Performing the same calculation for q_1 and q_3, one finds that W(X, IB, IB(1)) = 0 and W(X, IB, IB(3)) = 1. Therefore, it is possible to see that the suppression of q_2 has the most significant impact on the poset. Sensitivity is, thus, an important tool for partial order analysis, once it helps to find influential attributes for the object set. Figure 2 illustrates the Hasse diagrams for IB(1), IB(2), and IB(3), and Figure 3 shows the distribution of the sensitivity among the attributes:

Figure 2 – Hasse diagrams based on IB(1), IB(2), and IB(3)



Source: Brüggemann and Patil (2011)

Figure 3 – Distribution of attribute-related sensitivity of q_1, q_2, and q_3



Source: Brüggemann and Patil (2011)

IB(1), the first diagram in Figure 2, looks exactly the same as the original Hasse diagram because, as the sensitivity measure revealed, the exclusion of attribute q_1 has no impact on the poset (W(X, IB, IB(1)) = 0). On the other hand, IB(2) is the diagram which diverges the most from the initial one, since the impact of removing q_2 is the most significant one of the three.

Furthermore, it is possible to order a cumulative set of indicators—such as $\{q_1\} \subset \{q_1, q_2\} \subset \{q_1, q_2, \ldots, q_m\}$—according to the sensitivity measure of the attributes, $W(q_i)$, in a decreasing order. This series of attributes is called a canonical sequence, and the serial count of the cumulative subsets is called “natt.”

1.2 Ambiguity

Incomparabilities in a poset may generate ranking ambiguity. Being $|U(X, IB)|$ the number of incomparable pairs and n the total number of objects in the poset, the ambiguity measure can be calculated by

$$\mathrm{Am}(X, IB) = \frac{|U(X,IB)|}{n*(n-1)/2}$$

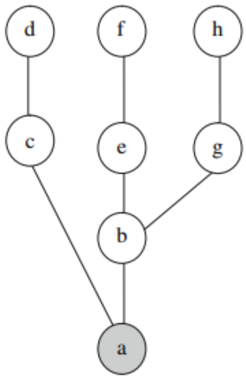
From this basis, it is possible to draw a cumulative ambiguity graph as a function of natt (described in the last subsection), denoted by Am(natt). If Am(natt) = 0, the indicators do not generate conflicting rankings, discarding the need for weights; if Am(natt) = 1, there are no comparabilities in the poset, and it is impossible to obtain a ranking without weights; finally, if 0 < Am(natt) < 1, there are comparabilities in the poset, making it possible to arrange a relative ranking of some objects. Moreover, if one is interested in analyzing an individual object rather than the entire object set, the local ambiguity graph may be of greater help, for it illustrates how the incomparabilities related to a single object vary with natt.

2 DEALING WITH COMPLEX HASSE DIAGRAMS

2.1 Separation of object subsets & Separability

When the number of attributes is too large and the visualization of the relations becomes difficult, it may be useful to try to separate subsets in the Hasse diagram. Distinguishing articulation points is a tool to detect separated object subsets, because their exclusion generally results in two components, called "approximate components." Figure 4 illustrates the concept. Object *a* is an example of an articulation point, while {*c*, *d*} and {*b, e, f, g, h*} represent approximate components.

Figure 4 – Hasse diagram depicting the concept of articulation point and approximate components



Source: Brüggemann and Patil (2011)

However, if two potentially separable subsets are found and one wants to evaluate their degree of separation, a separability analysis can be performed. Being the possible number of relations between the disjoint subsets X_1 and X_2 $N(X_1, X_2) = |X_1| * |X_2|$, and $|U(X_1, X_2, IB')|$ the number of incomparable relations between them, the separability is calculated by

$$\text{Sep}(X_1, X_2, IB') := \frac{|U(X1,X2,IB')|}{N(X1,X2)}$$

If $\text{Sep}(X_1, X_2, IB') = 1$, the subsets are separated; otherwise, they are not regarded as so.

2.2 Antagonism

Additionally, within this context, it may be of interest to find the properties of the data matrix responsible for such separation, that is, the set of antagonistic attributes. $AIB(X_1, X_2)$, thus, represents the antagonistic information base, for it contains those indicators which are causing the disarticulation between X_1 and X_2. The investigation of AIB is a computational task, and it is thereby necessary to resort to software such as PyHasse and WHASSE.

2.3 Dominance

Nevertheless, if a division method of the object set X is already provided by external knowledge, it is possible to calculate the degree of dominance between the subsets as well as the separability. Instead of examining order relations with respect to objects, the scale is expanded here, and relations among the subsets are the object of the analysis. Thus, X_i dominates X_j to the degree Dom(X_i, X_j). This measure is estimated by

$$\mathrm{Dom}(X_1, X_2) := |\{(x, y) \in X_1 * X_2, x \geq y\}| / (|X_1| * |X_2|)$$
$$\mathrm{Dom}(X_2, X_1) := |\{(x, y) \in X_1 * X_2, x \leq y\}| / (|X_1| * |X_2|)$$

Furthermore, one can draw a relation between the degree of dominance and that of the separability. Indeed, Dom(X_1, X_2) + Dom(X_2, X_1) + Sep(X_1, X_2) = 1.

2.4 The min-, median-, max (mmm) order

A possible approach to the issue of reducing the number of incomparabilities of a poset by making adjustments in the attribute set is the min-, median-, max- (mmm) order. It consists of replacing the original attributes of a data matrix by the minimal and maximal values, or minimal, median, and maximal values, for each object—that is, transform $\{q_1, ..., q_m\}$ into {min, max}, m^2 order, or {min, median, max}, m^3 order, both order preserving. In order to perform such a procedure, the attributes must have a common dimension or scale, and therefore need to be normalized.

2.6 p-Algorithm

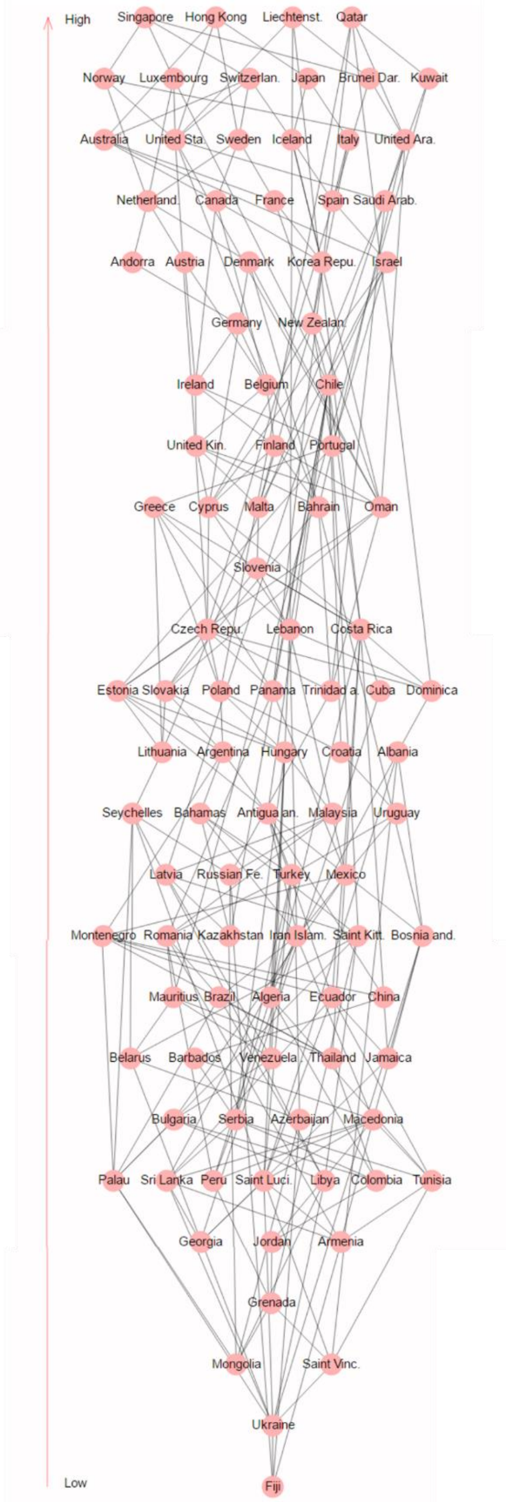
Furthermore, when Hasse diagrams are too complex, it can be useful to identify priority objects within the poset in which to focus the analysis, and the p-Algorithm is a method to detect such elements. Considering one of the attributes—for instance, q_i—one must determine a limiting value q_{i0} such that a percentage of the objects are excluded for having lower values, forming an equivalence class. This category of excluded objects is called “swamp,” denoted by SW_i.

If two attributes are correlated, selecting a limiting value for one of them would have the same effect as selecting such a value for the other, that is, it would exclude the same objects.

Therefore, the number of elements that may be put into the swamp is estimated by $|SW| = (P/100)^k * |X|$, being k a function of the correlation properties of the attributes ($k = 1$ if they are perfectly correlated and $k > 1$ in other cases).

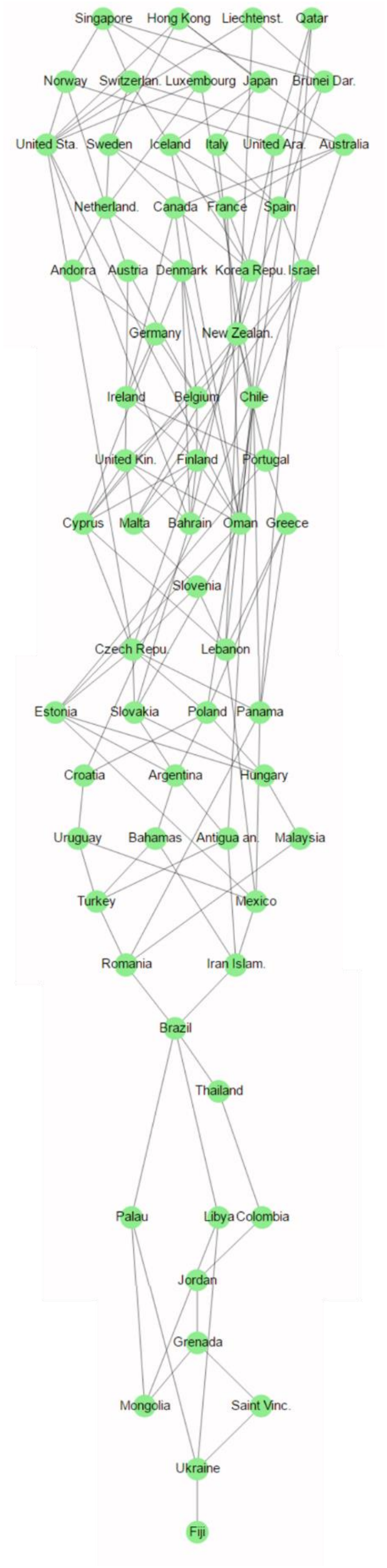
ANNEX A – Additional figures referenced on chapter 3

Figure I – Hasse diagram removing education



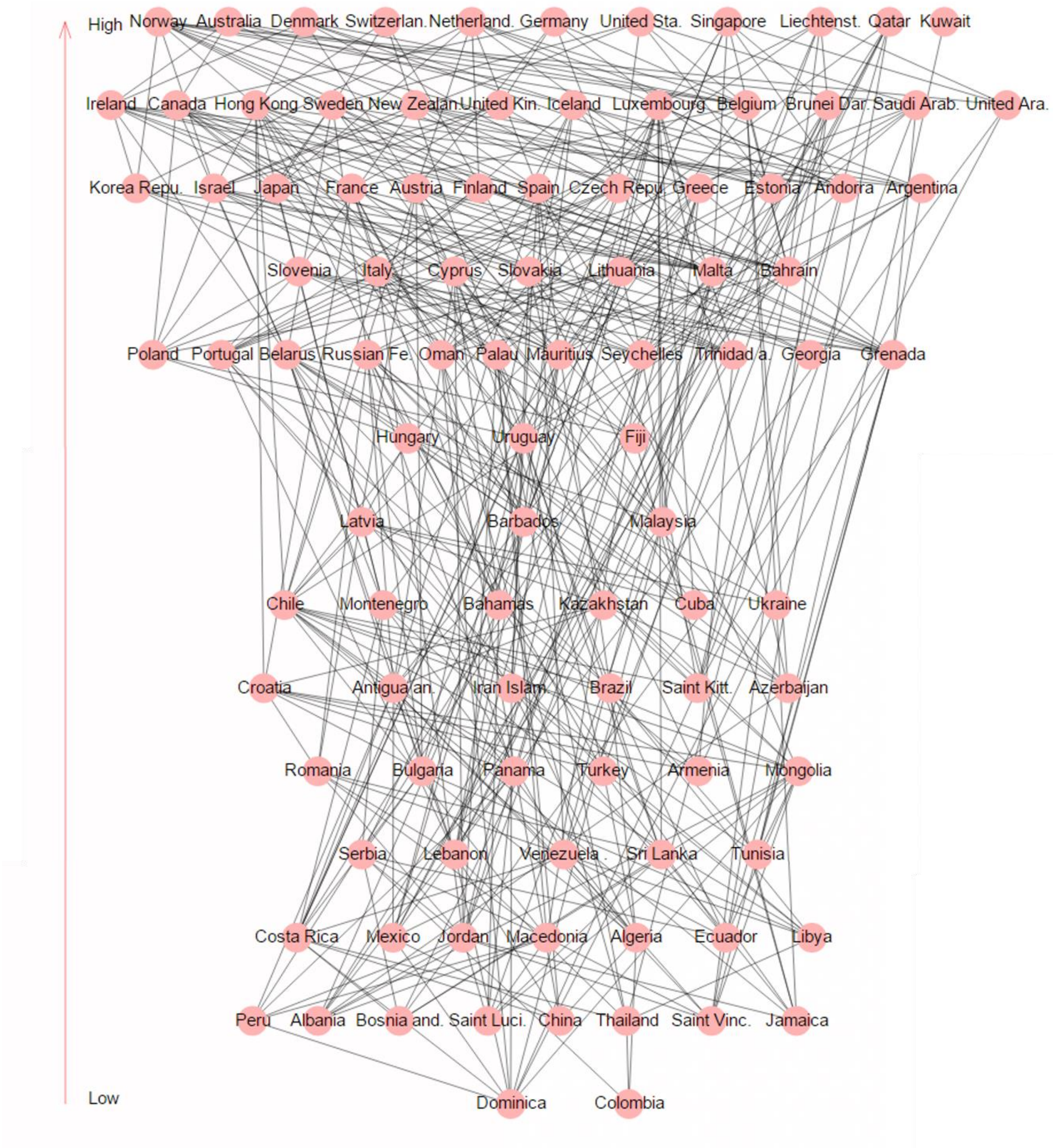
Source: UNDP (2016). Elaborated by the author on PyHasse software.

Figure II – Brazil's local Hasse diagram removing education



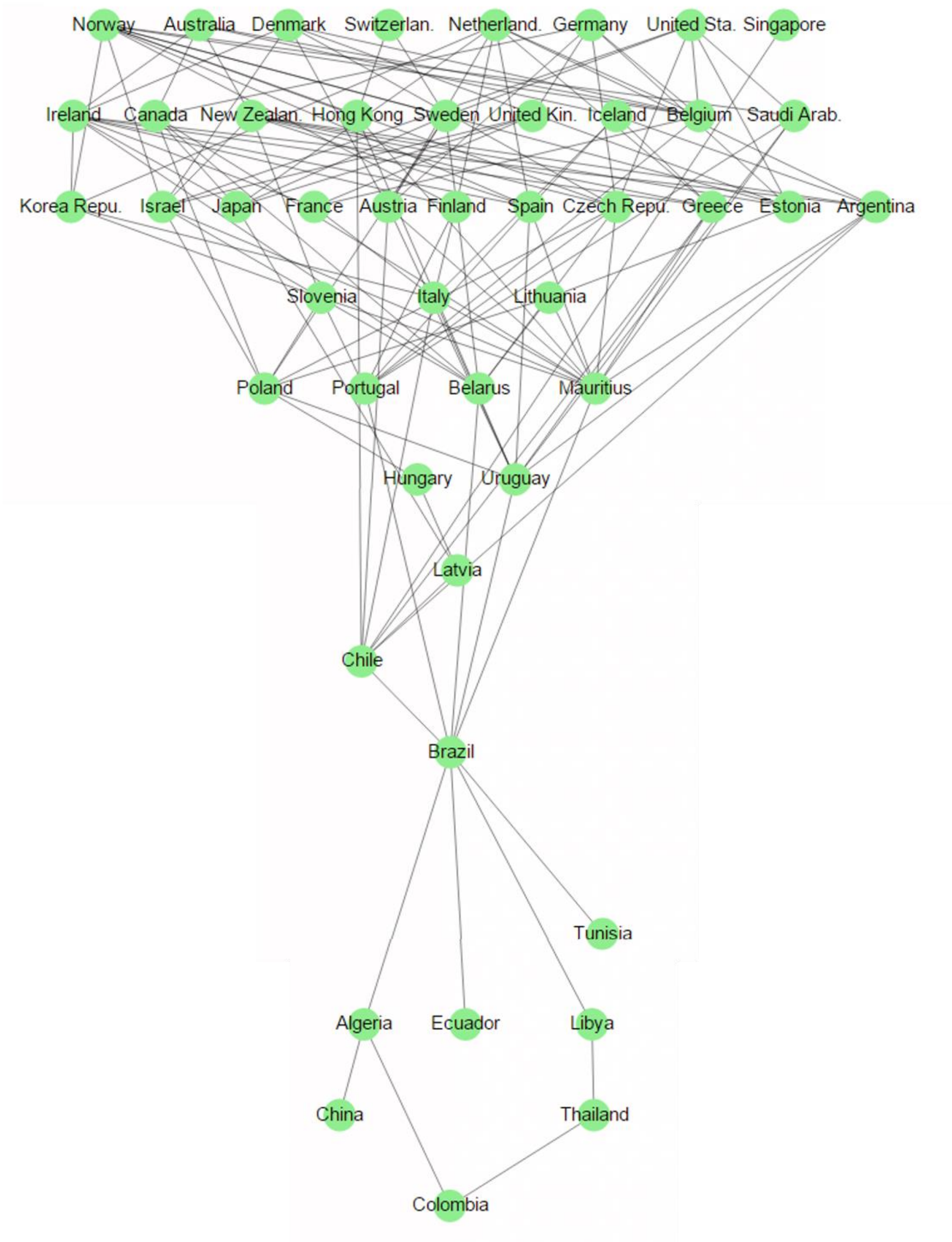
Source: UNDP (2016). Elaborated by the author on PyHasse software.

Figure III – Hasse diagram removing health



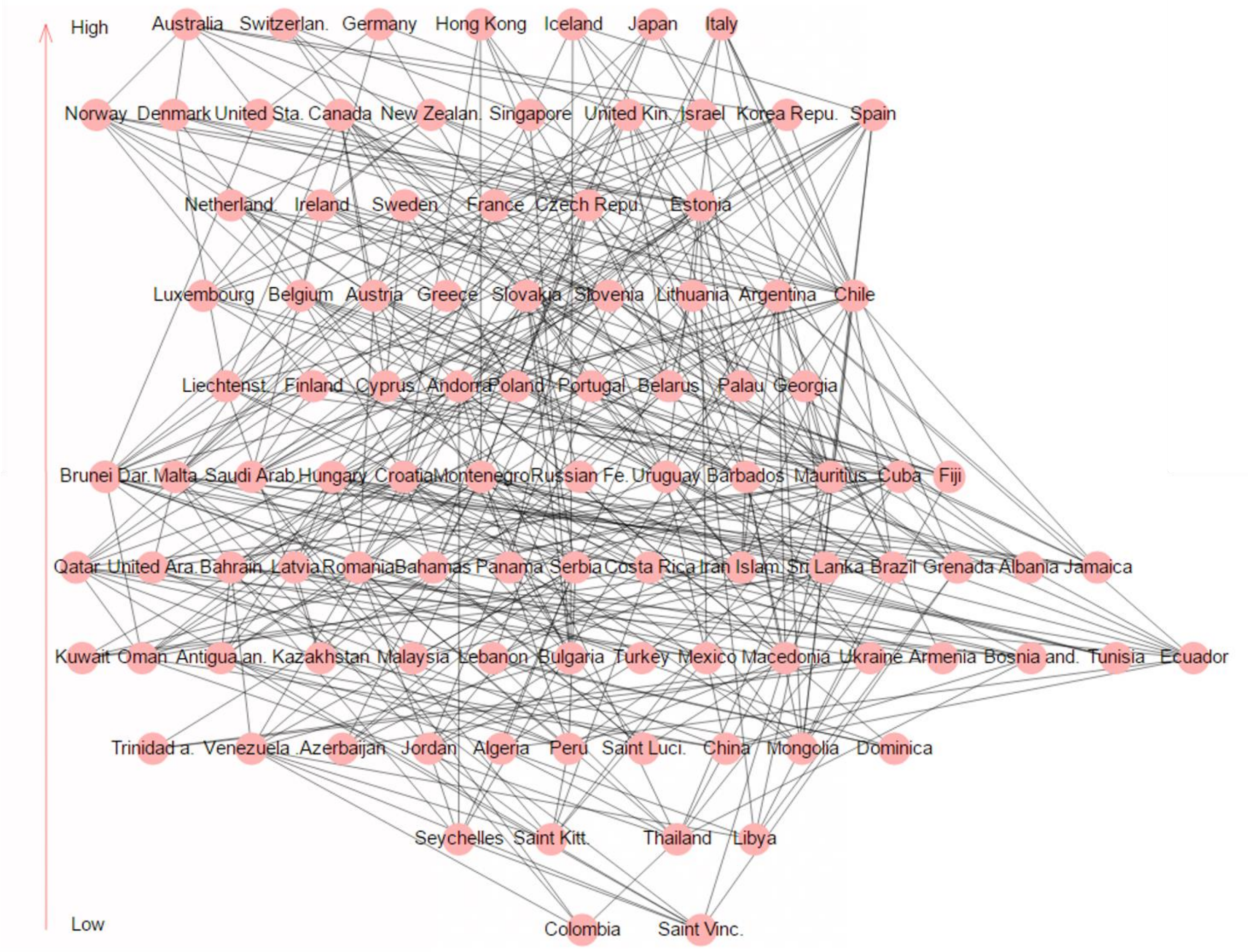
Source: UNDP (2016). Elaborated by the author on PyHasse software.

Figure IV – Brazil's local Hasse diagram removing health



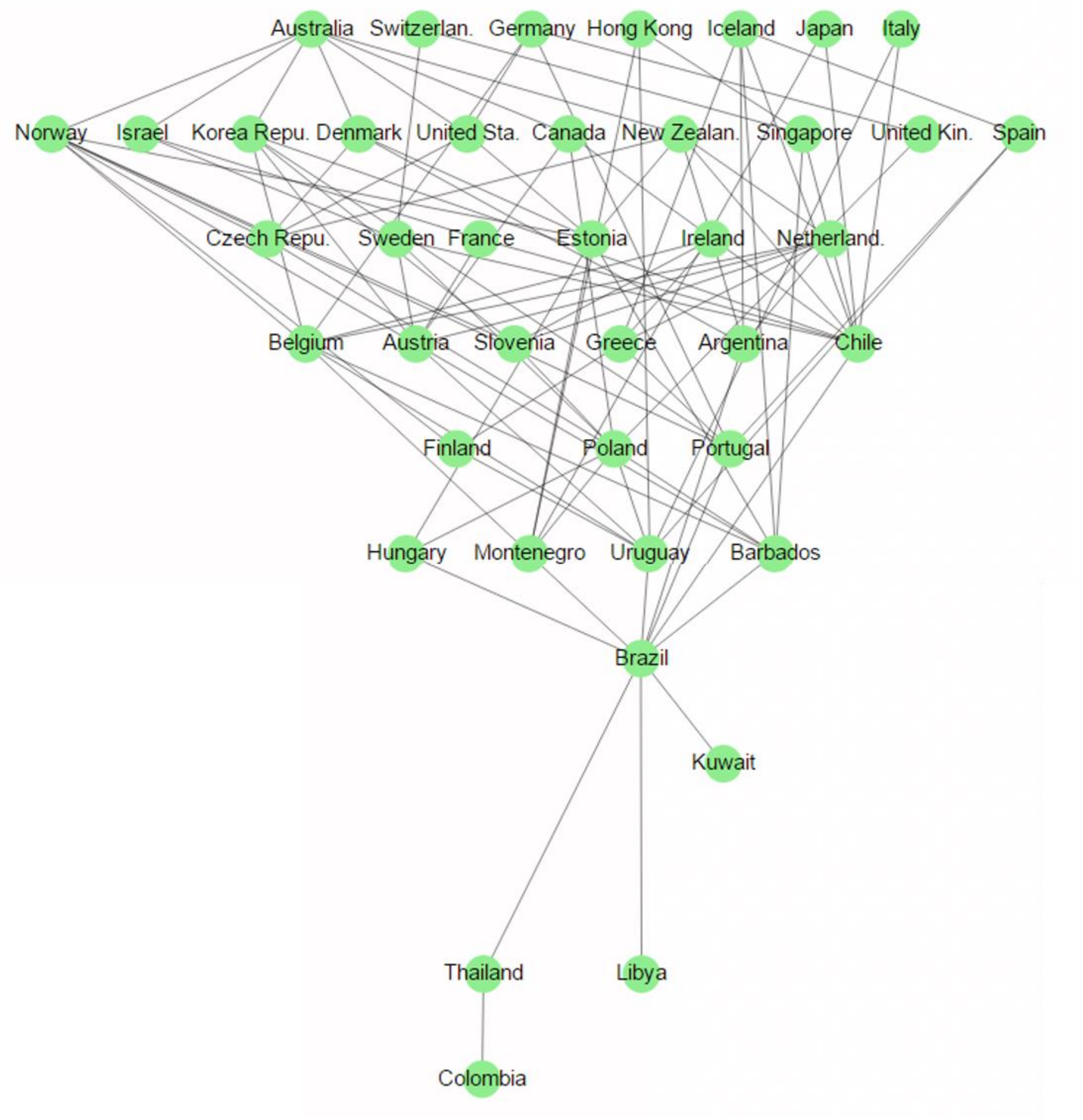
Source: UNDP (2016). Elaborated by the author on PyHasse software.

Figure V – Hasse diagram removing income



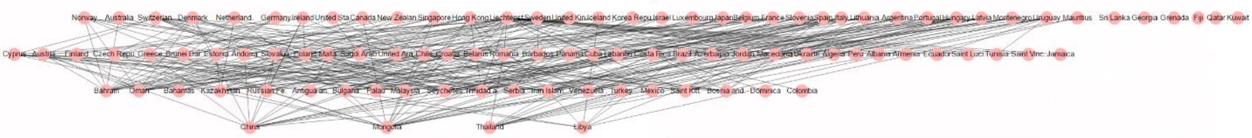
Source: UNDP (2016). Elaborated by the author on PyHasse software.

Figure VI – Brazil's local Hasse diagram removing income



Source: UNDP (2016). Elaborated by the author on PyHasse software.

Figure VII – Green HDI Hasse diagram



Source: UNDP (2016). Elaborated by the author on PyHasse software.

ANNEX B – Additional tables referenced on chapter 3

Table I - Complete HDI ranking

Country	Life Exp.	Exp. Years Sch.	Mean Years Sch.	GNI PC
Norway	81.6	17.5	12.6	64,992
Australia	82.4	20.2	13	42,261
Switzerland	83	15.8	12.8	56,431
Denmark	80.2	18.7	12.7	44,025
Netherlands	81.6	17.9	11.9	45,435
Germany	80.9	16.5	13.1	43,919
Ireland	80.9	18.6	12.2	39,568
United States	79.1	16.5	12.9	52,947
Canada	82	15.9	13	42,155
New Zealand	81.8	19.2	12.5	32,689
Singapore	83	15.4	10.6	76,628
Hong Kong	84	15.6	11.2	53,959
Liechtenstein	80	15	11.8	79,851
Sweden	82.2	15.8	12.1	45,636
United Kingdom	80.7	16.2	13.1	39,267
Iceland	82.6	19	10.6	35,182
Korea Republic of	81.9	16.9	11.9	33,890
Israel	82.4	16	12.5	30,676
Luxembourg	81.7	13.9	11.7	58,711
Japan	83.5	15.3	11.5	36,927
Belgium	80.8	16.3	11.3	41,187
France	82.2	16	11.1	38,056
Austria	81.4	15.7	10.8	43,869
Finland	80.8	17.1	10.3	38,695
Slovenia	80.4	16.8	11.9	27,852
Spain	82.6	17.3	9.6	32,045
Italy	83.1	16	10.1	33,030
Czech Republic	78.6	16.4	12.3	26,660
Greece	80.9	17.6	10.3	24,524
Estonia	76.8	16.5	12.5	25,214
Brunei Darussalam	78.8	14.5	8.8	72,570
Cyprus	80.2	14	11.6	28,633
Qatar	78.2	13.8	9.1	123,124
Andorra	81.3	13.5	9.6	43,978
Slovakia	76.3	15.1	12.2	25,845
Poland	77.4	15.5	11.8	23,177
Lithuania	73.3	16.4	12.4	24,500
Malta	80.6	14.4	10.3	27,930
Saudi Arabia	74.3	16.3	8.7	52,821
Argentina	76.3	17.9	9.8	22,050
United Arab Emirates	77	13.3	9.5	60,868

Chile	81.7	15.2	9.8	21,290
Portugal	80.9	16.3	8.2	25,757
Hungary	75.2	15.4	11.6	22,916
Bahrain	76.6	14.4	9.4	38,599
Latvia	74.2	15.2	11.5	22,281
Croatia	77.3	14.8	11	19,409
Kuwait	74.4	14.7	7.2	83,961
Montenegro	76.2	15.2	11.2	14,558
Belarus	71.3	15.7	12	16,676
Russian Federation	70.1	14.7	12	22,352
Oman	76.8	13.6	8	34,858
Romania	74.7	14.2	10.8	18,108
Uruguay	77.2	15.5	8.5	19,283
Bahamas	75.4	12.6	10.9	21,336
Kazakhstan	69.4	15	11.4	20,867
Barbados	75.6	15.4	10.5	12,488
Antigua and Barbuda	76.1	14	9.2	20,070
Bulgaria	74.2	14.4	10.6	15,596
Palau	72.7	13.7	12.3	13,496
Panama	77.6	13.3	9.3	18,192
Malaysia	74.7	12.7	10	22,762
Mauritius	74.4	15.6	8.5	17,470
Seychelles	73.1	13.4	9.4	23,300
Trinidad and Tobago	70.4	12.3	10.9	26,090
Serbia	74.9	14.4	10.5	12,190
Cuba	79.4	13.8	11.5	7,301
Lebanon	79.3	13.8	7.9	16,509
Costa Rica	79.4	13.9	8.4	13,413
Iran Islamic Republic of	75.4	15.1	8.2	15,440
Venezuela Bolivarian Republic of	74.2	14.2	8.9	16,159
Turkey	75.3	14.5	7.6	18,677
Sri Lanka	74.9	13.7	10.8	9,779
Mexico	76.8	13.1	8.5	16,056
Brazil	74.5	15.2	7.7	15,175
Georgia	74.9	13.8	12.1	7,164
Saint Kitts and Nevis	73.8	12.9	8.4	20,805
Azerbaijan	70.8	11.9	11.2	16,428
Grenada	73.4	15.8	8.6	10,939
Jordan	74	13.5	9.9	11,365
Macedonia	75.4	13.4	9.3	11,780
Ukraine	71	15.1	11.3	8,178
Algeria	74.8	14	7.6	13,054
Peru	74.6	13.1	9	11,015
Albania	77.8	11.8	9.3	9,943
Armenia	74.7	12.3	10.9	8,124
Bosnia and Herzegovina	76.5	13.6	8.3	9,638

Ecuador	75.9	14.2	7.6	10,605
Saint Lucia	75.1	12.6	9.3	9,765
China	75.8	13.1	7.5	12,547
Fiji	70	15.7	9.9	7,493
Mongolia	69.4	14.6	9.3	10,729
Thailand	74.4	13.5	7.3	13,323
Dominica	77.8	12.7	7.9	9,994
Libya	71.6	14	7.3	14,911
Tunisia	74.8	14.6	6.8	10,404
Colombia	74	13.5	7.3	12,040
Saint Vincent and the Grenadines	72.9	13.4	8.6	9,937
Jamaica	75.7	12.4	9.7	7,415

Source: UNDP (2016). Elaborated by the author.

Table II – Expected years of schooling ranking

Country	Expected Years of Schooling
Australia	20.2
New Zealand	19.2
Iceland	19
Denmark	18.7
Ireland	18.6
Netherlands	17.9
Argentina	17.9
Greece	17.6
Norway	17.5
Spain	17.3
Finland	17.1
Korea Republic of	16.9
Slovenia	16.8
Germany	16.5
United States	16.5
Estonia	16.5
Czech Republic	16.4
Lithuania	16.4
Belgium	16.3
Saudi Arabia	16.3
Portugal	16.3
United Kingdom	16.2
Israel	16
France	16
Italy	16
Canada	15.9
Switzerland	15.8
Sweden	15.8

Grenada	15.8
Austria	15.7
Belarus	15.7
Fiji	15.7
Hong Kong	15.6
Mauritius	15.6
Poland	15.5
Uruguay	15.5
Singapore	15.4
Hungary	15.4
Barbados	15.4
Japan	15.3
Chile	15.2
Latvia	15.2
Montenegro	15.2
Brazil	15.2
Slovakia	15.1
Iran Islamic Republic of	15.1
Ukraine	15.1
Liechtenstein	15
Kazakhstan	15
Croatia	14.8
Kuwait	14.7
Russian Federation	14.7
Mongolia	14.6
Tunisia	14.6
Brunei Darussalam	14.5
Turkey	14.5
Malta	14.4
Bahrain	14.4
Bulgaria	14.4
Serbia	14.4
Romania	14.2
Venezuela Bolivarian Republic of	14.2
Ecuador	14.2
Cyprus	14
Antigua and Barbuda	14
Algeria	14
Libya	14
Luxembourg	13.9
Costa Rica	13.9
Qatar	13.8
Cuba	13.8
Lebanon	13.8
Georgia	13.8
Palau	13.7

Sri Lanka	13.7
Oman	13.6
Bosnia and Herzegovina	13.6
Andorra	13.5
Jordan	13.5
Thailand	13.5
Colombia	13.5
Seychelles	13.4
Macedonia	13.4
Saint Vincent and the Grenadines	13.4
United Arab Emirates	13.3
Panama	13.3
Mexico	13.1
Peru	13.1
China	13.1
Saint Kitts and Nevis	12.9
Malaysia	12.7
Dominica	12.7
Bahamas	12.6
Saint Lucia	12.6
Jamaica	12.4
Trinidad and Tobago	12.3
Armenia	12.3
Azerbaijan	11.9
Albania	11.8

Source: UNDP (2016). Elaborated by the author.

Table III - Mean years of schooling ranking

Country	Mean Years of Schooling
Germany	13.1
United Kingdom	13.1
Australia	13
Canada	13
United States	12.9
Switzerland	12.8
Denmark	12.7
Norway	12.6
New Zealand	12.5
Israel	12.5
Estonia	12.5
Lithuania	12.4
Czech Republic	12.3
Palau	12.3
Ireland	12.2

Slovakia	12.2
Sweden	12.1
Georgia	12.1
Belarus	12
Russian Federation	12
Netherlands	11.9
Korea Republic of	11.9
Slovenia	11.9
Liechtenstein	11.8
Poland	11.8
Luxembourg	11.7
Cyprus	11.6
Hungary	11.6
Japan	11.5
Latvia	11.5
Cuba	11.5
Kazakhstan	11.4
Belgium	11.3
Ukraine	11.3
Hong Kong	11.2
Montenegro	11.2
Azerbaijan	11.2
France	11.1
Croatia	11
Bahamas	10.9
Trinidad and Tobago	10.9
Armenia	10.9
Austria	10.8
Romania	10.8
Sri Lanka	10.8
Singapore	10.6
Iceland	10.6
Bulgaria	10.6
Barbados	10.5
Serbia	10.5
Finland	10.3
Greece	10.3
Malta	10.3
Italy	10.1
Malaysia	10
Jordan	9.9
Fiji	9.9
Argentina	9.8
Chile	9.8
Jamaica	9.7
Spain	9.6

Andorra	9.6
United Arab Emirates	9.5
Bahrain	9.4
Seychelles	9.4
Panama	9.3
Macedonia	9.3
Albania	9.3
Saint Lucia	9.3
Mongolia	9.3
Antigua and Barbuda	9.2
Qatar	9.1
Peru	9
Venezuela Bolivarian Republic of	8.9
Brunei Darussalam	8.8
Saudi Arabia	8.7
Grenada	8.6
Saint Vincent and the Grenadines	8.6
Uruguay	8.5
Mauritius	8.5
Mexico	8.5
Costa Rica	8.4
Saint Kitts and Nevis	8.4
Bosnia and Herzegovina	8.3
Portugal	8.2
Iran Islamic Republic of	8.2
Oman	8
Lebanon	7.9
Dominica	7.9
Brazil	7.7
Turkey	7.6
Algeria	7.6
Ecuador	7.6
China	7.5
Thailand	7.3
Libya	7.3
Colombia	7.3
Kuwait	7.2
Tunisia	6.8

Source: UNDP (2016). Elaborated by the author.

Table IV - Life expectancy ranking

Country	Life Expectancy
Hong Kong	84
Japan	83.5
Italy	83.1
Switzerland	83
Singapore	83
Iceland	82.6
Spain	82.6
Australia	82.4
Israel	82.4
Sweden	82.2
France	82.2
Canada	82
Korea Republic of	81.9
New Zealand	81.8
Luxembourg	81.7
Chile	81.7
Norway	81.6
Netherlands	81.6
Austria	81.4
Andorra	81.3
Germany	80.9
Ireland	80.9
Greece	80.9
Portugal	80.9
Belgium	80.8
Finland	80.8
United Kingdom	80.7
Malta	80.6
Slovenia	80.4
Denmark	80.2
Cyprus	80.2
Liechtenstein	80
Cuba	79.4
Costa Rica	79.4
Lebanon	79.3
United States	79.1
Brunei Darussalam	78.8
Czech Republic	78.6
Qatar	78.2
Albania	77.8
Dominica	77.8
Panama	77.6
Poland	77.4
Croatia	77.3

Uruguay	77.2
United Arab Emirates	77
Estonia	76.8
Oman	76.8
Mexico	76.8
Bahrain	76.6
Bosnia and Herzegovina	76.5
Slovakia	76.3
Argentina	76.3
Montenegro	76.2
Antigua and Barbuda	76.1
Ecuador	75.9
China	75.8
Jamaica	75.7
Barbados	75.6
Bahamas	75.4
Iran Islamic Republic of	75.4
Macedonia	75.4
Turkey	75.3
Hungary	75.2
Saint Lucia	75.1
Serbia	74.9
Sri Lanka	74.9
Georgia	74.9
Algeria	74.8
Tunisia	74.8
Romania	74.7
Malaysia	74.7
Armenia	74.7
Peru	74.6
Brazil	74.5
Kuwait	74.4
Mauritius	74.4
Thailand	74.4
Saudi Arabia	74.3
Latvia	74.2
Bulgaria	74.2
Venezuela Bolivarian Republic of	74.2
Jordan	74
Colombia	74
Saint Kitts and Nevis	73.8
Grenada	73.4
Lithuania	73.3
Seychelles	73.1
Saint Vincent and the Grenadines	72.9

Palau	72.7
Libya	71.6
Belarus	71.3
Ukraine	71
Azerbaijan	70.8
Trinidad and Tobago	70.4
Russian Federation	70.1
Fiji	70
Kazakhstan	69.4
Mongolia	69.4

Source: UNDP (2016). Elaborated by the author.

Table V - Gross national income per capita ranking

Country	Gross National Income Per Capita
Qatar	123,124
Kuwait	83,961
Liechtenstein	79,851
Singapore	76,628
Brunei Darussalam	72,570
Norway	64,992
United Arab Emirates	60,868
Luxembourg	58,711
Switzerland	56,431
Hong Kong	53,959
United States	52,947
Saudi Arabia	52,821
Sweden	45,636
Netherlands	45,435
Denmark	44,025
Andorra	43,978
Germany	43,919
Austria	43,869
Australia	42,261
Canada	42,155
Belgium	41,187
Ireland	39,568
United Kingdom	39,267
Finland	38,695
Bahrain	38,599
France	38,056
Japan	36,927
Iceland	35,182
Oman	34,858
Korea Republic of	33,890

Italy	33,030
New Zealand	32,689
Spain	32,045
Israel	30,676
Cyprus	28,633
Malta	27,930
Slovenia	27,852
Czech Republic	26,660
Trinidad and Tobago	26,090
Slovakia	25,845
Portugal	25,757
Estonia	25,214
Greece	24,524
Lithuania	24,500
Seychelles	23,300
Poland	23,177
Hungary	22,916
Malaysia	22,762
Russian Federation	22,352
Latvia	22,281
Argentina	22,050
Bahamas	21,336
Chile	21,290
Kazakhstan	20,867
Saint Kitts and Nevis	20,805
Antigua and Barbuda	20,070
Croatia	19,409
Uruguay	19,283
Turkey	18,677
Panama	18,192
Romania	18,108
Mauritius	17,470
Belarus	16,676
Lebanon	16,509
Azerbaijan	16,428
Venezuela Bolivarian Republic of	16,159
Mexico	16,056
Bulgaria	15,596
Iran Islamic Republic of	15,440
Brazil	15,175
Libya	14,911
Montenegro	14,558
Palau	13,496
Costa Rica	13,413
Thailand	13,323
Algeria	13,054

China	12,547
Barbados	12,488
Serbia	12,190
Colombia	12,040
Macedonia	11,780
Jordan	11,365
Peru	11,015
Grenada	10,939
Mongolia	10,729
Ecuador	10,605
Tunisia	10,404
Dominica	9,994
Albania	9,943
Saint Vincent and the Grenadines	9,937
Sri Lanka	9,779
Saint Lucia	9,765
Bosnia and Herzegovina	9,638
Ukraine	8,178
Armenia	8,124
Fiji	7,493
Jamaica	7,415
Cuba	7,301
Georgia	7,164

Source: UNDP (2016). Elaborated by the author.

Table VI – Complete HDI levels

Level													
9	Nor	Aus	Swi	Den	Net	Ger	Uni	Sin	Hon	Lie	Ice	Lux	Jap
	Ita	Qat	Kuw										
8	Ire	Can	New	Swe	Uni	Kor	Isr	Bel	Fra	Spa	Bru	Sau	Uni
7	Aus	Fin	Slo	Cze	Gre	Est	Cyp	And	Arg	Chi	Por		
6	Slo	Pol	Lit	Mal	Bah	Oma	Mau	Tri	Cub	Gre			
5	Hun	Cro	Mon	Bel	Rus	Uru	Bah	Bar	Ant	Pal	Pan	Sey	Leb
	Cos	Geo	Alb										
4	Lat	Rom	Mal	Ser	Ira	Tur	Sri	Mex	Bra	Mac	Arm	Bos	Ecu
	Fij	Dom	Jam										
3	Kaz	Bul	Ven	Sai	Aze	Ukr	Alg	Per	Sai	Chi	Tha	Tun	
2	Jor	Mon	Lib	Col									
1	Sai												

Source: UNDP (2016). Elaborated by the author.

Table VII – Objects' connectivities for the complete HDI

Object	Down sets	Up sets	Incomparable objects
Norway	71	1	28
Australia	76	1	23
Switzerland	64	1	35
Denmark	63	1	36
Netherlands	63	1	36
Germany	66	1	33
Ireland	64	2	34
United States	59	1	40
Canada	61	2	37
New Zealand	66	2	32
Singapore	40	1	59
Hong Kong	48	1	51
Liechtenstein	43	1	56
Sweden	61	2	37
United Kingdom	60	2	38
Iceland	44	1	55
Korea Republic of	56	2	42
Israel	59	2	39
Luxembourg	28	1	71
Japan	47	1	52
Belgium	49	5	46
France	46	2	52
Austria	41	5	54
Finland	37	5	58
Slovenia	51	7	42
Spain	31	2	67
Italy	35	1	64
Czech Republic	53	7	40
Greece	34	6	60
Estonia	46	7	47
Brunei Darussalam	14	3	83
Cyprus	29	14	57
Qatar	11	1	88
Andorra	14	8	78
Slovakia	33	13	54
Poland	39	17	44
Lithuania	12	8	80
Malta	25	21	54
Saudi Arabia	6	3	91

Argentina	22	7	71
United Arab Emirates	6	4	90
Chile	27	14	59
Portugal	13	10	77
Hungary	24	19	57
Bahrain	16	18	66
Latvia	12	21	67
Croatia	24	23	53
Kuwait	1	1	98
Montenegro	17	22	61
Belarus	5	16	79
Russian Federation	2	17	81
Oman	4	24	72
Romania	8	28	64
Uruguay	13	27	60
Bahamas	3	27	70
Kazakhstan	2	25	73
Barbados	10	25	65
Antigua and Barbuda	8	37	55
Bulgaria	5	31	64
Palau	1	14	85
Panama	5	34	61
Malaysia	1	35	64
Mauritius	4	28	68
Seychelles	2	37	61
Trinidad and Tobago	1	24	75
Serbia	5	32	63
Cuba	1	19	80
Lebanon	5	32	63
Costa Rica	6	31	63
Iran Islamic Republic of	6	35	59
Venezuela Bolivarian Republic of	4	40	56
Turkey	5	38	57
Sri Lanka	1	31	68
Mexico	2	42	56
Brazil	4	35	61
Georgia	1	16	83
Saint Kitts and Nevis	1	45	54
Azerbaijan	1	30	69
Grenada	2	27	71
Jordan	2	42	56

Macedonia	4	42	54
Ukraine	1	26	73
Algeria	2	46	52
Peru	1	50	49
Albania	1	34	65
Armenia	1	31	68
Bosnia and Herzegovina	1	41	58
Ecuador	1	41	58
Saint Lucia	1	47	52
China	1	52	47
Fiji	1	27	72
Mongolia	1	43	56
Thailand	2	55	43
Dominica	1	39	60
Libya	1	53	46
Tunisia	1	41	58
Colombia	1	63	36
Saint Vincent and the Grenadines	1	57	42
Jamaica	1	38	61

Source: UNDP (2016). Elaborated by the author.

Table VIII – Levels removing education

Level	**Objects**						
25	Sin	Hon	Lie	Qat			
24	Nor	Swi	Lux	Jap	Bru	Kuw	
23	Aus	Uni	Swe	Ice	Ita	Uni	
22	Net	Can	Fra	Spa	Sau		
21	Den	Kor	Isr	Aus	And		
20	Ger	New					
19	Ire	Bel	Chi				
18	Uni	Fin	Por				
17	Gre	Cyp	Mal	Bah	Oma		
16	Slo						
15	Cze	Leb	Cos				
14	Est	Slo	Pol	Pan	Tri	Cub	Dom
13	Lit	Arg	Hun	Cro	Alb		
12	Uru	Bah	Ant	Mal	Sey		
11	Lat	Rus	Tur	Mex			
10	Mon	Rom	Kaz	Ira	Sai	Bos	
9	Mau	Bra	Alg	Ecu	Chi		
8	Bel	Bar	Ven	Tha	Jam		

7	Bul	Ser	Aze	Mac			
6	Pal	Sri	Per	Sai	Lib	Tun	Col
5	Geo	Jor	Arm				
4	Gre						
3	Mon	Sai					
2	Ukr						
1	Fij						

Source: UNDP (2016). Elaborated by the author.

Table IX – Objects' connectivities removing education

Object	Down sets	Up sets	Incomparable objects
Norway	79	2	19
Australia	77	4	19
Switzerland	88	2	10
Denmark	63	8	29
Netherlands	75	7	18
Germany	71	9	20
Ireland	69	13	18
United States	60	7	33
Canada	74	6	20
New Zealand	66	12	22
Singapore	93	1	6
Hong Kong	90	1	9
Liechtenstein	66	1	33
Sweden	81	4	15
United Kingdom	65	15	20
Iceland	71	5	24
Korea Republic of	67	10	23
Israel	66	9	25
Luxembourg	79	2	19
Japan	73	2	25
Belgium	67	13	20
France	69	6	25
Austria	71	8	21
Finland	65	15	20
Slovenia	60	26	14
Spain	67	7	26
Italy	69	3	28
Czech Republic	56	32	12
Greece	56	23	21

Estonia	47	37	16
Brunei Darussalam	61	3	36
Cyprus	60	26	14
Qatar	61	1	38
Andorra	72	8	20
Slovakia	45	37	18
Poland	47	36	17
Lithuania	12	43	45
Malta	61	25	14
Saudi Arabia	21	12	67
Argentina	38	42	20
United Arab Emirates	53	6	41
Chile	47	16	37
Portugal	58	22	20
Hungary	31	42	27
Bahrain	48	23	29
Latvia	16	46	38
Croatia	37	38	25
Kuwait	24	2	74
Montenegro	23	49	28
Belarus	5	60	35
Russian Federation	4	49	47
Oman	50	26	24
Romania	19	52	29
Uruguay	36	39	25
Bahamas	31	43	26
Kazakhstan	2	54	44
Barbados	16	53	31
Antigua And Barbuda	33	44	23
Bulgaria	10	60	30
Palau	4	67	29
Panama	35	37	28
Malaysia	24	43	33
Mauritius	15	54	31
Seychelles	11	44	45
Trinidad and Tobago	5	39	56
Serbia	13	58	29
Cuba	2	34	64
Lebanon	34	33	33
Costa Rica	26	33	41
Iran Islamic Republic of	21	51	28

Venezuela Bolivarian Republic of	11	58	31
Turkey	26	48	26
Sri Lanka	5	63	32
Mexico	29	45	26
Brazil	11	56	33
Georgia	1	68	31
Saint Kitts and Nevis	10	50	40
Azerbaijan	3	62	35
Grenada	5	75	20
Jordan	6	72	22
Macedonia	13	56	31
Ukraine	2	89	9
Algeria	11	56	33
Peru	6	63	31
Albania	10	40	50
Armenia	2	70	28
Bosnia and Herzegovina	6	50	44
Ecuador	10	52	38
Saint Lucia	5	62	33
China	18	52	30
Fiji	1	94	5
Mongolia	1	85	14
Thailand	8	61	31
Dominica	11	39	50
Libya	4	66	30
Tunisia	5	62	33
Colombia	7	70	23
Saint Vincent and the Grenadines	3	82	15
Jamaica	2	57	41

Source: UNDP (2016). Elaborated by the author.

Table X – Objects' average heights removing education

	Complete HDI	Removing education	Variation (%)
Norway	98.61	97.53	-1.10
Australia	98.70	95.06	-3.69
Germany	98.50	88.75	-9.90
Switzerland	98.46	97.77	-0.70
Denmark	98.43	88.73	-9.85
Netherlands	98.43	91.46	-7.08
United States	98.33	89.55	-8.93
Hong Kong	97.95	98.90	0.97
Japan	97.91	97.33	-0.59
Iceland	97.77	93.42	-4.45
Liechtenstein	97.72	98.50	0.80
Singapore	97.56	98.93	1.40
Italy	97.22	95.83	-1.43
New Zealand	97.05	84.61	-12.82
Ireland	96.96	84.14	-13.22
Canada	96.82	92.50	-4.46
Sweden	96.82	95.29	-1.58
United Kingdom	96.77	81.25	-16.04
Israel	96.72	88.00	-9.02
Korea Republic of	96.55	87.01	-9.88
Luxembourg	96.55	97.53	1.02
France	95.83	92.00	-4.00
Spain	93.93	90.54	-3.61
Qatar	91.66	98.38	7.33
Belgium	90.74	83.75	-7.70
Austria	89.13	89.87	0.83
Czech Republic	88.33	63.63	-27.96
Finland	88.09	81.25	-7.76
Slovenia	87.93	69.76	-20.66
Estonia	86.79	55.95	-35.53
Greece	85.00	70.88	-16.61
Brunei Darussalam	82.35	95.31	15.74
Argentina	75.86	47.50	-37.38
Slovakia	71.73	54.87	-23.50
Poland	69.64	56.62	-18.70
Cyprus	67.44	69.76	3.44
Saudi Arabia	66.66	63.63	-4.55
Chile	65.85	74.60	13.29
Andorra	63.63	90.00	41.44

Lithuania	60.00	21.81	-63.65
United Arab Emirates	60.00	89.83	49.72
Portugal	56.52	72.50	28.27
Hungary	55.81	42.46	-23.92
Malta	54.34	70.93	30.53
Croatia	51.06	49.33	-3.39
Kuwait	50.00	92.30	84.60
Bahrain	47.05	67.60	43.68
Montenegro	43.58	31.94	-26.71
Latvia	36.36	25.80	-29.04
Uruguay	32.50	48.00	47.69
Barbados	28.57	23.18	-18.87
Belarus	23.80	7.69	-67.69
Romania	22.22	26.76	20.43
Antigua and Barbuda	17.77	42.85	141.14
Costa Rica	16.21	44.06	171.81
Iran Islamic Republic of	14.63	29.16	99.32
Oman	14.28	65.78	360.64
Bulgaria	13.88	14.28	2.88
Serbia	13.51	18.30	35.46
Lebanon	13.51	50.74	275.57
Panama	12.82	48.61	279.17
Mauritius	12.50	21.73	73.84
Turkey	11.62	35.13	202.32
Russian Federation	10.52	7.54	-28.33
Brazil	10.25	16.41	60.10
Bahamas	10.00	41.89	318.90
Venezuela Bolivarian Republic of	9.09	15.94	75.36
Macedonia	8.69	18.84	116.80
Kazakhstan	7.40	3.57	-51.76
Grenada	6.89	6.25	-9.29
Palau	6.66	5.63	-15.47
Georgia	5.88	1.44	-75.51
Seychelles	5.12	20.00	290.63
Cuba	5.00	5.55	11.00
Mexico	4.54	39.18	763.00
Jordan	4.54	7.69	69.38
Algeria	4.16	16.41	294.47
Trinidad and Tobago	4.00	11.36	184.00
Ukraine	3.70	2.19	-40.81
Fiji	3.57	1.05	-70.59

Thailand	3.50	11.59	231.14
Azerbaijan	3.22	4.61	43.17
Sri Lanka	3.12	7.35	135.58
Armenia	3.12	2.77	-11.22
Albania	2.85	20.00	601.75
Malaysia	2.77	35.82	1,193.14
Jamaica	2.56	3.38	32.03
Dominica	2.50	22.00	780.00
Bosnia and Herzegovina	2.38	10.71	350.00
Ecuador	2.38	16.12	577.31
Tunisia	2.38	7.46	213.45
Mongolia	2.27	1.16	-48.90
Saint Kitts and Nevis	2.17	16.66	667.74
Saint Lucia	2.08	7.46	258.65
Peru	1.96	8.69	343.37
China	1.88	25.71	1,267.55
Libya	1.85	5.71	208.65
Saint Vincent and the Grenadines	1.72	3.52	104.65
Colombia	1.56	9.09	482.69

Source: UNDP (2016). Elaborated by the author.

Table XI - Levels removing health

Level	**Objects**											
14	Nor	Aus	Swi	Den	Net	Ger	Uni	Sin	Lie	Qat	Kuw	
13	Ire	Can	New	Hon	Swe	Uni	Ice	Lux	Bel	Bru	Sau	Uni
12	Kor	Isr	Jap	Fra	Aus	Fin	Spa	Cze	Gre	Est	And	Arg
11	Slo	Ita	Cyp	Slo	Lit	Mal	Bah					
10	Pol	Por	Bel	Rus	Oma	Pal	Mau	Sey	Tri	Geo	Gre	
9	Hun	Uru	Fij									
8	Lat	Bar	Mal									
7	Chi	Mon	Bah	Kaz	Cub	Ukr						
6	Cro	Ant	Ira	Bra	Sai	Aze						
5	Rom	Bul	Pan	Tur	Arm	Mon						
4	Ser	Leb	Ven	Sri	Tun							
3	Cos	Mex	Jor	Mac	Alg	Ecu	Lib					
2	Per	Alb	Bos	Sai	Chi	Tha	Sai	Jam				
1	Dom	Col										

Source: UNDP (2016). Elaborated by the author.

Table XII – Objects' connectivities removing health

Object	Down sets	Up sets	Incomparable objects
Norway	81	1	18
Australia	80	1	19
Switzerland	66	1	33
Denmark	79	1	20
Netherlands	69	1	30
Germany	73	1	26
Ireland	70	3	27
United States	74	1	25
Canada	62	3	35
New Zealand	68	2	30
Singapore	40	1	59
Hong Kong	48	3	49
Liechtenstein	47	1	52
Sweden	62	4	34
United Kingdom	65	2	33
Iceland	45	2	53
Korea Republic of	57	6	37
Israel	59	8	33
Luxembourg	28	3	69
Japan	47	12	41
Belgium	53	7	40
France	47	10	43
Austria	42	8	50
Finland	40	6	54
Slovenia	53	8	39
Spain	31	8	61
Italy	35	14	51
Czech Republic	58	7	35
Greece	35	7	58
Estonia	56	7	37
Brunei Darussalam	16	3	81
Cyprus	29	16	55
Qatar	12	1	87
Andorra	14	11	75
Slovakia	42	13	45
Poland	46	19	35
Lithuania	55	8	37
Malta	25	24	51
Saudi Arabia	23	3	74
Argentina	31	7	62

United Arab Emirates	9	4	87
Chile	27	33	40
Portugal	13	17	70
Hungary	44	20	36
Bahrain	23	18	59
Latvia	41	22	37
Croatia	29	29	42
Kuwait	2	1	97
Montenegro	21	26	53
Belarus	34	16	50
Russian Federation	34	17	49
Oman	5	26	69
Romania	21	32	47
Uruguay	16	31	53
Bahamas	5	31	64
Kazakhstan	33	25	42
Barbados	15	28	57
Antigua and Barbuda	13	42	45
Bulgaria	18	35	47
Palau	15	14	71
Panama	7	47	46
Malaysia	5	36	59
Mauritius	15	30	55
Seychelles	11	37	52
Trinidad and Tobago	3	24	73
Serbia	12	38	50
Cuba	1	29	70
Lebanon	5	53	42
Costa Rica	6	54	40
Iran Islamic Republic of	9	41	50
Venezuela Bolivarian Republic of	12	44	44
Turkey	7	44	49
Sri Lanka	4	38	58
Mexico	3	57	40
Brazil	8	40	52
Georgia	1	17	82
Saint Kitts and Nevis	2	49	49
Azerbaijan	2	32	66
Grenada	6	28	66
Jordan	7	47	46
Macedonia	6	53	41

Ukraine	3	26	71
Algeria	3	57	40
Peru	2	59	39
Albania	1	62	37
Armenia	1	39	60
Bosnia and Herzegovina	1	62	37
Ecuador	1	59	40
Saint Lucia	1	61	38
China	1	69	30
Fiji	2	27	71
Mongolia	8	43	49
Thailand	2	65	33
Dominica	1	75	24
Libya	3	56	41
Tunisia	1	52	47
Colombia	1	69	30
Saint Vincent and the Grenadines	1	62	37
Jamaica	1	55	44

Source: UNDP (2016). Elaborated by the author.

Table XIII – Objects' average height removing health

	Complete HDI	**Removing health**	**Variation (%)**
Norway	98.61	98.78	0.17
Australia	98.70	98.76	0.06
Germany	98.50	98.64	0.14
Switzerland	98.46	98.50	0.04
Denmark	98.43	98.75	0.33
Netherlands	98.43	98.57	0.14
United States	98.33	98.66	0.34
Hong Kong	97.95	94.11	-3.92
Japan	97.91	79.66	-18.64
Iceland	97.77	95.74	-2.08
Liechtenstein	97.72	97.91	0.19
Singapore	97.56	97.56	0.00
Italy	97.22	71.42	-26.54
New Zealand	97.05	97.14	0.09
Ireland	96.96	95.89	-1.10
Canada	96.82	95.38	-1.49

Sweden	96.82	93.93	-2.98
United Kingdom	96.77	97.01	0.25
Israel	96.72	88.05	-8.96
Korea Republic of	96.55	90.47	-6.30
Luxembourg	96.55	90.32	-6.45
France	95.83	82.45	-13.96
Spain	93.93	79.48	-15.38
Qatar	91.66	92.30	0.70
Belgium	90.74	88.33	-2.66
Austria	89.13	84.00	-5.76
Czech Republic	88.33	89.23	1.02
Finland	88.09	86.95	-1.29
Slovenia	87.93	86.88	-1.19
Estonia	86.79	88.88	2.41
Greece	85.00	83.33	-1.96
Brunei Darussalam	82.35	84.21	2.26
Argentina	75.86	81.57	7.53
Slovakia	71.73	76.36	6.45
Poland	69.64	70.76	1.61
Cyprus	67.44	64.44	-4.45
Saudi Arabia	66.66	88.46	32.70
Chile	65.85	45.00	-31.66
Andorra	63.63	56.00	-11.99
Lithuania	60.00	87.30	45.50
United Arab Emirates	60.00	69.23	15.38
Portugal	56.52	43.33	-23.34
Hungary	55.81	68.75	23.19
Malta	54.34	51.02	-6.11
Croatia	51.06	50.00	-2.08
Kuwait	50.00	66.66	33.32
Bahrain	47.05	56.09	19.21
Montenegro	43.58	44.68	2.52
Latvia	36.36	65.07	78.96
Uruguay	32.50	34.04	4.74
Barbados	28.57	34.88	22.09
Belarus	23.80	68.00	185.71
Romania	22.22	39.62	78.31
Antigua and Barbuda	17.77	23.63	32.98
Costa Rica	16.21	10.00	-38.31
Iran Islamic Republic of	14.63	18.00	23.03
Oman	14.28	16.12	12.89
Bulgaria	13.88	33.96	144.67

Serbia	13.51	24.00	77.65
Lebanon	13.51	8.62	-36.20
Panama	12.82	12.96	1.09
Mauritius	12.50	33.33	166.64
Turkey	11.62	13.72	18.07
Russian Federation	10.52	66.66	533.65
Brazil	10.25	16.66	62.54
Bahamas	10.00	13.88	38.80
Venezuela Bolivarian Republic of	9.09	21.42	135.64
Macedonia	8.69	10.16	16.92
Kazakhstan	7.40	56.89	668.78
Grenada	6.89	17.64	156.02
Palau	6.66	51.72	676.58
Georgia	5.88	5.55	-5.61
Seychelles	5.12	22.91	347.46
Cuba	5.00	3.33	-33.40
Mexico	4.54	5.00	10.13
Jordan	4.54	12.96	185.46
Algeria	4.16	5.00	20.19
Trinidad and Tobago	4.00	11.11	177.75
Ukraine	3.70	10.34	179.46
Fiji	3.57	6.89	93.00
Thailand	3.50	2.98	-14.86
Azerbaijan	3.22	5.88	82.61
Sri Lanka	3.12	9.52	205.13
Armenia	3.12	2.50	-19.87
Albania	2.85	1.58	-44.56
Malaysia	2.77	12.19	340.07
Jamaica	2.56	1.78	-30.47
Dominica	2.50	1.31	-47.60
Bosnia and Herzegovina	2.38	1.58	-33.61
Ecuador	2.38	1.66	-30.25
Tunisia	2.38	1.88	-21.01
Mongolia	2.27	15.68	590.75
Saint Kitts and Nevis	2.17	3.92	80.65
Saint Lucia	2.08	1.61	-22.60
Peru	1.96	3.27	66.84
China	1.88	1.42	-24.47
Libya	1.85	5.08	174.59
Saint Vincent and The Grenadines	1.72	1.58	-8.14

Colombia	1.56	1.42	-8.97

Source: UNDP (2016). Elaborated by the author.

Table XIV – Levels removing income

Level												
11	Aus	Swi	Ger	Hon	Ice	Jap	Ita					
10	Nor	Den	Uni	Can	New	Sin	Uni	Kor	Isr	Spa		
9	Net	Ire	Swe	Fra	Cze	Est						
8	Lux	Bel	Aus	Slo	Gre	Slo	Lit	Arg	Chi			
7	Lie	Fin	Cyp	And	Pol	Por	Bel	Pal	Geo			
6	Bru	Mal	Sau	Hun	Cro	Mon	Rus	Uru	Bar	Mau	Cub	Fij
5	Qat	Uni	Bah	Lat	Rom	Bah	Pan	Ser	Cos	Ira	Sri	Bra
	Gre	Alb	Jam									
4	Kuw	Oma	Kaz	Ant	Bul	Mal	Leb	Tur	Mex	Mac	Ukr	Arm
	Bos	Ecu	Tun									
3	Tri	Ven	Aze	Jor	Alg	Per	Sai	Chi	Mon	Dom		
2	Sey	Sai	Tha	Lib								
1	Col	Sai										

Source: UNDP (2016). Elaborated by the author.

Table XV – Objects' connectivities removing income

Object	Down sets	Up sets	Incomparable objects
Norway	81	1	18
Australia	80	1	19
Switzerland	66	1	33
Denmark	79	1	20
Netherlands	69	1	30
Germany	73	1	26
Ireland	70	3	27
United States	74	1	25
Canada	62	3	35
New Zealand	68	2	30
Singapore	40	1	59
Hong Kong	48	3	49
Liechtenstein	47	1	52
Sweden	62	4	34
United Kingdom	65	2	33
Iceland	45	2	53

Korea Republic of	57	6	37
Israel	59	8	33
Luxembourg	28	3	69
Japan	47	12	41
Belgium	53	7	40
France	47	10	43
Austria	42	8	50
Finland	40	6	54
Slovenia	53	8	39
Spain	31	8	61
Italy	35	14	51
Czech Republic	58	7	35
Greece	35	7	58
Estonia	56	7	37
Brunei Darussalam	16	3	81
Cyprus	29	16	55
Qatar	12	1	87
Andorra	14	11	75
Slovakia	42	13	45
Poland	46	19	35
Lithuania	55	8	37
Malta	25	24	51
Saudi Arabia	23	3	74
Argentina	31	7	62
United Arab Emirates	9	4	87
Chile	27	33	40
Portugal	13	17	70
Hungary	44	20	36
Bahrain	23	18	59
Latvia	41	22	37
Croatia	29	29	42
Kuwait	2	1	97
Montenegro	21	26	53
Belarus	34	16	50
Russian Federation	34	17	49
Oman	5	26	69
Romania	21	32	47
Uruguay	16	31	53
Bahamas	5	31	64
Kazakhstan	33	25	42
Barbados	15	28	57
Antigua and Barbuda	13	42	45

Bulgaria	18	35	47
Palau	15	14	71
Panama	7	47	46
Malaysia	5	36	59
Mauritius	15	30	55
Seychelles	11	37	52
Trinidad and Tobago	3	24	73
Serbia	12	38	50
Cuba	1	29	70
Lebanon	5	53	42
Costa Rica	6	54	40
Iran Islamic Republic of	9	41	50
Venezuela Bolivarian Republic of	12	44	44
Turkey	7	44	49
Sri Lanka	4	38	58
Mexico	3	57	40
Brazil	8	40	52
Georgia	1	17	82
Saint Kitts and Nevis	2	49	49
Azerbaijan	2	32	66
Grenada	6	28	66
Jordan	7	47	46
Macedonia	6	53	41
Ukraine	3	26	71
Algeria	3	57	40
Peru	2	59	39
Albania	1	62	37
Armenia	1	39	60
Bosnia and Herzegovina	1	62	37
Ecuador	1	59	40
Saint Lucia	1	61	38
China	1	69	30
Fiji	2	27	71
Mongolia	8	43	49
Thailand	2	65	33
Dominica	1	75	24
Libya	3	56	41
Tunisia	1	52	47
Colombia	1	69	30

Saint Vincent and the Grenadines	1	62	37
Jamaica	1	55	44

Source: UNDP (2016). Elaborated by the author.

Table XVI – Objects' average heights removing income

	Complete HDI	**Removing income**	**Variation (%)**
Norway	9861	97.40	-1.23
Australia	98.70	98.90	0.20
Germany	98.50	98.64	0.14
Switzerland	98.46	98.61	0.15
Denmark	98.43	97.18	-1.27
Netherlands	98.43	95.83	-2.64
United States	98.33	95.45	-2.93
Hong Kong	97.95	98.14	0.19
Japan	97.91	98.14	0.23
Iceland	97.77	98.11	0.35
Liechtenstein	97.72	75.00	-23.25
Singapore	97.56	93.33	-4.34
Italy	97.22	97.67	0.46
New Zealand	97.05	97.59	0.56
Ireland	96.96	95.94	-1.05
Canada	9682	97.18	0.37
Sweden	96.82	94.36	-2.54
United Kingdom	96.77	97.01	0.25
Israel	96.72	97.26	0.56
Korea Republic of	96.55	97.14	0.61
Luxembourg	96.55	78.94	-18.24
France	95.83	94.64	-1.24
Spain	93.93	95.12	1.27
Qatar	91.66	24.44	-73.34
Belgium	90.74	87.09	-4.02
Austria	89.13	80.70	-9.46
Czech Republic	88.33	89.39	1.20
Finland	88.09	84.00	-4.64
Slovenia	87.93	89.55	1.84
Estonia	86.79	88.13	1.54
Greece	85.00	88.00	3.53
Brunei Darussalam	82.35	32.55	-60.47
Argentina	75.86	78.12	2.98

Slovakia	71.73	71.42	-0.43
Poland	69.64	72.58	4.22
Cyprus	67.44	68.08	0.95
Saudi Arabia	66.66	24.00	-64.00
Chile	65.85	71.42	8.46
Andorra	63.63	42.85	-32.66
Lithuania	60.00	61.90	3.17
United Arab Emirates	60.00	13.95	-76.75
Portugal	56.52	57.69	2.07
Hungary	55.81	58.69	5.16
Malta	54.34	56.86	4.64
Croatia	51.06	59.64	16.80
Kuwait	50.00	2.27	-95.46
Bahrain	47.05	32.00	-31.99
Montenegro	43.58	59.25	35.96
Latvia	36.36	40.00	10.01
Uruguay	32.50	37.20	14.46
Barbados	28.57	45.65	59.78
Belarus	23.80	33.33	40.04
Romania	22.22	27.50	23.76
Antigua and Barbuda	17.77	18.75	5.51
Costa Rica	16.21	22.50	38.80
Iran Islamic Republic of	14.63	17.77	21.46
Oman	14.28	8.69	-39.15
Bulgaria	13.88	20.00	44.09
Serbia	13.51	27.27	101.85
Lebanon	13.51	12.82	-5.11
Panama	12.82	14.63	14.12
Mauritius	12.50	17.64	41.12
Turkey	11.62	10.86	-6.54
Russian Federation	10.52	10.00	-4.94
Brazil	10.25	11.90	16.10
Bahamas	10.00	11.76	17.60
Venezuela Bolivarian Republic of	9.09	10.20	12.21
Macedonia	8.69	10.41	19.79
Kazakhstan	7.40	7.14	-3.51
Grenada	6.89	10.00	45.14
Palau	6.66	17.64	164.86
Georgia	5.88	44.82	662.24

Seychelles	5.12	3.70	-27.73
Cuba	5.00	58.69	1,073.80
Mexico	4.54	6.52	43.61
Jordan	4.54	10.00	120.26
Algeria	4.16	7.54	81.25
Trinidad and Tobago	4.00	2.50	-37.50
Ukraine	3.70	13.33	260.27
Fiji	3.57	6.89	93.00
Thailand	3.50	3.07	-12.29
Azerbaijan	3.22	2.77	-13.98
Sri Lanka	3.12	21.42	586.54
Armenia	3.12	5.71	83.01
Albania	2.85	2.77	-2.81
Malaysia	2.77	2.27	-18.05
Jamaica	2.56	2.50	-2.34
Dominica	2.50	2.43	-2.80
Bosnia and Herzegovina	2.38	8.69	265.13
Ecuador	2.38	12.76	436.13
Tunisia	2.38	2.38	0.00
Mongolia	2.27	2.17	-4.41
Saint Kitts and Nevis	2.17	1.53	-29.49
Saint Lucia	2.08	2.04	-1.92
Peru	1.96	3.63	85.20
China	1.88	1.78	-5.32
Libya	1.85	1.66	-10.27
Saint Vincent and the Grenadines	1.72	1.63	-5.23
Colombia	1.56	1.42	-8.97

Source: UNDP (2016). Elaborated by the author.

Table XVII – Carbon dioxide emissions per country (2013)

Country	Carbon Dioxide Emission PC	People per 100 Tons of Carbon Dioxide Emitted
Qatar	40.5	2.5
Trinidad and Tobago	34.5	2.9
Kuwait	27.3	3.7
Bahrain	23.7	4.2
Brunei Darussalam	18.9	5.3
Luxembourg	18.7	5.3
United Arab Emirates	18.7	5.3

Saudi Arabia	17.9	5.6
United States	16.4	6.1
Australia	16.3	6.1
Oman	15.7	6.4
Kazakhstan	15.4	6.5
Estonia	15.1	6.6
Mongolia	14.5	6.9
Canada	13.5	7.4
Russian Federation	12.5	8.0
Korea Republic of	11.8	8.5
Norway	11.7	8.5
Palau	10.7	9.3
Netherlands	10.1	9.9
Japan	9.8	10.2
Singapore	9.4	10.6
Czech Republic	9.4	10.6
Germany	9.2	10.9
Israel	8.8	11.4
Finland	8.5	11.8
Belgium	8.4	11.9
Bahamas	8.2	12.2
Libya	8.1	12.3
Malaysia	8	12.5
Iran Islamic Republic of	8	12.5
Poland	7.9	12.7
Ireland	7.6	13.2
New Zealand	7.6	13.2
China	7.6	13.2
Austria	7.4	13.5
Seychelles	7.2	13.9
United Kingdom	7.1	14.1
Slovenia	7	14.3
Denmark	6.8	14.7
Belarus	6.7	14.9
Andorra	6.5	15.4
Hong Kong	6.3	15.9
Greece	6.3	15.9
Serbia	6.3	15.9
Slovakia	6.2	16.1
Iceland	6.1	16.4
Venezuela Bolivarian Republic of	6.1	16.4
Ukraine	6	16.7
Antigua and Barbuda	5.8	17.2
Italy	5.7	17.5
Bosnia and Herzegovina	5.7	17.5
Bulgaria	5.4	18.5

Cyprus	5.2	19.2
Malta	5.2	19.2
France	5.1	19.6
Spain	5.1	19.6
Barbados	5.1	19.6
Saint Kitts and Nevis	5.1	19.6
Switzerland	5	20.0
Chile	4.7	21.3
Sweden	4.6	21.7
Argentina	4.5	22.2
Thailand	4.5	22.2
Portugal	4.4	22.7
Lithuania	4.3	23.3
Lebanon	4.3	23.3
Hungary	4.2	23.8
Croatia	4.2	23.8
Turkey	4.2	23.8
Macedonia	4	25.0
Mexico	3.9	25.6
Azerbaijan	3.8	26.3
Montenegro	3.6	27.8
Latvia	3.5	28.6
Romania	3.5	28.6
Cuba	3.5	28.6
Algeria	3.5	28.6
Jordan	3.4	29.4
Mauritius	3	33.3
Grenada	2.9	34.5
Ecuador	2.8	35.7
Jamaica	2.8	35.7
Panama	2.7	37.0
Brazil	2.5	40.0
Tunisia	2.5	40.0
Uruguay	2.2	45.5
Saint Lucia	2.2	45.5
Georgia	2	50.0
Peru	1.9	52.6
Fiji	1.9	52.6
Colombia	1.9	52.6
Saint Vincent and the Grenadines	1.9	52.6
Armenia	1.8	55.6
Dominica	1.8	55.6
Albania	1.7	58.8
Costa Rica	1.6	62.5
Liechtenstein	1.4	71.4
Sri Lanka	0.8	12.0

Source: World Bank (2016). Elaborated by the author.

Table XVIII – Levels adding sustainability

Level	Objects											
4	Nor	Aus	Swi	Den	Net	Ger	Ire	Uni	Can	New	Sin	Hon	Lie
	Swe	Uni	Ice	Kor	Isr	Lux	Jap	Bel	Fra	Slo	Spa	Ita	Qat
	Lit	Arg	Por	Hun	Lat	Kuw	Mon	Uru	Mau	Sri	Geo	Gre	Fij
3	Aus	Fin	Cze	Gre	Est	Bru	Cyp	And	Slo	Pol	Mal	Sau	Uni
	Chi	Cro	Bel	Rom	Bar	Pan	Cub	Leb	Cos	Bra	Aze	Jor	Mac
	Ukr	Alg	Per	Alb	Arm	Ecu	Sai	Tun	Sai	Jam			
2	Bah	Rus	Oma	Bah	Kaz	Ant	Bul	Pal	Mal	Sey	Tri	Ser	Ira
	Ven	Tur	Mex	Sai	Bos	Dom	Col						
1	Chi	Mon	Tha	Lib									

Source: World Bank (2016). Elaborated by the author.

Table XIX – Objects' connectivities adding sustainability

Object	Down sets	Up sets	Incomparable objects
Norway	10	1	89
Australia	3	1	96
Switzerland	29	1	70
Denmark	17	1	82
Netherlands	6	1	93
Germany	10	1	89
Ireland	14	1	85
United States	4	1	95
Canada	6	1	93
New Zealand	14	1	85
Singapore	6	1	93
Hong Kong	13	1	86
Liechtenstein	42	1	57
Sweden	28	1	71
United Kingdom	15	1	84
Iceland	11	1	88
Korea Republic of	4	1	95
Israel	6	1	93
Luxembourg	2	1	97
Japan	5	1	94
Belgium	5	1	94
France	18	1	81
Austria	8	3	89

Finland	4	2	94
Slovenia	11	1	88
Spain	10	1	89
Italy	10	1	89
Czech Republic	6	4	90
Greece	7	2	91
Estonia	2	5	93
Brunei Darussalam	1	3	96
Cyprus	9	3	88
Qatar	1	1	98
Andorra	3	4	93
Slovakia	11	2	87
Poland	7	8	85
Lithuania	9	1	90
Malta	8	4	88
Saudi Arabia	1	3	96
Argentina	9	1	90
United Arab Emirates	1	4	95
Chile	9	2	89
Portugal	5	1	94
Hungary	11	1	88
Bahrain	1	18	81
Latvia	9	1	90
Croatia	10	2	88
Kuwait	1	1	98
Montenegro	6	1	93
Belarus	2	4	94
Russian Federation	2	13	85
Oman	1	19	80
Romania	4	2	94
Uruguay	12	1	87
Bahamas	1	14	85
Kazakhstan	1	23	76
Barbados	3	4	93
Antigua and Barbuda	3	11	86
Bulgaria	2	8	90
Palau	1	9	90
Panama	3	2	95
Malaysia	1	20	79
Mauritius	3	1	96
Seychelles	1	18	81
Trinidad and Tobago	1	24	75

Serbia	1	12	87
Cuba	1	2	97
Lebanon	3	2	95
Costa Rica	6	2	92
Iran Islamic Republic of	2	21	77
Venezuela Bolivarian Republic of	2	15	83
Turkey	3	4	93
Sri Lanka	1	1	98
Mexico	2	4	94
Brazil	3	2	95
Georgia	1	1	98
Saint Kitts and Nevis	1	10	89
Azerbaijan	1	3	96
Grenada	1	1	98
Jordan	1	2	97
Macedonia	1	3	96
Ukraine	1	6	93
Algeria	1	3	96
Peru	1	2	97
Albania	1	2	97
Armenia	1	2	97
Bosnia and Herzegovina	1	13	86
Ecuador	1	3	96
Saint Lucia	1	2	97
China	1	31	68
Fiji	1	1	98
Mongolia	1	39	60
Thailand	1	14	85
Dominica	1	3	96
Libya	1	36	63
Tunisia	1	3	96
Colombia	1	3	96
Saint Vincent and the Grenadines	1	2	97
Jamaica	1	2	97

Source: World Bank (2016). Elaborated by the author.

Table XX – Objects' average heights adding sustainability

	HDI	Green HDI	Variation (%)
Norway	98.61	90.90	-7.82
Australia	98.70	75.00	-24.01
Germany	98.50	90.90	-7.72
Switzerland	98.46	96.66	-1.83
Denmark	98.43	94.44	-4.05
Netherlands	98.43	85.71	-12.92
United States	98.33	80.00	-18.64
Hong Kong	97.95	92.85	-5.21
Japan	97.91	83.33	-14.89
Iceland	97.77	91.66	-6.25
Liechtenstein	97.72	97.67	-0.05
Singapore	97.56	85.71	-12.15
Italy	97.22	90.90	-6.50
New Zealand	97.05	93.33	-3.83
Ireland	96.96	93.33	-3.74
Canada	96.82	85.71	-11.47
Sweden	96.82	96.55	-0.28
United Kingdom	96.77	93.75	-3.12
Israel	96.72	85.71	-11.38
Korea Republic of	96.55	80.00	-17.14
Luxembourg	96.55	66.66	-30.96
France	95.83	94.73	-1.15
Spain	93.93	90.90	-3.23
Qatar	91.66	50.00	-45.45
Belgium	90.74	83.33	-8.17
Austria	89.13	72.72	-18.41
Czech Republic	88.33	60.00	-32.07
Finland	88.09	66.66	-24.33
Slovenia	87.93	91.66	4.24
Estonia	86.79	28.57	-67.08
Greece	85.00	77.77	-8.51
Brunei Darussalam	82.35	25.00	-69.64
Argentina	75.86	90.00	18.64
Slovakia	71.73	84.61	17.96
Poland	69.64	46.66	-33.00
Cyprus	67.44	75.00	11.21
Saudi Arabia	66.66	25.00	-62.50
Chile	65.85	81.81	24.24
Andorra	63.63	42.85	-32.66
Lithuania	60.00	90.00	50.00

United Arab Emirates	60.00	20.00	-66.67
Portugal	56.52	83.33	47.43
Hungary	55.81	91.66	64.24
Malta	54.34	66.66	22.67
Croatia	51.06	83.33	63.20
Kuwait	50.00	50.00	0.00
Bahrain	47.05	5.26	-88.82
Montenegro	43.58	85.71	96.67
Latvia	36.36	90.00	147.52
Uruguay	32.50	92.30	184.00
Barbados	28.57	42.85	49.98
Belarus	23.80	33.33	40.04
Romania	22.22	66.66	200.00
Antigua and Barbuda	17.77	21.42	20.54
Costa Rica	16.21	75.00	362.68
Iran Islamic Republic of	14.63	8.69	-40.60
Oman	14.28	5.00	-64.99
Bulgaria	13.88	20.00	44.09
Serbia	13.51	7.69	-43.08
Lebanon	13.51	60.00	344.12
Panama	12.82	60.00	368.02
Mauritius	12.50	75.00	500.00
Turkey	11.62	42.85	268.76
Russian Federation	10.52	13.33	26.71
Brazil	10.25	60.00	485.37
Bahamas	10.00	6.66	-33.40
Venezuela Bolivarian Republic of	9.09	11.76	29.37
Macedonia	8.69	25.00	187.69
Kazakhstan	7.40	4.16	-43.78
Grenada	6.89	50.00	625.69
Palau	6.66	10.00	50.15
Georgia	5.88	50.00	750.34
Seychelles	5.12	5.26	2.73
Cuba	5.00	33.33	566.60
Mexico	4.54	33.33	634.14
Jordan	4.54	33.33	634.14
Algeria	4.16	25.00	500.96
Trinidad and Tobago	4.00	4.00	0.00
Ukraine	3.70	14.28	285.95
Fiji	3.57	50.00	1,300.56
Thailand	3.50	6.66	90.29
Azerbaijan	3.22	25.00	676.40

Sri Lanka	3.12	50.00	1,502.56
Armenia	3.12	33.33	968.27
Albania	2.85	33.33	1,069.47
Malaysia	2.77	4.76	71.84
Jamaica	2.56	33.33	1,201.95
Dominica	2.50	25.00	900.00
Bosnia and Herzegovina	2.38	7.14	200.00
Ecuador	2.38	25.00	950.42
Tunisia	2.38	25.00	950.42
Mongolia	2.27	2.50	10.13
Saint Kitts and Nevis	2.17	9.09	318.89
Saint Lucia	2.08	33.33	1,502.40
Peru	1.96	33.33	1,600.51
China	1.88	3.12	65.96
Libya	1.85	2.70	45.95
Saint Vincent and the Grenadines	1.72	33.33	1,837.79
Colombia	1.56	25.00	1,502.56

Source: World Bank (2016). Elaborated by the author.