

Electrostatic waves in dusty plasmas with variable charge on dust particles

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Abstract.

In this work we present results obtained using a kinetic approach for parallel propagation of waves in a homogeneous magnetized dusty plasma. In this magnetized plasma we consider embedded spherical dust grains with constant radius and variable charge. We consider that the principal process of dust charging is by capture of electrons and ions of the plasma, and use a cross-section derived from the orbital motion limited theory to describe this process. We concentrate on electrostatic waves and obtain the dispersion relation, which is applied to Langmuir oscillations, taking into account those effects due to charge fluctuation which do not contain terms with the derivative of the charging cross-section, and using an approximation for the collision frequency between plasma particles and dust particles. We also analyse the competition between Landau damping and the damping that results from the electric charge variation of the dust particles.

Keywords: dusty plasmas, dust charge fluctuations, Langmuir waves

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THE MODEL

We consider a plasma in a homogeneous external magnetic field $\vec{B}_0 = B_0 \vec{e}_z$. In this magnetized plasma we take into account the presence of spherical dust grains with constant radius a and variable charge q_d ; this charge originates from inelastic collisions between the dust particles and particles of species β (electrons and ions), with charge q_β and mass m_β . For simplicity, we will consider simply charged ions.

The charging model for the dust particles must in principle take into account the presence of an external magnetic field. This field must influence the characteristics of charging of the dust particles, because the path described by electrons and ions is modified in the presence of the field. In the case of an homogeneous field we have cyclotron motion of electrons and ions around the magnetic field lines. However, it has been shown by Chang and Spariosu, through numerical calculation, that for $a \ll \rho_G$, where $\rho_G = (\pi/2)^{1/2} r_{Le}$ and r_{Le} is the electron Larmor radius, the effect of the magnetic field on the charging of the dust particles can be neglected [1]. For the parameter values used in the present work the relation $a \ll \rho_G$ is always satisfied.

We will consider the dust grain charging process to occur by the capture of plasma electrons and ions during inelastic collisions between these particles and dust particles. Since the electron thermal speed is much larger than the ion thermal speed, the dust charge will be preferentially negative. As a cross-section for the charging process of

the dust particles, we use expressions derived from the OML theory (orbital motion limited theory) [2, 3]. In the present work, dust particles are assumed to be immobile, and consequently the validity of the proposed model will be restricted to waves with frequency much higher than the characteristic dust frequencies. In particular we will consider the regime in which $\omega \gg |\Omega_d|$, where Ω_d is the cyclotron frequency of the dust particles. This modelling excludes the modes that can arise from the dust dynamics.

As we will see, in this range of frequencies the dust particles modify the dispersion relation, through modifications of the quasineutrality condition and through effects due to dust charge fluctuation. In particular, these dust charge fluctuations provide an additional damping mechanism for the Langmuir waves, beyond the well-known Landau damping mechanism.

PROPAGATION PARALLEL TO \vec{B}_0 AND MAXWELLIAN DISTRIBUTION FUNCTION

In order to study electrostatic waves, we will use the general form of the dielectric tensor previously obtained. The dielectric tensor for a magnetized dusty plasma, homogeneous, fully ionized, with identical immobile dust particles and charge variable in time, could be written in the following way [4, 5]:

$$\epsilon_{ij} = \epsilon_{ij}^C + \epsilon_{ij}^N. \quad (1)$$

The term ϵ_{ij}^C is formally identical, except for the iz components, to the dielectric tensor of a magnetized homogeneous conventional plasma of electrons and ions, with the resonant denominator changed by the addition of a purely imaginary term which contains the collision frequency of electrons and ions with the dust particles. In the case of iz components of the dielectric tensor, in addition to the term obtained with the prescription above, there is a term which is proportional to the collision frequency of electrons and ions with the dust particles.

The term ϵ_{ij}^N is entirely new and only exists in the presence of dust particles with variable charge. Its form is strongly dependent on the model used to describe the charging process of the dust particles. The general expressions for ϵ_{ij}^C and ϵ_{ij}^N are given in [4, 6].

In the case of propagation parallel to the external magnetic field and Maxwellian equilibrium distributions for the electrons and ions, the dielectric tensor assumes the form

$$\vec{\epsilon} = \begin{pmatrix} \epsilon_{xx}^C & \epsilon_{xy}^C & 0 \\ -\epsilon_{xy}^C & \epsilon_{xx}^C & 0 \\ 0 & 0 & \epsilon_{zz}^C + \epsilon_{zz}^N \end{pmatrix}, \quad (2)$$

where

$$\epsilon_{xx}^C = 1 + \frac{1}{4} \sum_{\beta} X_{\beta} \left[\widehat{I}_{\beta}^{+} + \widehat{I}_{\beta}^{-} \right], \quad (3)$$

$$\epsilon_{xy}^C = -\frac{i}{4} \sum_{\beta} X_{\beta} \left[\widehat{I}_{\beta}^{+} - \widehat{I}_{\beta}^{-} \right], \quad (4)$$

$$\epsilon_{zz}^C = 1 + \sum_{\beta} X_{\beta} \hat{I}_{\beta}^0, \quad (5)$$

with $X_{\beta} \equiv \omega_{p\beta}^2 / \omega^2$, and

$$\hat{I}_{\beta}^s \equiv \frac{1}{n_{\beta 0}} \int d^3 p \frac{p_{\perp} \partial f_{\beta 0} / \partial p_{\perp}}{1 - \frac{k_{\parallel} p_{\parallel}}{m_{\beta} \omega} + s \frac{\Omega_{\beta}}{\omega} i + i \frac{v_{\beta d}^0(p)}{\omega}},$$

where $s = \pm 1$, and

$$\hat{I}_{\beta}^0 \equiv \frac{1}{n_{\beta 0}} \int d^3 p \frac{p_{\perp} \partial f_{\beta 0} / \partial p_{\perp} (p_{\parallel} / p_{\perp})^2}{1 - \frac{k_{\parallel} p_{\parallel}}{m_{\beta} \omega} + i \frac{v_{\beta d}^0(p)}{\omega}}.$$

We also have

$$\epsilon_{zz}^N = -\frac{4\pi i n_{d0}}{\omega} U_z S_z, \quad (6)$$

where

$$U_z \equiv \frac{i}{\omega + i(v_{ch} + v_1)} \sum_{\beta} \frac{q_{\beta}}{m_{\beta}^2} \int d^3 p \frac{p_{\perp} \sigma'_{\beta}(p) f_{\beta 0} p}{1 - \frac{k_{\parallel} p_{\parallel}}{m_{\beta} \omega} + i \frac{v_{\beta d}^0(p)}{\omega}} \left(\frac{p_{\parallel}}{p_{\perp}} \right),$$

$$S_z \equiv \frac{1}{\omega n_{d0}} \sum_{\beta} q_{\beta}^2 \int d^3 p \frac{v_{\beta d}^0(p)}{\omega} \frac{\partial f_{\beta 0} / \partial p_{\perp}}{1 - \frac{k_{\parallel} p_{\parallel}}{m_{\beta} \omega} + i \frac{v_{\beta d}^0(p)}{\omega}} \left(\frac{p_{\parallel}}{p_{\perp}} \right),$$

with

$$v_{ch} = -\sum_{\beta} \frac{q_{\beta}}{m_{\beta}} \int d^3 p \sigma'_{\beta}(p) p f_{\beta 0},$$

$$v_1 = \sum_{\beta} \frac{q_{\beta}}{m_{\beta}} \int d^3 p \frac{[i v_{\beta d}^0(p) / \omega] \sigma'_{\beta}(p) f_{\beta 0} p}{1 - \frac{k_{\parallel} p_{\parallel}}{m_{\beta} \omega} + i \frac{v_{\beta d}^0(p)}{\omega}}.$$

THE DISPERSION RELATION FOR ELECTROSTATIC MODES

In the case we are considering, the dispersion relation for $\vec{k} = k_{\parallel} \vec{e}_z$, follows from the determinant

$$\det \begin{pmatrix} \epsilon_{xx}^C - N_{\parallel}^2 & \epsilon_{xy}^C & 0 \\ -\epsilon_{xy}^C & \epsilon_{xx}^C - N_{\parallel}^2 & 0 \\ 0 & 0 & \epsilon_{zz}^C + \epsilon_{zz}^N \end{pmatrix} = 0. \quad (7)$$

In this expression, $N_{\parallel} = k_{\parallel} c / \omega$ is the refractive index in the parallel direction to external magnetic field.

In the case of wave propagation with electric field parallel to \vec{B}_0 , that is $\widehat{E}_x = \widehat{E}_y = 0$, we obtain

$$\varepsilon_{zz} = \varepsilon_{zz}^C + \varepsilon_{zz}^N = 0, \quad (8)$$

which is the dispersion relation for longitudinal electrostatic modes.

In this preliminary approach we will neglect the contributions to the dielectric properties arising from the term denoted as ε_{zz}^N in equation (8). Therefore, the dispersion relation is reduced to the following,

$$\varepsilon_{zz}^C = 1 + \sum_{\beta} X_{\beta} \widehat{I}_{\beta}^0 = 0, \quad (9)$$

with

$$\widehat{I}_{\beta}^0 \equiv \frac{1}{n_{\beta 0}} \int d^3 p \frac{p_{\parallel}^2 / p_{\perp} \partial f_{\beta 0} / \partial p_{\perp}}{1 - \frac{k_{\parallel} p_{\parallel}}{m_{\beta} \omega} + i \frac{v_{\beta d}^0(p)}{\omega}}.$$

In order to evaluate the integral \widehat{I}_{β}^0 we replace the functions $v_{\beta d}^0(p)$, which represent the rate of capture of particles of species β by dust particles in the equilibrium state, by their average values in momentum space:

$$v_{\beta} \equiv \frac{1}{n_{d0}} \int d^3 p v_{\beta d}^0(p) f_{\beta 0},$$

where $v_{\beta d}^0(p)$ is given by [4]

$$v_{\beta d}^0(p) = \frac{\pi a^2 n_{d0} (p^2 + C_{\beta})}{m_{\beta} p} H(p^2 + C_{\beta}).$$

For Maxwellian distributions we obtain:

$$v_i = 2\sqrt{2\pi} a^2 n_{d0} v_{T_i} (1 + \chi_i),$$

$$v_e = 2\sqrt{2\pi} a^2 n_{d0} v_{T_e} e^{\chi_e},$$

where $\chi_i \equiv Z_d e^2 / (a T_i)$, $\chi_e \equiv -(T_i / T_e) \chi_i$, and $v_{T_e} = (T_{\beta} / m_{\beta})^{1/2}$. The number of charges in each dust particle, Z_d , is calculated from the equation of balance of current in the dust particles, in the equilibrium state, and from the quasi-neutrality condition, which also gives n_{e0} , if we give the ion and dust densities n_{i0} and n_{d0} , respectively.

Using the average value of the collision frequency, the relevant integral for the dispersion relation can be evaluated and gives

$$\widehat{I}_{\beta}^0 = 2\zeta_{\beta} \widehat{\zeta}_{\beta} \left[1 + \widehat{\zeta}_{\beta} Z \left(\widehat{\zeta}_{\beta} \right) \right],$$

where Z is the plasma dispersion function, defined by [8]

$$Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} dt \frac{e^{-t^2}}{t - \zeta},$$

and

$$\widehat{\zeta}_\beta = \zeta_\beta + i \frac{v_\beta}{\sqrt{2}k_\parallel v_{T\beta}}, \quad \text{where } \zeta_\beta = \frac{\omega}{\sqrt{2}k_\parallel v_{T\beta}}.$$

Then the dispersion relation, given by equation (9), assumes the form

$$1 + 2 \sum_\beta X_\beta \zeta_\beta \widehat{\zeta}_\beta \left[1 + \widehat{\zeta}_\beta Z \left(\widehat{\zeta}_\beta \right) \right] = 0. \quad (10)$$

In order to find the numerical solution we introduce dimensionless variables, defined as follows:

$$\begin{aligned} z &\equiv \frac{\omega}{\omega_{pe}}, & u_\beta &\equiv \frac{v_{T\beta}}{C_S}, & \text{with } C_S &= \sqrt{\frac{T_e}{m_i}}, \\ r_\beta &\equiv \frac{\omega p_\beta}{\omega_{pe}}, & r_i^2 &= \frac{m_e}{m_i} \equiv \alpha, \\ q &\equiv \frac{k_\parallel C_S}{\omega_{pe}}, & \widetilde{v}_\beta &\equiv \frac{v_\beta}{\omega_{pe}}. \end{aligned}$$

Using these variables, the dispersion relation, given by equation (10), assumes the form

$$1 + \sum_\beta \frac{r_\beta^2}{q^2 u_\beta^2} \left(1 + i \frac{\widetilde{v}_\beta}{z} \right) \left[1 + \widehat{\zeta}_\beta Z \left(\widehat{\zeta}_\beta \right) \right] = 0, \quad (11)$$

where

$$\widehat{\zeta}_\beta \equiv \zeta_\beta + i \frac{\widetilde{v}_\beta}{\sqrt{2}q u_\beta}, \quad \text{and where } \zeta_\beta \equiv \frac{z}{\sqrt{2}q u_\beta}.$$

The expressions for \widetilde{v}_i and \widetilde{v}_e are given by

$$\begin{aligned} \widetilde{v}_i &= 2\sqrt{2\pi\gamma\varepsilon\tilde{a}^2} u_i (1 + \chi_i), \\ \widetilde{v}_e &= 2\sqrt{2\pi\gamma\varepsilon\tilde{a}^2} u_e e^{\chi_e}, \end{aligned}$$

where

$$\begin{aligned} \varepsilon &\equiv \frac{n_{d0}}{n_{i0}}, & \tilde{a} &\equiv \frac{a}{\lambda}, & \lambda &\equiv \frac{e^2}{T_i}, \\ \gamma &\equiv \frac{\lambda^2 n_{i0} C_S}{\omega_{pe}}, & \chi_i &= \frac{z_d}{\tilde{a}}, & \chi_e &= -\frac{\chi_i}{\tau_e}, & \tau_e &\equiv \frac{T_e}{T_i}. \end{aligned}$$

In the absence of dust, $n_{d0} = 0$, we have $v_\beta = 0$, which gives the dispersion relation for an electron-ion plasma, in the form

$$1 + 2 \sum_\beta X_\beta \zeta_\beta^2 \left[1 + \zeta_\beta Z \left(\zeta_\beta \right) \right] = 0, \quad (12)$$

which, in dimensionless variables, can be written as

$$1 + \sum_\beta \frac{r_\beta^2}{q^2 u_\beta^2} \left[1 + \zeta_\beta Z \left(\zeta_\beta \right) \right] = 0. \quad (13)$$

Analytical solution for Langmuir waves in dustless plasmas

For Langmuir oscillations we must have $\omega/k_{\parallel} \gg v_{T\beta}$, which implies that $\zeta_{\beta} \gg 1$. Using the asymptotic series to express the Z function, and retaining the three first terms of the expansion, the dispersion relation given by equation (11) assumes the form

$$1 + \sum_{\beta} \frac{r_{\beta}^2}{q^2 u_{\beta}^2} \left(1 + i \frac{\tilde{v}_{\beta}}{z}\right) \left[i\sqrt{\pi} \hat{\zeta}_{\beta} e^{-\hat{\zeta}_{\beta}^2} - \frac{1}{2\hat{\zeta}_{\beta}^2} - \frac{3}{4\hat{\zeta}_{\beta}^4} \right] = 0. \quad (14)$$

In the absence of dust, the above expression becomes the dispersion relation for Langmuir oscillations in an electron-ion plasmas, given by

$$1 + \sum_{\beta} \frac{r_{\beta}^2}{q^2 u_{\beta}^2} \left[i\sqrt{\pi} \zeta_{\beta} e^{-\zeta_{\beta}^2} - \frac{1}{2\zeta_{\beta}^2} - \frac{3}{4\zeta_{\beta}^4} \right] = 0. \quad (15)$$

In this case it is possible to assume that the imaginary part of the frequency is much smaller than the real part, and use a traditional approach [9] to obtain

$$z_r^2 \simeq 1 + 3q^2 u_e^2 \quad \text{or} \quad \omega_r^2 = \omega_{p_e}^2 + 3k_{\parallel}^2 v_{T_e}^2,$$

$$z_i \simeq -\sqrt{\frac{\pi}{8}} \frac{e^{-3/2} e^{-1/2 q^2 u_e^2}}{q^3 u_e^3} \quad \text{or} \quad \frac{\omega_i}{\omega_{p_e}} \simeq -\sqrt{\frac{\pi}{8}} \frac{\omega_{p_e}^3}{k_{\parallel}^3 v_{T_e}^3} \exp\left(-\frac{3}{2} - \frac{\omega_{p_e}^2}{2k_{\parallel}^2 v_{T_e}^2}\right).$$

NUMERICAL ANALYSIS

We consider the following parameters: ambient magnetic field $B = 1.0 \times 10^{-4}$ T, ion temperature $T_i = 1.0 \times 10^4$ K, ion density $n_{i0} = 1.0 \times 10^9$ cm $^{-3}$, ion charge number $Z_i = 1.0$, and ion mass $m_i = m_p$, the proton mass. The electron temperature is assumed to be the same as the ion temperature ($T_e = T_i$). For the radius of the dust particles, we assume $a = 1.0 \times 10^{-4}$ cm. For the classical distance of minimum approach, measured in cm, we use the value $\lambda = 1.44 \times 10^{-7}/T_i(\text{eV})$, where $T_i(\text{eV})$ means the ion temperature expressed in unit of eV.

Initially, we consider the solution of the dispersion relation given by equation (11) for several values of ε , starting from $\varepsilon = 0$. In the limit $\varepsilon = 0$, ion and electron densities are equal and $\tilde{v}_e = \tilde{v}_i = 0$, and therefore the dispersion relation becomes the usual dispersion relation for Langmuir waves in the absence of dust.

Fig. 1(a) shows the real part of the normalized frequency (z_r), as a function of q , for five values of ε . The dust density n_{d0} is obtained from $\varepsilon = n_{d0}/n_{i0}$, and the values of ε utilized are $\varepsilon = 0.0, 2.5 \times 10^{-5}, 5.0 \times 10^{-5}, 7.5 \times 10^{-5}$, and 1.0×10^{-4} . Fig. 1 shows that, for this range of variation of ε , the quantity z_r is almost independent of the dust density, so that the five curves appear superposed.

The corresponding values of the imaginary part of the normalized frequency (z_i), are shown as a function of q in Fig. 1(b), for the same five values of ε considered for Fig. 1(a). As in the case of the real part z_r , the imaginary part appears to be insensitive to the

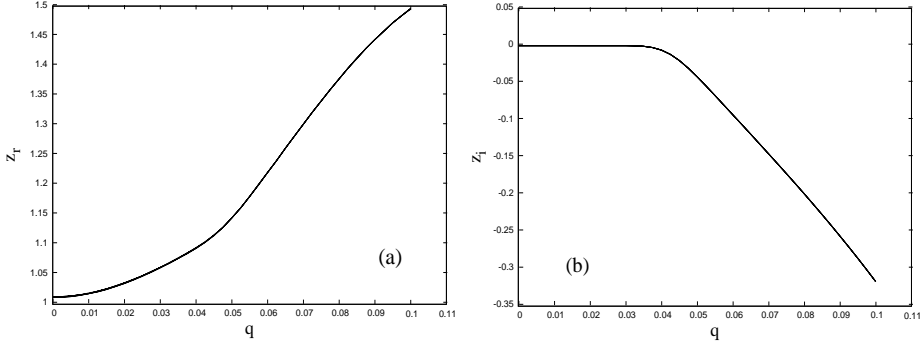


FIGURE 1. (a) Real part of the normalized frequency (z_r), as a function of q , for five values of ϵ , obtained from solving the dispersion relation given by equation (11). $B = 1.0 \times 10^{-4}$ T, $T_i = 1.0 \times 10^4$ K, $n_{i0} = 1.0 \times 10^9$ cm $^{-3}$, $Z_i = 1.0$, $m_i = m_p$, and $T_e = T_i$. Radius of dust particles, $a = 1.0 \times 10^{-4}$ cm. The dust density n_{d0} is obtained from $\epsilon = n_{d0}/n_{i0}$. The values of ϵ utilized are $\epsilon = 0.0, 2.5 \times 10^{-5}, 5.0 \times 10^{-5}, 7.5 \times 10^{-5}$, and 1.0×10^{-4} . For this range of variation of ϵ , the quantity z_r is almost independent of the dust density, so that the five curves appear superposed. (b) Imaginary part of the normalized frequency (z_i), as a function of q , for five values of ϵ , for the same parameters used for panel (a), and from the same dispersion relation. In the scale of the figure the five curves for the quantity z_i also appear to be superposed.

presence of dust. It is seen that large damping rates appear for $q \geq 0.03$, due to Landau damping by the electron population.

However, a closer look is more revealing. Fig. 2 shows an enlarged picture of the region of small q , from $q = 0$ up to $q \simeq 0.027$, where Landau damping starts to become significant. Fig. 2(a) shows that the real part of the frequency is indeed quite insensitive to the presence of the dust. However, the situation is different for the imaginary part z_i , seen at Fig. 2(b). The curve for $\epsilon = 0.0$ indeed shows vanishing damping rate for q up to nearly 0.025, where Landau damping starts to occur. In the presence of dust, however, it is seen that a damping rate of order 10^{-6} appears in the whole interval between $q = 0.0$ and $q \simeq 0.025$, approximately independent of q , and increasing linearly with the value of ϵ , in the range of values considered.

The subject of wave propagation and absorption in dusty plasmas is frequently treated in the literature with the approximation that neglects charge fluctuation phenomena, keeping charge imbalance between ions and electrons as the dominant effect associated to the presence of the dust. This approximation can be easily compared with our more complete approach which takes into account the charge fluctuations. In Fig. 3(a) and 3(b) we show respectively the values of z_r and z_i obtained from dispersion relation (11), but considering $\tilde{v}_\beta = 0$, for $\beta = e, i$. Other parameters are the same used for Fig. 2.

It is seen from Fig. 3(a) that a very small effect due to the dust density occurs in the value of z_r , similar to the effect seen in Fig. 2(a). The conclusion is that the small effect on the real part of the wave frequency occurs due to charge imbalance between ions and electrons. On the other hand, Fig. 3(b) shows absence of damping rates for q between 0.0 and 0.025, indicating that the damping rate for this range, depicted in Fig. 2, is entirely due to the charge fluctuations, and not to charge imbalance. This is an interesting novel

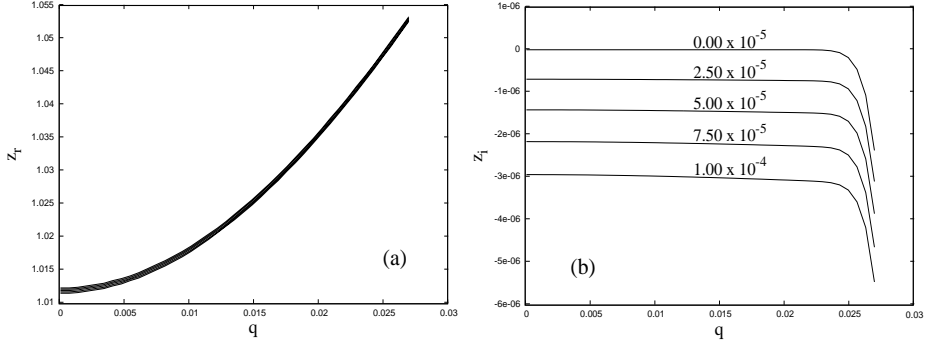


FIGURE 2. (a) Amplified vision of the real part of the normalized frequency (z_r), for the range of small values of q , for the same five values of ϵ used for Fig. 1. Other parameters are also the same as those used for Fig. 1. In the scale of this amplified vision, the quantity z_r shows very small effect of the dust density. (b) Amplified vision of the imaginary part of the normalized frequency (z_i), as a function of q , for the same five values of ϵ and the same parameters used for panel (a). In the scale of the figure the five curves for the quantity z_i appear well separated and clearly display the effect of the dust on the wave damping.

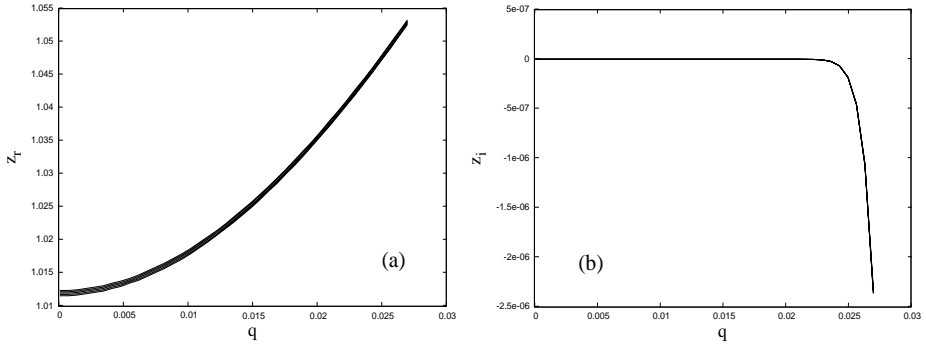


FIGURE 3. (a) Amplified vision of the real part of the normalized frequency (z_r), for the range of small values of q , obtained from dispersion relation (11), but neglecting the charge variation \tilde{v}_β and keeping the charge imbalance between ions and electrons as the sole effect of the presence of the dust. The figure has been obtained considering the same five values of ϵ used for Fig. 1. Other parameters are also the same as those used for Fig. 1. The effect of the dust on the quantity z_r is the same depicted by Fig. 2(a). (b) Corresponding values of the imaginary part of the normalized frequency (z_i), as a function of q , from the same approximated dispersion relation and for the same five values of ϵ and the same parameters used for panel (a). There is no damping of the wave for very small q , indicating that the damping obtained in Fig. 2(b) occurs due to charge variation, and that the charge imbalance alone does not produce meaningful effect on the wave damping of Langmuir waves.

result which we intend to explore more thoroughly in a forthcoming publication.

It is interesting to notice that our results show damping of Langmuir waves due to the presence of dust particles, based on the approximation utilized, equation (9), which takes into account only effects due to the dust included on the conventional part of the dielectric tensor (the term denoted as ϵ_{zz}^C), including some effects due to charge fluctuation. However, the analytical results, which are partially reproduced here, indicate

that there is yet another contribution due to the dust to the dielectric tensor, which occurs only due to charge fluctuation, and which has been neglected in the present approach. This novel contribution is the term denoted as ϵ_{zz}^N appearing in equation (8). In a future publication, along with a more complete parametric investigation of the effects of charge fluctuations hitherto investigated, we intend to examine the contribution of this new term.

CONCLUSIONS

In the present work, we have discussed the behavior of electrostatic waves, in particular the Langmuir waves, in a magnetized plasma whose constituents are ions, electrons and charged dust grains with variable electrostatic charge.

Our principal purpose was to analyse the importance and influence of the charge fluctuation of the dust particles in the propagation and damping of Langmuir waves. We have used a kinetic approach to obtain the dispersion relation and damping rates of these waves.

We have shown that the presence of dust particles with variable charge modifies the plasma properties and affects the propagation and the damping of Langmuir waves. In particular the charge fluctuation of the dust particles produces an additional damping of the Langmuir waves, which occurs in a region where conventional Landau damping doesn't occur.

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REFERENCES

1. J. S. Chang and K. Spariosu, *J. Phys. Soc. Japan*, **62**, 97, (1993).
2. J. E. Allen, *Phys. Scr.*, **45**, 497, (1992).
3. V. N. Tsytovich, *Phys. Usp.*, **40**, 53, (1997).
4. M. C. de Juli and R. S. Schneider, *J. Plasma Phys.*, **60**, 243, (1998).
5. M. C. de Juli and R. S. Schneider, *J. Plasma Phys.*, **64**, 57, (2000).
6. M.C. de Juli, R.S. Schneider, D. Falceta-Gonçalves, Jatenco-Pereira, V., *IEEE Transaction on Plasma Science*, textbf32, 542, (2004).
7. M. C. de Juli, R. S. Schneider, L. F. Ziebell, V. Jatenco-Pereira, *Electrostatic Waves in a Maxwellian Dusty Plasma with Variable Charge on Dust Particles*, in preparation.
8. B. D. Fried and S. D. Conte, *The Plasma Dispersion Function* (Academic Press, New York, 1961).
9. N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics* (McGraw-Hill, New York, 1973).