Front Form Approach to $q\bar{Q}$ Mesons with Harmonic Confinement

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Abstract. We investigate mesons as unequal mass $q\bar{Q}$ objects in the front form representation (equivalently, on the light cone (LC) or in the infinite momentum frame (IMF)). Our starting point is the manifestly covariant Bethe-Salpeter equation (BS) in the instant form, which we reduce to a 3-dimensional covariant LC equation by restricting the quark and sea antiquark system symmetrically off their mass shells. We discuss analytical solutions to the LC amplitude and compare our findings with current parameterizations of the distribution amplitude emphasizing the characteristic x-dependence for unequal quark masses.

INTRODUCTION

The common approach to hadronic two- and many-body systems is their formulation based either on the nonrelativistic many-body Schrödinger equation or the minimal relativistic extension towards the corresponding Dirac equation; in practical calculations, effective two-body forces and more or less severe mean-field approximations are employed to make the system tractable. The common framework for the many-body problem is the instant form: in the language of second quantization the commutators (anti-commutators) of the underlying field operators are quantized along the hyperplane with $t = const$, resulting in the standard commutation and anti-commutation relations. The advantage of this approach is the appearance of a single (unique) time-variable t for all interacting particles together with standard Feynmann contributions in perturbative expansions. As a characteristic feature the relation for a free particle $E = \pm\sqrt{\vec{p}+m^2}$ involves the intimate coupling of positive and negative energy states even without interaction i.e. the existence of a complicated "nontrivial" Dirac vacuum for fermions.

However, particularly with the advent of recent systematic experiments at high energy and/or momentum transfers, such as elastic or deep-inelastic hard scattering processes, a different frame for such processes is suggested[1]. As with increasing energy the bulk of scattering events are focussed on a small forward cone along the direction of the projectile (the beam), as characterized by $q_0^2, q_z^2 >> \vec{q}_\perp^2$, then, with the transversal momentum component being neglected, the dispersion relation is truncated to an (effectively) 2-dimensional form $q^2 = q_0^2 - q_z^2 - \vec{q}_\perp{}^2 \simeq (q_0 - q_z)(q_0 + q_z) = q_- q_+$: the

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physics becomes 2-dimensional (we exhibit the symmetry between q_0 and q_z by introducing quantities $q_\pm$ discussed below). Though rather simplistic, already this intuitive picture suggests a different system of reference for relativistic high energy processes. Of course, observables in any field theoretical calculation should be independent of the specific frame of quantization: the quantization along the light-like hyperplane $t+z=const$ is equally acceptable. In momentum space — as intuitively envisioned — this leads to a transition to LC variables $(p_0, p_z, \vec{p}_\perp) \to (p_-, p_+, \vec{p}_\perp)$ with a rotation of $\theta = \pi/4$ around the perpendicular $\vec{p}_\perp$ axis:

$$\begin{pmatrix} p_- \\ p_+ \\ \vec{p}_\perp \end{pmatrix} = \begin{pmatrix} \sqrt{2}cos(\pi/4) & -\sqrt{2}sin(\pi/4) & 0 \\ \sqrt{2}sin(\pi/4) & \sqrt{2}cos(\pi/4) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_0 \\ p_z \\ \vec{p}_\perp \end{pmatrix}, \tag{1}$$

ending up in a new representation of the dispersion relation of a free particle $p^2 = p_0^2 - p_z^2 - \vec{p}_\perp{}^2$. Representing p_- as the new light cone energy and p_+ as the longitudinal LC momentum, this new dispersion relation $p_- = \frac{m^2+\vec{p}_\perp{}^2}{p_+}$ develops a unique new feature: being single-valued in the energy variable p_-, it strictly decouples particles from antiparticles and thus the free (and, as can be shown, interacting) vacuum becomes "trivial"[2]. Immediate consequences are, as shown in the pioneering work of Dirac[3], the appearance of 7 (instead of 6) kinematical generators for the Poincaré group, specifically the kinematical nature of Lorentz-boosts, as already anticipated for the standard Lorentz transformations

$$\begin{matrix} p_0' = \gamma(p_0 + \beta p_z) \\ p_z' = \gamma(\beta p_0 + p_z) \end{matrix} \to p_\pm' = \gamma(1 \pm \beta) p_\pm \tag{2}$$

(with $\beta = v$ and $\gamma = 1/\sqrt{1-\beta^2}$; as equivalently seen from $p_z \to \infty$) in the IMF) or the suppression of pair-creation out of the vacuum (which, in turn allows an appropriate decomposition of the Fock-space respecting baryon number conservation within each Fock component). Finally the corresponding Feynman rules are recovered in the old-fashioned perturbation theory basically from the substitution of the LC energies[4]

$$p_0 = \sqrt{\vec{p}^2 + m^2} \to p_- = \frac{m^2 + \vec{p}_\perp{}^2}{x} \tag{3}$$

(when we introduce the momentum fraction $x = \frac{p_+}{P_+}$ with P_+ being the total momentum of the system).

Thus going to the LC we have recovered the intuitive picture sketched at the beginning for scattering processes at high energy and momentum transfers with an effectively 2-dimensional $p_\pm$-world. As expected, a price has to be paid for this simplification: breaking the a priori spherical symmetry in the transition to the LC variables, rotational invariance is lost; similarly, the implementation of parity conservation in the 3-dimensional Hamiltonian approach, upon integrating out the time component in the LC amplitude

$$\psi(x, \vec{p}_\perp) = \int dp_- \psi(x, \vec{p}_\perp, p_-), \tag{4}$$

evidently destroys reflection symmetry $p_- \stackrel{P}{\longleftrightarrow} p_+$, present in manifestly covariant formulations[5]. However, in spite of these deceases, getting the implementation of covariance under Lorentz boosts practically for free on the LC together with the trivial vacuum, renders the relativistic front form one of the most promising attempts to bound systems at large total 4-momenta.

In this note, we apply the covariant LC formalism to two body-systems in QCD. Quite explicitly, motivated by numerous studies of mesonic $q\bar{Q}$ systems in the recent literature, we investigate in this note the structure of mesons on the LC in the framework of Quasi-Potential (QP) equations[6]. As we are looking for analytical solutions for the corresponding LC wave-functions, we incorporate the genuine feature of QCD by a covariant scalar harmonic confinement potential (attempts in the instant form in this direction have been followed, for example, by Mitra and coworkers [7]). In view of recent activities at charm (C) and beauty (B) factories, we focus on momentum distributions both of $q\bar{q}$ mesons with equal masses and light-heavy $q\bar{Q}$ mesons with a light u, d quark and a heavy s, c, b anti-quark.

FROM THE BETHE-SALPETER TO THE QUASI-POTENTIAL EQUATION FOR THE $Q\bar{Q}$ SYSTEM ON THE LIGHT CONE

Our starting point for a quark q (with mass m_1 and momentum p_1) and an anti-quark $\bar{Q}$ (with mass m_2 and momentum p_2) is the manifestly covariant BSE in the instant form[8]

$$\begin{aligned}\Psi(p_1,p_2) &= (\not{p}+m)_{(1)}(\not{p}+m)_{(2)}G(p_1,p_2)\\ &\times\int K(p_1,p_2,k_1,k_2)\Psi(k_1,k_2)\delta(k_1+k_2-p_1-p_2)dk_1dk_2 \qquad (5)\end{aligned}$$

with the two-particle propagator

$$G(p_1,p_2)=(D_1(p_1)D_2(p_2))^{-1}=((p_1^2-m_1^2-i\varepsilon)(p_2^2-m_2^2-i\varepsilon))^{-1}. \qquad (6)$$

It is clear that the transition of the 4-dimensional BSE from the instant form to the LC is merely a formal change in the explicit representation, without modifying its physical content. Both from conceptional and practical reasons (such as to provide for example an appropriate one-body limit to the Dirac equation for phenomenological kernel, a feature which is in general not exhibited by the BS equation with truncated kernels, such as in the ladder approximation[9]) we integrate out the relative p_- component, by restricting the corresponding retardation in a covariant way. As a consequence the resulting equation is still covariant and avoids the conventional static (Salpeter) limit.

We eliminate the dependence of eq. (5) on the relative energy variable and convert the BSE into a QPE by projecting the propagation of the $q\bar{Q}$ system on the mass shell as [10]

$$G(p,P)=i\pi\frac{\delta_+(\eta D_1(p,P)-(1-\eta)D_2(p,P))}{D_1(p,P)+D_2(p,P)}, \qquad (7)$$

with the η parameter restricted to the interval $[0,1]$; above we denote the total and relative momenta for the $q\bar{Q}$ system by p and P, respectively; furthermore in eq. (7) we keep only the positive energy pole $(+)$ in the δ function. We remark that the observables calculated should be independent of η.

Even after integrating out the dependence on the internal energy variable, the resulting equations is in its full structure involve a coupled system of 16 differential equations due to the spin of the interacting fermions and particle - anti-particle mixing, schematically

$$\psi(p_1,p_2) = \begin{pmatrix} \psi_{++} \\ \psi_{+-} \\ \psi_{-+} \\ \psi_{--} \end{pmatrix} |[j_1 j_2] J J_3 > . \tag{8}$$

Both in the instant and front form we reduce this complex system to a single radial equation by an appropriate simplification of the spin structure of the system

$$\Psi(p,P)_{\lambda\lambda' J J_3} = R(\lambda\lambda' J J_3)\Phi(p,P); \tag{9}$$

where in an helicity representation $R(\lambda\lambda' J J_3)$ is determined in the Mock representation (assuming no interaction in the spin sector [11]), or on the LC by a free Melosh rotation from the $q\bar{Q}$ rest system to the IMF [12]. Without further details we mention that we are currently exploring the detailed spin-structure of the LC by employing a one-rank separable interaction kernel, which allows to reduce the complex system of coupled differential equations to a set of 16 ordinary algebraic equations.

Upon elimination of the complexity of the spin dependence the covariant nature of the BSE allows an immediate transition to LC variables[2-5]

$$\begin{aligned} P &= (P_-, P_+, \underline{0}); P_- P_+ = M^2; P_+ = 1; \\ p &= (p, x, p_\perp); x_i = \frac{p_i^+}{P^+}; x_1 + x_2 = 1; \end{aligned} \tag{10}$$

The resulting radial QPE for $\Phi(x, p_\perp, M^2)$ on the LC is then obtained with the appropriate LC projection for the two-particle Green's function [13]

$$G(P_-; x, \underline{p}_\perp, M^2) \sim \delta(p_- - \frac{(1-\eta)(1-x)M^2 + \eta m_{1\perp}^2 - (1-\eta) m_{2\perp}^2}{1-\eta+(2\eta-1)x}), \tag{11}$$

which leads to

$$(M^2 - \frac{m_{1\perp}^2}{x} - \frac{m_{2\perp}^2}{(1-x)})\Phi(x, \underline{p}_\perp; M^2) = \int K(x, \underline{p}_\perp, y, \underline{k}_\perp)\Phi(y, \underline{k}_\perp, M^2) dy d\underline{k}_\perp, \tag{12}$$

with M^2 being the invariant squared $q\bar{Q}$ mass and $m_{i\perp}^2 = m_i^2 + \vec{p}_\perp^{\,2}$ (we absorb all irrelevant factors from the projection of the Green's function in the kernel K).

For a given interaction kernel the solution of the equations above directly results in the light cone distribution of $q\bar{Q}$ system. As so far a detailed derivation of the LC amplitude from QCD is till lacking (only results in the large $Q^2 \to \infty$ limit or estimates from QCD sum rules exist [2, 14]), the main two goals of this short note are very modest: on the one side we would like to use a kernel which can be derived explicitly from the corresponding covariant representation in the instant form and we aim for an analytical solution for the momentum distribution. A QCD inspired kernel, which includes confinement as the unique property of QCD, is a covariant harmonic confining kernel

$$K(Z^2) \sim Z_\mu Z^\mu ; \tag{13}$$

within yields, in momentum space, in the instant form

$$K(p,\kappa) = \frac{1}{g^2}\frac{\partial^2}{\partial p^2}\delta(p-\kappa) \tag{14}$$

or, equivalently, in LC coordinates

$$K(p_-,x,\vec{p}_\perp;k_-,y,\vec{k}_\perp) = \frac{1}{g^2}\left(\frac{\partial^2}{\partial p_-\partial x} - \frac{\partial^2}{\partial \vec{p}_\perp^{\,2}}\right)\delta(p_- - k_-)\delta(x-y)\delta(\vec{p}_\perp - \vec{k}_\perp) \tag{15}$$

(the strength parameter g is related to the running quark-gluon coupling constant; its value is fixed in the non-relativistic limit from meson spectroscopy). Following the same steps for the reduction to the QPE as sketched above, we obtain explicitly

$$(M^2 - \frac{m^2_{1\perp}}{x} - \frac{m^2_{2\perp}}{(1-x)})\Phi(x,\underline{p}_\perp;M^2) = \frac{1}{g^2}(\frac{\partial}{\partial p_-}\frac{\partial}{\partial x} + \frac{\partial^2}{\partial \underline{p}_\perp^2})\Phi(x,\underline{p}_\perp,M^2)\,, \tag{16}$$

where the projection on the relative energy variable fixes p_- from eq. (11). Upon eliminating p_- in the last equation with

$$\frac{\partial}{\partial p_-} = \left(\frac{\partial x}{\partial p_-}\right)_{\vec{p}_\perp = const}\frac{\partial}{\partial x} + \left(\frac{\partial \vec{p}_\perp}{\partial p_-}\right)_{x=const}\frac{\partial}{\partial \vec{p}_\perp}, \tag{17}$$

from equation (16) we obtain for an arbitrary parameter η the explicit equation for $\phi_\eta(x,\vec{p}_\perp;M^2)$:

$$\begin{aligned}
&(M^2 - \frac{m^2_{1\perp}}{x} - \frac{m^2_{2\perp}}{1-x})\phi_\eta(x,\vec{p}_\perp;M^2) = \\
&\frac{1}{g^2}\left(\frac{(1-\eta+(2\eta-1)x)^2}{(\eta(\eta-1)M^2+(1-2\eta)(\eta m^2_{1\perp}-(1-\eta)m^2_{2\perp})}\frac{\partial^2}{\partial x^2}\right. \\
&\left.\frac{(1-\eta+(2\eta-1)x)}{2(2\eta-1)}\frac{1}{\vec{p}_\perp}\frac{\partial}{\partial \vec{p}_\perp}\frac{\partial}{\partial x} + \frac{\partial^2}{\partial \vec{p}_\perp^{\,2}}\right)\phi_\eta(x,\vec{p}_\perp;M^2)
\end{aligned} \tag{18}$$

As expected, for arbitrary values of η, the longitudinal and the transversal components of the light cone amplitude are intimately coupled both through the effective masses $m^2_{i\perp} = m_i^2 + \vec{p}_\perp^{\,2}$ in the coefficient proportional to $\partial^2/\partial x^2$ and through the mixed derivative $\left(\vec{p}_\perp \partial/\partial \vec{p}_\perp\right)\frac{\partial}{\partial x}$.

In its most general form, i.e. for an arbitrary η, equation (18) admits only numerical solutions. As we in this note we are interested in analytically solvable equations, we explore the range of η to separate longitudinal and transversal components. Going back to equation (18) we immediately identify the two appropriate limits for η:

- $\eta = 1/2$ (the Blankenblecker-Sugar BBS projection [15]), yielding $p_- = (1-\eta)M^2 + \frac{1}{2}(m_1^2 - m_2^2)$ and
- $\eta = 0$ (the Gross projection [16]) in the limit $m_2 = m_Q >> m_1 = m_q$ giving $p_- = M^2 - \frac{m_{2\perp}^2}{1-x}$ for $M^2 \sim m_2^2 \to \infty, m_{2\perp}^2 \to M^2$).

Both projections stress different physics for the off-shell propagation of the bound particles; $\eta = 1/2$ restricts the propagation of the particles symmetrically off-shell and is thus a natural choice for interacting particles with equal mass; $\eta = 0$ puts the heavy particle on its mass shell and enforces by this restriction the appropriate one-body limes (resulting in a single Dirac equation for the light particle in the limit $m_2 \to \infty$; this limit bears a strong relation to the heavy quark effective theory[17]).

In the following we focus on the BBS solution for the general case of unequal masses. We mention that with $\eta = 1/2$, but $m_q \neq m_Q$, p_- is only shifted by the difference of the squared masses and does not effect the radial solution; only the spin dependence, which we do not consider here, in detail, is modified.

The next steps are now well defined. The radial LC equation (18) in the BBS limit is explicitly given as

$$\left[\left(\frac{1}{M^2}\frac{\partial^2}{\partial x^2} + g^2 x(1-x)M^2\right) + \left(\frac{\partial^2}{\partial \vec{p}_\perp^2} - g^2\vec{p}_\perp^2\right) - g^2 m_q^2\right]\Phi_{BBS}(x,\vec{p}_\perp,M^2) \tag{19}$$

As the longitudinal and the transversal components factorize we obtain with the ansatz (we drop the index BBS for simplicity)

$$\phi(x,\vec{p}_\perp,M^2) = \phi(x,M^2)\varphi(\vec{p}_\perp,M^2)\,, \tag{20}$$

$$\left[\left(\frac{1}{M^2}\frac{\partial^2}{\partial x^2} + g^2 x(1-x)M^2 - E^2\right)\right]\Phi(x,M^2) = 0\,, \tag{21}$$

$$\left(\frac{\partial^2}{\partial \vec{p}_\perp^2} - g^2\vec{p}_\perp^2 - \varepsilon^2\right)\varphi(\vec{p}_\perp,M^2) = 0\,, \tag{22}$$

and $E^2 + \varepsilon^2 = g^2 m_q^2$.

The explicit solution of the differential equation in the BBS limit is given as[18]

$$\varphi(\vec{p}_\perp,M^2) = H_{nx}(p_x)H_{ny}(p_y)e^{-\frac{a^2}{2}\vec{p}_\perp^2} \tag{23}$$

and

$$\Phi(x,M^2) = e^{-\frac{1}{2}\lambda x^2}\left(A\,{}_1F_1(a,\frac{1}{2};\lambda x^2) + B(\lambda x^2)^{\frac{1}{2}}\,{}_1F_1(a+\frac{1}{2},\frac{3}{2};\lambda x^2)\right)\,, \tag{24}$$

with

$$a \equiv \frac{1}{4} - \frac{k^2}{4\lambda} = \frac{1}{4} - \frac{M^2(\frac{gM^2}{4} - E^2)}{4g^2M^4}\,, \tag{25}$$

with $-1/2 \leq x \leq 1/2$. In these expressions, the functions $F(a,b,z)$ denote confluent hypergeometrical functions, $H_{ni}(p_i)$ are Hermite polynomials. The eigenvalues for the invariant mass M^2 , i. e. the mesonic spectrum, are then defined by the eigenvalue conditions

$$\Phi(x=-1/2,M^2) = \Phi(x=1/2,M^2) = 0 \tag{26}$$

Together with the normalization condition $1 = \frac{1}{2}\int dxd\vec{p}_\perp|\Phi(x,\vec{p}_\perp,M^2)|^2$ this fixes the LC wave function completely as a function of the strength g of the harmonic kernel.

We compare our derivation with current parameterizations in the literature, where various simple analytical forms can be found[19]. In order to exhibit the influence of unequal quark masses we compare with two typical momentum distributions (for the variable x shifted to $\rightarrow x+1/2$)

- the Lepage-Brodsky parameterization [20]

$$\Phi(x,\underline{p}_\perp,M^2) = Nexp\left\{-\frac{Q^2}{2}\left(\frac{m_q^2+\vec{p}_\perp^2}{x}+\frac{m_Q^2+\vec{p}_\perp^2}{1-x}\right)\right\}; \tag{27}$$

- the parameterization as a simple power law [21]

$$\Phi(x,M^2) = Nx^{\frac{m_q}{m_Q}}(1-x)^{\frac{m_Q}{m_q}} \tag{28}$$

(we extended the standard equal-mass parameterization to unequal quark masses). Insight into the x-dependence of the BBS solution, particularly for unequal particle masses, is obtained from a comparison of the invariant longitudinal distribution amplitude (IDA) [2, 21],

$$\Phi(x,M^2) = \int d\vec{p}_\perp|\Phi(x,\vec{p}_\perp,M^2)|\Theta(\Lambda^2-\vec{p}_\perp^2). \tag{29}$$

(for the parameterizations compared the transversal cut-off at a scale Λ affects only the overall normalization of the distribution amplitude; due to the Gaussian form the contribution from the perpendicular components is always finite, even without a cut-off).

Exceeding the limited scope of this brief note, a detailed discussion of our results is presented in a forthcoming publication. Here we stress only various characteristic features of the invariant amplitudes derived. As a general trend we find, that the various current parameterizations in the literature show similar gross features, but differ mainly around the end points $x \rightarrow 0$ and $x \rightarrow 1$ and in the width of the distribution around the maximum $x_{max} = m_q/m_Q$. Furthermore we confirm a systematic shift of the dominant momentum components towards $x=0$ for $q\bar{Q}$ mesons with increasing heavy quark mass, a trend which is expected qualitatively from a minimization of the kinetic energy in the LC Hamiltonian:

$$\frac{m_1^2}{x_{max}} - \frac{m_2^2}{1-x_{max}} \simeq 0 \tag{30}$$

yielding $x_{max} \sim m_1^2/m_2^2$ for $m_2^2 >> m_1^2$[22].

Let us summarize our short contribution. In this note we investigate the momentum distribution of mesons with unequal quark masses on the light cone. Without considering spin effects in detail, the radial dependence was derived in a covariant fashion starting from the BSE, by projecting it in a still covariant and physically motivated way onto the light cone. Rigorous analytical solutions were derived and compared for a covariant confining kernel for the BBS constraint of the relative energy variable of the $q\bar{Q}$ system.

We find, that our results reproduce qualitatively the trends of purely phenomenological parameterizations of LC distribution amplitudes. In detail, however, particularly in the endpoint behaviour both in the longitudinal and in the transversal components we find serious quantitative differences due to the implementation of our boundary conditions. Though very simple, our model shows some advantages. First of all it starts from a covariant interaction kernel, which can be easily extended to more realistic cases (for example, to parameterize the influence of the one-gluon exchange allowing still an analytical solution). Furthermore, the transition from the BSE to QPE allows a direct transition from the instant form to the front form for arbitrary systems, such as to two very heavy quarks in the non-relativistic limit.

Of course, the rather selective ideas presented here yield only little insight in the details of the model. Presently we are extending our approach (including additional elements of the $q\bar{Q}$ interaction and comparing different recipes for the $4 \to 3$-dim reduction); beyond that we test the model for other quantities, such as for moments of the distribution functions, decay constants and form factors [23].

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