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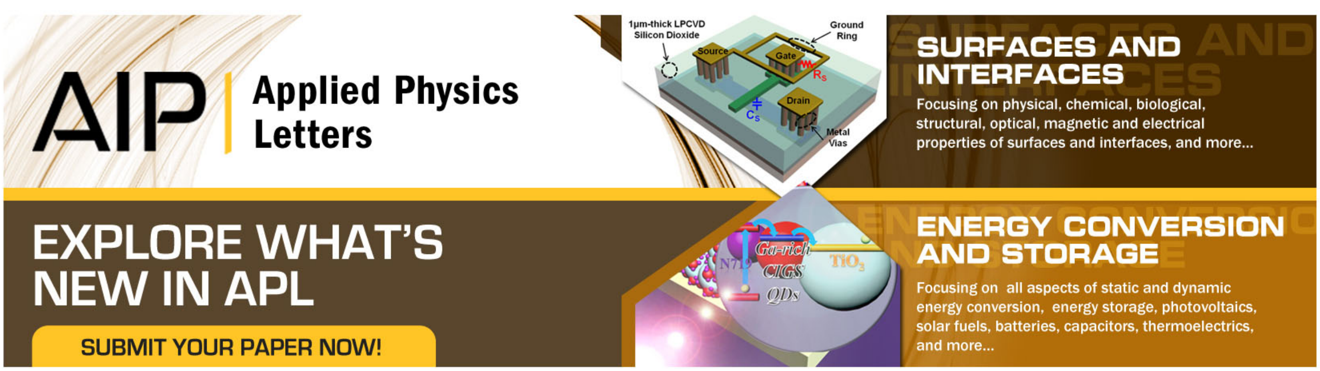

Nonlinear dynamics of inhomogeneous mismatched charged particle beams

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This work analyzes the transversal dynamics of an inhomogeneous and mismatched charged particle beam. The beam is azimuthally symmetric, initially cold, and evolves in a linear channel permeated by an external constant magnetic field. Based on a Lagrangian approach, a low-dimensional model for the description of the beam dynamics has been obtained. The small set of nonlinear dynamical equations provided results that are in reasonable agreement with that ones observed in full self-consistent N-particle beam numerical simulations. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4746395]

It is well-known that completely homogeneous beams evolve inside the magnetic focusing structures with no emittance growth.[1] The beam constituents cannot individually couple and then be excited by macroscopic oscillations such as that introduced by the initial mismatch of the envelope. As a consequence, the orbit of each particle stays limited to the beam boundary established by the envelope, for any given time.[2] In this situation, the halo is not formed and thence the beam emittance, which is a macroscopic quantity associated with the beam heating, remains unchanged.[3,4]

However, this is not the case of initially inhomogeneous beams. In such systems, particles can be expelled from the beam core due to the breaking of density waves.[5–7] For beams with matched envelopes, the ejected particles, although not anymore inside, stay outside but around and close to the beam limits.[8,9] However, if the beam envelope is also mismatched, these ejected particles can be captured and driven by large dynamical resonant islands.[2,10,11] A rarefied but extended population of particles – a halo – is thus developed around the remaining particles – the core – of the beam, directing the system to its equilibrium. In fact, halo formation is a macroscopic transcription of microscopic instabilities acting inside the beam over its constituent particles. The halo formation arises as a mechanism of stabilization, which conducts the beam to its equilibrium.

Although for cold and regular inhomogeneous beams the time scale of halo formation can be adequately predicted by the time with which the first density wave breaks,[5] for quasi-homogeneous cold beams, or for homogeneous thermal beams, this is not possible, since the results diverges from the ones obtained from numerical simulations. Particle jets are pretty less prominent and particles, instead of leaving the beam in large groups, leave the beam almost individually, one by one. In this situation, for an accurate description of quantities associated with halo formation, it is interesting to describe the interaction between particles and the whole beam.[11,12] And for a viable modeling, it is interesting to develop an analytical description for the beam dynamics with, as minor as possible, degrees of freedom.

Previous works have investigated individually the role of the initial envelope mismatch and the role of the initial magnitude of inhomogeneity in the beam transport inside the confining channel. However, it is clear that in real implemented beams both exist together. It is almost impossible inject the beam inside the accelerator structure with a flat-top density of particles and without any even small envelope mismatch. In this sense, the main purpose of this work is to consider concomitantly the effects of both, extending the previous developed Lagrangian description to account further the envelope mismatch. A more accurate description of real beams is expected.

The system of interest consists of an initially inhomogeneous and continuous beam of charged particles evolving inside a linear channel. The beam is surrounded by a cylindric conducting pipe and focused by a constant magnetic field aligned to the pipe symmetry axis. All beam particles propagates with constant longitudinal velocity $\dot{z}$. z is the axis along the propagation direction, while x and y axes pertain to the beam transversal section.

The inhomogeinity of the (transversal) beam density of particles is chosen to follow a parabolic shape

$$n_b = \begin{cases} \dfrac{N_b}{\pi r_b^2}\left[1+\eta\left(\dfrac{2R^2}{r_b^2}-1\right)\right], & \text{if } 0 \le R \le r_b \\ 0, & \text{if } r_b < R \le r_w, \end{cases} \quad (1)$$

from which can be readily observes that also azimuthal symmetry has been considered. The transversal beam section is explored by the radial coordinate R. N_b is the number of beam particles per unit axial length. The quantity r_b and the parameter η refer, respectively, to the beam envelope and inhomogeneity. If $\eta = 0$, the beam is perfectly homogeneous. The quantity s although not time, scales with it, in the sense of $s = s_0 + \dot{z}t$. Informally, the dependence of s thus means dynamics. r_w is the radial position of the conducting pipe. It was adopted the Gaussian unit system. In this work, the dot $\dot{}$ over any quantity represents its time derivative.

Notwithstanding the absence of explicit time dependence of n_b in Eq. (1), the dependence of s can be introduced indirectly through r_b and η. In fact, both are naturally

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functions of time, since envelope oscillations and charge redistribution processes normally exist in mismatched inhomogeneous beams. In this way, the first step to develop the intended model is to consider that $r_b = r_b(s)$ and $\eta = \eta(s)$, which imposes the beam density n_b to implicitly depend of coordinate s.

To derive the dynamical equations for r_b and η, one strategy is to obtain the Lagrangian L associated to the – beam – transversal section, which can be expressed as

$$L = \int_0^{r_b(s)} \mathcal{L} n_b(\mathbf{R}_0) d\mathbf{R}_0, \tag{2}$$

being $\mathbf{R}_0$ the position vector and

$$\mathcal{L} = \frac{1}{2}V^2 - \frac{1}{2}\kappa_{z0}R^2 + Q(R_0)\ln(R) \tag{3}$$

the transversal Lagrangian of a – beam ring – initially positioned at the coordinate $R(s=0) = R_0$. κ_{z0} is the coefficient of magnetic focusing, which is a constant.

The Eq. (3) contains essentially two other quantities that have to be determined. One is the beam velocity profile V, which accounts how each beam ring evolves with the time s. The other is the dimensionless fraction of charge $Q(R_0)$, which specifies the charge trapped by a Gauss surface at $R = R_0$. Since the beam under inspection is inhomogeneous, it seems clear that both should be functions of radial coordinate R.

By the use of the continuity equation, one can determine the beam velocity profile V

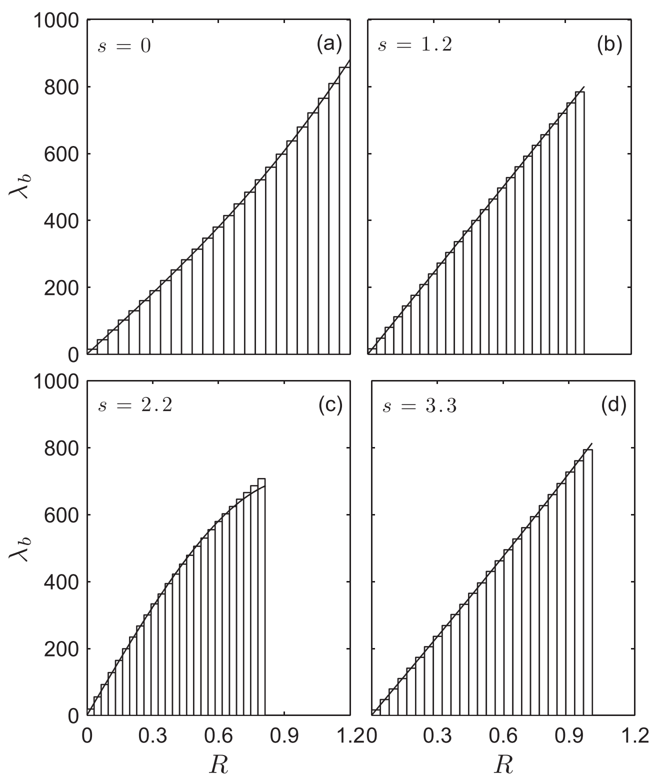

FIG. 1. Dynamics of the linear beam density λ_b for an initially 20% mismatched beam. The solid lines and bars refer, respectively, to the model and numerical simulations.

$$V = -\frac{1}{2\pi R n_b}\int \frac{\partial}{\partial s} n_b d\mathbf{R}. \tag{4}$$

The dimensionless fraction of charge $Q(R_o)$ is given by

$$Q(R_0) = -\frac{K_b}{N_b}\int_0^{\mathbf{R}_0} n_b(\mathbf{R}_0')d\mathbf{R}_0'. \tag{5}$$

Inserting the initial beam density of Eq. (1) into Eqs. (4) and (5), one respectively obtains for the beam velocity profile

$$V(R) = \frac{1}{2}\frac{(r_b\dot{\eta} - 4\eta\dot{r}_b)R^3 + (-2r_b^2\dot{r}_b - r_b^3\dot{\eta} + 2r_b^2\eta\dot{r}_b)R}{-r_b^3 - 2\eta r_b R^2 + r_b^3\eta}; \tag{6}$$

and for the fraction of charge,

$$Q(R_0) = \eta\frac{R_0^4}{r_b^4} + (1-\eta)\frac{R_0^2}{r_b^2}. \tag{7}$$

With Eqs. (4) and (5), the transversal Lagrangian $\mathcal{L}$ of Eq. (3) for the beam rings is now completely determined. As consequence, the Eq. (2) can be readily integrated. Proceeding in this way, it is found

$$\begin{aligned}L &= \frac{1}{2}\ln(r_b) + \frac{1}{4}(-r_b^2 + \dot{r}_b^2) - \frac{1}{8} + \frac{1}{12}(\eta + \eta\dot{r}_b^2 - r_b^2\eta + r_b\dot{\eta}\dot{r}_b) \\ &\quad - \frac{1}{48}\eta^2 + \frac{1}{64}\frac{r_b^2\dot{\eta}^2}{\eta^2}\left(1 + \frac{1}{\eta}\right) - \frac{1}{96}\frac{r_b^2\dot{\eta}^2}{\eta} \\ &\quad + \frac{1}{128}\frac{r_b^2\dot{\eta}^2}{\eta}\left[\ln(\eta+1)\left(1 + \frac{1}{\eta} - \frac{1}{\eta^2} - \frac{1}{\eta^3}\right)\right. \\ &\quad \left. + \ln(1-\eta)\left(-1 - \frac{1}{\eta} + \frac{1}{\eta^2} + \frac{1}{\eta^3}\right)\right].\end{aligned} \tag{8}$$

Observe that L is a function of the magnitude of the inhomogeneity η, of the beam envelope r_b and of its derivatives. With the aid of the corresponding Euler-Lagrange equations, it is possible to obtain second-order nonlinear ordinary differential equation (ODEs) for r_b and η, which have, respectively, the form of $\ddot{r}_b = F_{r_b}(r_b, \dot{r}_b, \eta, \dot{\eta})$ and $\ddot{\eta} = F_\eta(r_b, \dot{r}_b, \eta, \dot{\eta})$. The expressions for F_{r_b} and F_η have been omitted for compactness, being their derivation a result of straightforward algebra.

As stated by Eq. (1), the superficial beam density n_b depends of just r_b and η. To know how n_b evolves with the time s, one has just to know the dynamics of r_b and η, which are the only free parameters of the modeling. The beam density n_b is thus governed by r_b and η, which are functions that can be obtained by the direct numerical integration of the dynamical equations for $\ddot{r}_b$ and $\ddot{\eta}$.

Since a model for the dynamical behavior of the density n_b has been developed, many macroscopic beam quantities can be then promptly calculated. And between the many possible, one of them is the emittance. Emittance has great interest in beam physics, once it contains information about how much kinetic energy in average the beam earns. Essentially, the emittance ϵ is defined in the form $\epsilon \equiv \sqrt{4(\langle \mathbf{V}^2\rangle\langle \mathbf{R}^2\rangle - \langle \mathbf{R}\cdot\mathbf{V}\rangle^2)}$, in which the angle brackets $\langle\rangle$ denotes phase space average. $\mathbf{R}$ and $\mathbf{V}$ are, respectively,

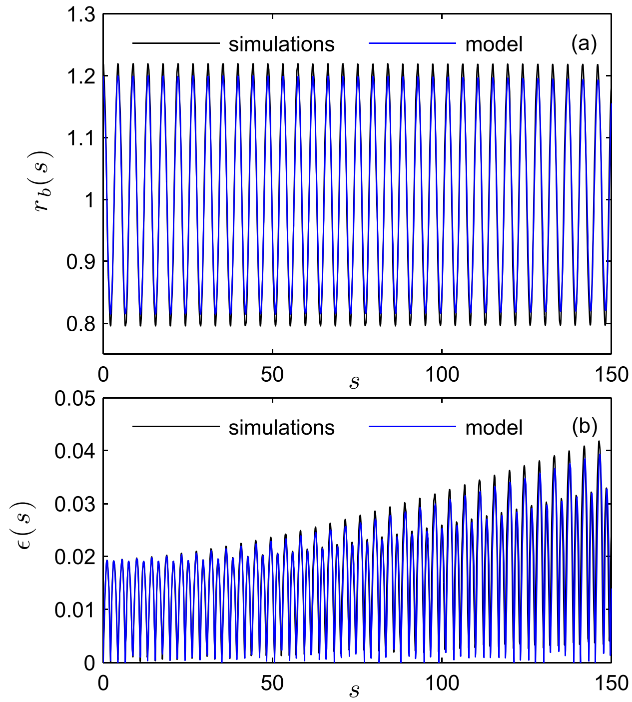

FIG. 2. Comparison between the model and the numerical simulations for (a) the envelope r_b and (b) the emittance ϵ.

the vector position and the velocity of each beam particle in a given time s.

While in breathing homogeneous beams, the emittance is a constant, in the current case it is not. Because r_b and η are functions of s, the beam emittance ϵ is also expected to be dependent of the time s. From the previously mentioned definition of emittance and the beam density n_b, the following ODE is obtained for ϵ:

$$\frac{d}{ds}\epsilon^2 = -\frac{1}{18} r_b^2 \eta(\eta+1)\dot{\eta}, \quad (9)$$

which is a function of the degrees of freedom of the proposed model, r_b and η.

The results provided by the model are confronted with the numerical simulations in Figure 1, for the linear beam density λ_b, and in Figure 2, for the envelope r_b and emittance ϵ, considering $r_b(s=0)=1.2$ and $\eta(s=0)=0.1$. The comparison occurs for the linear beam density λ_b because the numerical simulations histograms output particles/length instead of particles/area. But both are directly related through $\lambda_b = 2\pi R n_b$. The beam envelope is initially matched if $r_b(s=0) \equiv r_{beq} = 1$. Inspecting both figures, it is found a good correspondence. Reasonable agreement has been also found between the model and the numerical simulations for other values of $r_b(s=0)$ and $\eta(s=0)$. Numerical simulations are based on Gauss law[2] and employed $N_b = 10\,000$ macroparticles.

Since the low-dimensional model for the beam dynamics is in satisfactory accordance with the numerical simulations, it is then possible to describe the interaction of individual particles with the whole beam through test-charge methods. Simulating halo particles as test-charge ones will improve the description of the beam for long times. This will be the subject of future works.

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