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# Effects of dust-charge fluctuation on the damping of Alfvén waves in dusty plasmas

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Using a completely kinetic description to analyze wave propagation in dusty plasmas, the case of propagation of waves exactly parallel to the external magnetic field and Maxwellian distributions for electrons and ions in the equilibrium is considered. A model for the charging process of dust particles which depends on the frequency of inelastic collisions between dust particles and electrons and ions is used. The dispersion relation and damping rates for Alfvén waves are obtained. For the numerical solutions, the average value of the inelastic collision frequency is used as an approximation. The results show that the presence of dust particles with variable charge in the plasma produces significant additional damping of the Alfvén wave. A novel process of mode coupling of low-frequency waves is demonstrated to occur due to the presence of dust particles. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1899647]

## I. INTRODUCTION

Alfvén and magnetoacoustic wave propagation in a dusty plasma has been investigated previously by a number of authors. Some of them[1–4] focused on the ultralow-frequency waves, with frequencies much below the dust-cyclotron frequency. These waves are affected by the dynamic of the dust grains. Others authors considered the circularly polarized electromagnetic waves propagating parallel to the magnetic field in a plasma with static dust grains, in particular, the case of frequencies much below the ion-cyclotron frequency.[5–7]

More specifically, Pillip *et al.* showed that even if the fraction of negative charge on the dust grains is quite small compared to that carried by free electrons (typically $\simeq 10^{-4}$ in interstellar clouds), it can have a large effect on hydromagnetic Alfvén waves propagating at frequencies well below the ion-cyclotron frequency.[1] Mendis and Rosenberg,[5] and Shukla[7] showed that the dispersion properties of Alfvén and magnetoacoustic waves in a dusty plasma at frequencies below the ion-cyclotron frequency, but well above the dust-cyclotron frequency, are modified when a fraction of the negative charge is on the dust grains.

Salimullah *et al.*, using a kinetic description, have studied the circularly polarized electromagnetic waves propagating along homogeneous magnetic field in a dusty plasma.[8] They have found that in addition to the usual Landau damping, a novel mechanism of damping of the Alfvén wave, due to the correlation between dust particles, comes into play. However, the dust-charge fluctuation is not considered. Also, Salimullah and Rosenberg investigated the modification of the damping mechanism for kinetic Alfvén waves originated by the misbalance of electron and ion densities, for the case in which the dust particles may capture a great amount of electrons.[9]

Kotsarenko *et al.* have studied low-frequency kinetic Alfvén waves in a dusty plasma using a fluid analysis, which does not include Landau damping.[10] Das *et al.* have considered kinetic Alfvén waves in the frequency regime below the dust-cyclotron frequency, in a plasma with magnetized massive dust grains, and have studied damping due to charge fluctuations.[11] Also using a fluid model, Shukla and Rahman investigated shear Alfvén waves and other low-frequency electromagnetic waves in nonuniform dusty magnetoplasmas.[12]

In addition, low frequency, long wavelength Alfvén waves in multibeam dusty plasmas, with application to comets and planetary rings, have been considered.[13] Also effects of dust on absorption of Alfvén waves in tokamak edge plasmas has been discussed.[14] Alam *et al.*, using a multifluid theory, considered the growth rates of propagating waves in a magnetized dusty plasma in the presence of dust-charge fluctuation and of a weak magnetic field.[15] Another example to be mentioned is that dust-charge fluctuations as well as the dynamics of the dust charging process can lead to the appearance of an imaginary part in the dispersion equation for ion-sound waves,[16] leading to a damping of the wave in addition to the Landau damping.

The conclusion to be drawn is that the presence of dust particles can lead to the modification of the usual Landau

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damping of low-frequency waves, or to the appearance of additional damping mechanisms for these waves. It is therefore important to take into account the presence of dust when discussing the dispersion relation for low-frequency waves. However, to the best of our knowledge, Alfvén waves in a magnetized dusty plasma with variable charge on the dust particles have not previously been considered in the context of the kinetic theory. The point is that in kinetic theory the distribution function of electrons and ions satisfies an equation with a collision term with dust particles, which assures the possibility of variation of the charge of the dust particles. In this paper we will consider this question in the study of the Alfvén wave propagation in a dusty plasma.

The structure of the paper is as follows. In Sec. II we discuss the model used to describe the dusty plasma. In Sec. III we present the dielectric tensor for wave propagation exactly parallel to the external magnetic field, derived assuming Maxwellian distributions for the electrons and ions in the equilibrium. In Sec. IV the dispersion relation for Alfvén waves is obtained, the inelastic collision frequency is modeled, and some limiting cases to this dispersion relation are discussed, in order to clarify the numerical procedure used. In Sec. V the numerical results for the damping of Alfvén waves modified by the dust are presented and discussed. The conclusions are presented in Sec. VI. In the Appendix, we give the general expression for the dielectric tensor of a homogeneous magnetized dusty plasma, fully ionized, with dust particles assumed to be identical, immobile, and with variable charge.

## II. THE DUSTY PLASMA MODEL

We consider a plasma in a homogeneous external magnetic field $\mathbf{B_0}=B_0\mathbf{e_z}$. In this magnetized plasma we take into account the presence of spherical dust grains with constant radius $a$ and variable charge $q_d$; this charge originates from inelastic collisions between the dust particles and particles of species $\beta$ (electrons and ions) with charge $q_\beta$ and mass $m_\beta$. For simplicity, we will consider simply charged ions.

The charging model for the dust particles must in principle take into account the presence of an external magnetic field. This field must influence the characteristics of charging of the dust particles, because the path described by electrons and ions is modified: in this case we have cyclotron motion of electrons and ions around the magnetic field lines. However, it has been shown by Chang and Spariosu, through numerical calculation, that for $a\ll\rho_G$, where $\rho_G=(\pi/2)^{1/2}r_{Le}$ and $r_{Le}$ is the electron Larmor radius, the effect of the magnetic field on the charging of the dust particles can be neglected.[17] For the values of parameters used in the present work the relation $a\ll\rho_G$ is always satisfied.

We will consider the dust grain charging process to occur by the capture of plasma electrons and ions during inelastic collisions between these particles and the dust particles. Since the electron thermal speed is much larger than the ion thermal speed, the dust charge will be preferentially negative. As a cross section for the charging process of the dust particles, we use expressions derived from the orbital motion limited theory.[18,19]

In the present work we focus our attention on low-frequency waves in a weakly coupled dusty magnetoplasma. Dust particles are assumed to be immobile, and consequently the validity of the proposed model will be restricted to waves with frequency much higher than the characteristic dust frequencies. In particular, we will consider the regime in which $|\Omega_d|\ll\omega\ll|\Omega_\beta|$, where $\Omega_d$ and $\Omega_\beta$ are the cyclotron frequencies of the dust particles and of electrons and ions, respectively. This modeling excludes the modes that can arise from the dust dynamics.

As we will see, in this range of frequencies the dust particles modify the dispersion relation through modifications of the quasineutrality condition and through effects due to dust-charge fluctuation. These dust-charge fluctuations provide an additional damping mechanism for the Alfvén wave, beyond the well-known Landau damping mechanism.

## III. PROPAGATION PARALLEL TO $B_0$ AND MAXWELLIAN DISTRIBUTION FUNCTION

If $f_{\beta 0}$ is a Maxwellian distribution, we get, from Eq. (A10) in the Appendix, $L(f_{\beta 0})=0$, and the effects of charge variation in the term $\epsilon_{ij}^C$, given by Eq. (A2), only occurs in the resonant denominator, being this result independent of the direction of $\mathbf{k}$. The resonant denominator is modified by the addition of a purely imaginary term which contains the inelastic collision frequency of electrons and ions with dust particles.

In the case of propagation exactly parallel to the external magnetic field, the term $\epsilon_{ij}^N$, given by Eq. (A13), only occurs for $i=j=3$ (whether $f_{\beta 0}$ is or is not a Maxwellian distribution).

In the case of propagation parallel to the external magnetic field and Maxwellian distributions for the electrons and ions, the dielectric tensor assumes the form

$$\overleftrightarrow{\epsilon}=\begin{pmatrix}\epsilon_{11}^C & \epsilon_{12}^C & 0\\ -\epsilon_{12}^C & \epsilon_{11}^C & 0\\ 0 & 0 & \epsilon_{33}^C+\epsilon_{33}^N\end{pmatrix}, \tag{1}$$

where

$$\epsilon_{11}^C=1+\tfrac{1}{4}\sum_\beta X_\beta[\hat{I}_\beta^+ +\hat{I}_\beta^-], \tag{2}$$

$$\epsilon_{12}^C=-\frac{i}{4}\sum_\beta X_\beta[\hat{I}_\beta^+ -\hat{I}_\beta^-], \tag{3}$$

$$\epsilon_{33}^C=1+\sum_\beta X_\beta\hat{I}_\beta^0, \tag{4}$$

with

$$\hat{I}_\beta^s\equiv\frac{1}{n_{\beta 0}}\int d^3p\,\frac{p_\perp\,\partial f_{\beta 0}/\partial p_\perp}{1-\dfrac{k_\parallel p_\parallel}{m_\beta\omega}+s\dfrac{\Omega_\beta}{\omega}+i\dfrac{\nu_{\beta d}^0(p)}{\omega}}, \tag{5}$$

where $s=\pm 1$ and

$$\hat{I}^0_\beta \equiv \frac{1}{n_{\beta 0}} \int d^3p \frac{p_\perp \, \partial f_{\beta 0}/\partial p_\perp (p_\parallel/p_\perp)^2}{1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} + i \frac{\nu^0_{\beta d}(p)}{\omega}}. \tag{6}$$

We also have

$$\epsilon^N_{33} = -\frac{4\pi i n_{d0}}{\omega} U_3 S_3, \tag{7}$$

where

$$U_3 \equiv \frac{i}{\omega + i(\nu_{ch} + \nu_1)} \times \sum_\beta \frac{q_\beta}{m^2_\beta} \int d^3p \frac{p_\perp \sigma'_\beta(p) p \, f_{\beta 0}}{1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} + i \frac{\nu^0_{\beta d}(p)}{\omega}} \left(\frac{p_\parallel}{p_\perp}\right), \tag{8}$$

$$S_3 \equiv \frac{1}{\omega n_{d0}} \sum_\beta q^2_\beta \int d^3p \frac{\nu^0_{\beta d}(p)}{\omega} \times \frac{\partial f_{\beta 0}/\partial p_\perp}{1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} + i \frac{\nu^0_{\beta d}(p)}{\omega}} \left(\frac{p_\parallel}{p_\perp}\right), \tag{9}$$

with

$$\nu_{ch} = -\sum_\beta \frac{q_\beta}{m_\beta} \int d^3p \; \sigma'_\beta(p) p f_{\beta 0}, \tag{10}$$

$$\nu_1 = \sum_\beta \frac{q_\beta}{m_\beta} \int d^3p \frac{[i\nu^0_{\beta d}(p)/\omega] \sigma'_\beta(p) p \, f_{\beta 0}}{1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} + i \frac{\nu^0_{\beta d}(p)}{\omega}}. \tag{11}$$

## IV. THE DISPERSION RELATION FOR ALFVÉN WAVES

The general dispersion relation for $\mathbf{k} = k_\parallel \mathbf{e_z}$ and Maxwellian distributions of electrons and ions follows from the determinant,

$$\det \begin{pmatrix} \epsilon^C_{11} - N^2_\parallel & \epsilon^C_{12} & 0 \\ -\epsilon^C_{12} & \epsilon^C_{11} - N^2_\parallel & 0 \\ 0 & 0 & \epsilon^C_{33} + \epsilon^N_{33} \end{pmatrix} = 0. \tag{12}$$

In this expression, $N_\parallel = k_\parallel c/\omega$ is the refractive index in the parallel direction to the external magnetic field. The dispersion relation for Alfvén waves is obtained retaining only the components in the upper left $2\times 2$ determinant in Eq. (12), that is, by imposing $E_z = 0$,

$$[N^2_\parallel - \epsilon^C_{11}]^2 + (\epsilon^C_{12})^2 = 0$$

or

$$[N^2_\parallel]_\pm = \epsilon^C_{11} \pm i\epsilon^C_{12}. \tag{13}$$

Using Eqs. (2) and (3), and denoting by $s = \pm 1$:

$$[N^2_\parallel]_s = 1 + \tfrac{1}{2} \sum_\beta X_\beta \hat{I}^s_\beta. \tag{14}$$

In order to evaluate the integrals $\hat{I}^s_\beta$ we replace the functions $\nu^0_{\beta d}(p)$ by their average values in momentum space,

$$\nu_\beta \equiv \frac{1}{n_{\beta 0}} \int d^3p \; \nu^0_{\beta d}(p) f_{\beta 0}, \tag{15}$$

where $\nu^0_{\beta d}(p)$ is given by Eq. (A8). The frequency given by Eq. (15) represents the rate of capture of particles of species $\beta$ by dust particles in equilibrium state. For Maxwellian distributions we obtain

$$\nu_i = 2\sqrt{2\pi} a^2 n_{d0} v_{Ti}(1 + \chi_i),$$

$$\nu_e = 2\sqrt{2\pi} a^2 n_{d0} v_{Te} e^{\chi_e},$$

where $\chi_i \equiv Z_d e^2/(aT_i)$, $\chi_e \equiv -(T_i/T_e)\chi_i$, and $v_{T\beta} = (T_\beta/m_\beta)^{1/2}$. The number of charges in each dust particle, $Z_d$, is calculated from the equation of balance of current in the dust particles, in the equilibrium state, and from the quasineutrality condition, which gives also $n_{e0}$ if we fix the ion and dust densities $n_{i0}$ and $n_{d0}$.

This approximation has been adopted in order to arrive at a relatively simple estimate of the effect of dust-charge fluctuation, which is usually neglected in analysis of the dispersion relation for low-frequency waves. If the dispersion relation obtained using the averaged effect of collisions between electrons, ions, and dust particles shows significant effect due to the charge fluctuations, there will be reason for further research in order to improve the approximation. For the present approach, the approximation shall provide at least qualitatively a fair idea about the effect which is under investigation.

Therefore, using this approximation, the relevant integral for the dispersion relation can be evaluated and gives

$$\hat{I}^s_\beta = 2\zeta^0_\beta Z(\hat{\zeta}^s_\beta),$$

where $Z$ is the plasma dispersion function, defined by

$$Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} dt \frac{e^{-t^2}}{t - \zeta}$$

and

$$\zeta^0_\beta \equiv \frac{\omega}{\sqrt{2} k_\parallel v_{T\beta}}, \quad \hat{\zeta}^s_\beta \equiv \frac{\omega + s\Omega_\beta + i\nu_\beta}{\sqrt{2} k_\parallel v_{T\beta}}.$$

Then the dispersion relation, given by Eq. (14), assumes the form

$$[N^2_\parallel]_s = 1 + \sum_\beta X_\beta \zeta^0_\beta Z(\hat{\zeta}^s_\beta). \tag{16}$$

The $Z$ function can be substituted by its asymptotic series.[20] This dispersion relation in the absence of dust, for the conditions of existence of Alfvén waves, has only one branch, meaning that the two modes solutions of Eq. (16) collapse to only one, therefore reproducing the traditional dispersion relation for an electron-ion plasma. The modifications introduced by the dust particles occur via the quasineutrality condition ($n_{i0} \neq n_{e0}$) that must be used to evaluate

$X_\beta \equiv \omega_{p\beta}^2/\omega^2$, and also in the dust-charge fluctuation present in terms which contain the average inelastic collision frequency $\nu_\beta$.

In order to find the numerical solution we introduce dimensionless quantities, in the following way:

$$z = \frac{\omega}{\Omega_i}, \quad \delta = \frac{n_{i0}}{n_{e0}}, \quad \epsilon = \frac{n_{d0}}{n_{i0}},$$

$$u_\beta = \frac{v_{T\beta}}{v_A}, \quad \gamma = \frac{\lambda^2 n_{i0} v_A}{\Omega_i}, \quad a = \lambda \tilde{a},$$

$$\lambda = \frac{e^2}{T_i}, \quad \tau_e = \frac{T_e}{T_i}, \quad q = \frac{k_\parallel v_A}{\Omega_i},$$

$$\tilde{\nu}_\beta = \frac{\nu_\beta}{\Omega_i}, \quad \eta_\beta = \frac{\omega_{p\beta}}{\Omega_i}, \quad r_\beta = \frac{\Omega_\beta}{\Omega_i}, \tag{17}$$

where $v_A$ is the Alfvén velocity,

$$v_A = \frac{B_0^2}{4\pi n_{i0} m_i}.$$

The relevant results can therefore be cast in terms of these dimensionless quantities. The dimensionless collision frequencies are given by

$$\tilde{\nu}_i = 2\sqrt{2\pi}\epsilon\gamma\tilde{a}^2 u_i(1+\chi_i),$$

$$\tilde{\nu}_e = 2\sqrt{2\pi}\epsilon\gamma\tilde{a}^2 u_e e^{\chi_e}, \tag{18}$$

where $\chi_i = Z_d/\tilde{a}$ and $\chi_e = -\chi_i/\tau_e$.

The complete dispersion relation becomes

$$\frac{q^2c^2}{v_A^2 z^2} = 1 + \sum_\beta \frac{\eta_\beta^2}{\sqrt{2} q u_\beta z} Z(\hat{\zeta}_\beta^s), \tag{19}$$

where

$$\hat{\zeta}_\beta^s = \frac{z + s r_\beta + i\tilde{\nu}_\beta}{\sqrt{2} q u_\beta}.$$

Taking into account that in the range of parameters of interest the argument of the $Z$ function is very large, it is easy to obtain the expanded dispersion relation

$$\frac{q^2c^2}{v_A^2 z^2} = 1 + \sum_\beta \frac{\eta_\beta^2}{\sqrt{2} q u_\beta z}\left[ -\frac{\sqrt{2} q u_\beta}{z + s r_\beta + i\tilde{\nu}_\beta} + i\sqrt{\pi}\exp\left(-\frac{(z + s r_\beta + i\tilde{\nu}_\beta)^2}{2q^2u_\beta^2}\right)\right]. \tag{20}$$

If the effect of the charge fluctuation is neglected the quantities $\tilde{\nu}_\beta$ will vanish ($\tilde{\nu}_\beta = 0$, for $\beta = i, e$). In that case, the real part of Eq. (20) becomes the same as Eq. (8) of Ref. 5, in the range $\omega_{pd} \ll \omega \ll \Omega_i$. It also reproduces dispersion relations appearing in other well-known works on low-frequency waves, for instance, in Refs. 3 and 7. This choice of the frequency range is necessary because the dynamics of the dust particles has been neglected in the derivation of Eq. (20).

The numerical calculations done using the complete and the expanded dispersion relation give the same results. We tried also to calculate $z_i$ using the approximate expression,

$$z_i = -\frac{\Lambda_i(z_r)}{\left.\dfrac{\partial \Lambda_r}{\partial z}\right|_{z=z_r}},$$

where $\Lambda_r$ and $\Lambda_i$ are, respectively, the real and the imaginary parts of the dispersion relation, which is valid if $|z_i| \ll |z_r|$, but the results show that this condition is not always satisfied in the presence of dust. In the absence of dust this condition is satisfied and we get, as usual,

$$z_r = \frac{cq}{v_A\sqrt{1+\eta_i^2}}, \tag{21}$$

$$z_i = -\sqrt{\frac{\pi}{8}}\frac{\eta_i^2}{q(1+\eta_i^2)u_i}\exp\left(-\frac{1}{2q^2u_i^2}\right). \tag{22}$$

## V. NUMERICAL ANALYSIS

We consider the following parameters: Ambient magnetic field $B = 1.0 \times 10^{-4}$ T, ion temperature $T_i = 1.0 \times 10^4$ K, ion density $n_{i0} = 1.0 \times 10^9$ cm$^{-3}$, ion charge number $Z_i = 1.0$, and ion mass $m_i = m_p$, the proton mass. For most of the numerical results, unless explicitly stated otherwise, we consider electron temperature $T_e = T_i$. For the radius of the dust particles, we assume $a = 1.0 \times 10^{-4}$ cm.

For the classical distance of minimum approach, measured in centimeters, we use the value $\lambda = 1.44 \times 10^{-7}/T_i(\mathrm{eV})$, where $T_i(\mathrm{eV})$ means the ion temperature expressed in units of eV.

One of the motivations of our choice of parameters is the following. A long standing issue in the theory of the loss of mass in stars is the explanation of the low terminal velocities and the high rate of mass loss occurring in the stellar winds. Alfvén waves have been utilized by many authors when attempting to explain the characteristics of stellar winds.[21–25] Since the damping and propagation of Alfvén waves may be affected by the presence of dust, we have chosen parameters which are in the range of parameters of interest for the stellar winds. For instance, the densities in stellar winds may be $n_n \simeq 1 \times 10^{10}$ cm$^{-3}$, with $n_i \simeq n_n$ ($n_n$ is the density of neutral particles; assuming slightly smaller ion density, we have used $n_i = 1 \times 10^9$ cm$^{-3}$). The diameter of dust grains may be $a \simeq 1$ $\mu$m$= 1 \times 10^{-4}$cm, and the electron temperature $T_e \simeq 1$ eV.[26,27] For the magnetic field, the value which we have assumed is representative of fields in the interstellar medium. Moreover, it is possible to show by numerical analysis that the results which we have obtained with our choice of parameters are also obtained, qualitatively, within a range of values around those which were chosen as example.

Initially, we consider the solution of the dispersion relation given by Eq. (19) for several values of $\epsilon$, starting from $\epsilon = 0$. In the limit $\epsilon = 0$, ion and electron densities are equal and $\tilde{\nu}_e = \tilde{\nu}_i = 0$, and therefore the dispersion relation becomes the usual dispersion relation for Alfvén waves in the absence of dust.

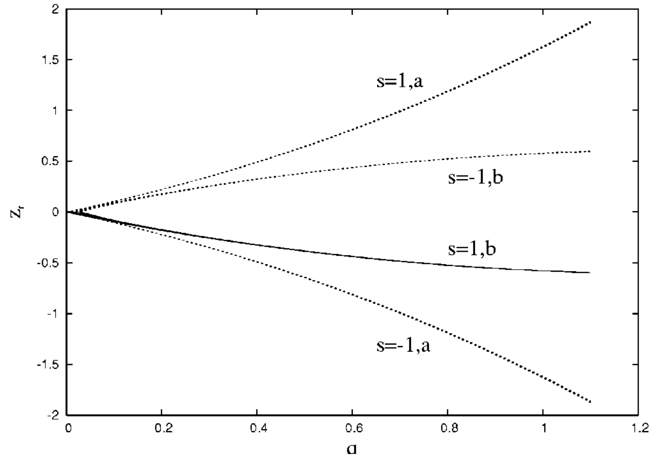


FIG. 1. Real part of the normalized frequency ($z_r$) for the two roots obtained using $s=1$ and for the two roots obtained using $s=-1$, as a function of $q$, for five values of $\epsilon$. $B=1.0\times10^{-4}$ T, $T_i=1.0\times10^{4}$ K, $n_{i0}=1.0\times10^{9}$ cm$^{-3}$, $Z_i=1.0$, $m_i=m_p$, and $T_e=T_i$. Radius of dust particles $a=1.0\times10^{-4}$ cm. The dust density $n_{d0}$ is obtained from $\epsilon=n_{d0}/n_{i0}$, and the values of $\epsilon$ utilized are $\epsilon=0.0$, $1.25\times10^{-6}$, $2.50\times10^{-6}$, $3.75\times10^{-6}$, and $5.0\times10^{-6}$.

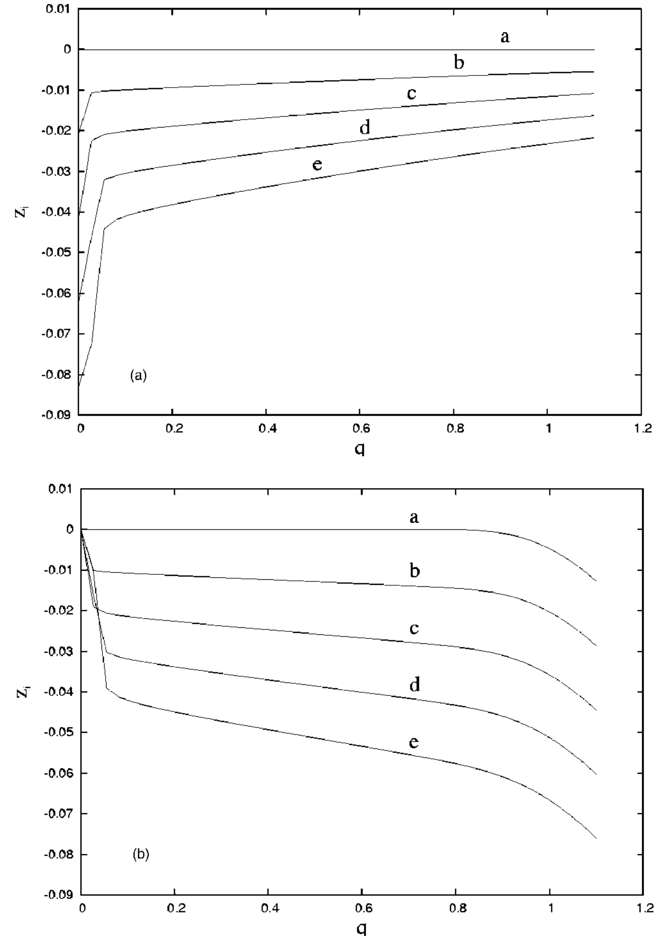


FIG. 2. (a) Imaginary part $z_i$ of the roots labeled as $s=1$, $a$ and $s=-1$, $a$ in Fig. 1, as a function of $q$, for five values of $\epsilon$. (b) Imaginary part $z_i$ of the roots labeled as $s=1$, $b$ and $s=-1$, $b$ in Fig. 1, as a function of $q$, for five values of $\epsilon$. The curves are identified by "a" $\epsilon=0.0$, "b" $\epsilon=1.25\times10^{-6}$, "c" $\epsilon=2.50\times10^{-6}$, "d" $\epsilon=3.75\times10^{-6}$, and "e" $\epsilon=5.0\times10^{-6}$. The parameters are the same as in Fig. 1.

We start by investigating the range of small values of dust density, considering $\epsilon$ between 0 and $5.0\times10^{-6}$. In Fig. 1 we observe the real part of the two roots obtained from Eq. (19) for each of the signs $s=1$ and $s=-1$, as a function of $q$. There are two curves with positive values of $z_r$, describing waves propagating in the positive direction. The uppermost curve is obtained with $s=1$, and corresponds to the so-called *whistler* branch, while the lower curve in the positive side is obtained with $s=-1$ and corresponds to the branch identified with circularly polarized waves propagating along the ambient magnetic field. These curves, labeled, respectively, as $s=1$, $a$ and $s=-1$, $b$ in Fig. 1, are easily recognized from well-known textbooks.[28] For negative values of $z_r$ we have perfectly symmetrical solutions propagating in negative direction, obtained, respectively, with $s=1$ (the upper curve, labeled as $s=1$, $b$) and $s=-1$ (the lower curve, labeled as $s=-1$, $a$). For small values of $q$, $q\leqslant0.2$, the two branches of waves propagating in a given direction collapse together in a single branch known as the branch of the Alfvén waves, approximately described by Eq. (21). Each of the curves appearing in Fig. 1 indeed corresponds to the superposition of five curves, obtained with $\epsilon=0.0$, $1.25\times10^{-6}$, $2.50\times10^{-6}$, $3.75\times10^{-6}$, and $5.0\times10^{-6}$. These results show that the presence of a small density of dust particles has negligible effect on the real part of the roots obtained from the dispersion relation of low-frequency waves propagating along the magnetic field.

In Fig. 2 we see the corresponding imaginary parts. Figure 2(a) shows the imaginary part of the root corresponding to the whistler branch appearing in Fig. 1, either for $s=1$ or $s=-1$. It is seen that the damping rate for these waves is negligible in the absence of dust ($\epsilon=0$), but becomes significant in the presence of even a small amount of dust, specially in the range of small values of $q$. Figure 2(b) depicts the imaginary part corresponding to the branch of the circularly polarized waves, labeled as $s=1$, $b$ and $s=-1$, $b$ in Fig. 1. In the region of small values of $q$, where the waves are identified with Alfvén waves, the damping rate is negligible for $\epsilon=0$, becoming significant for $q$ approaching $q=1$. This damping is due to wave-particle resonance and is known as the ion-cyclotron damping, which occurs for $|\omega|\simeq\Omega_i$. In the presence of dust this mode detaches from the mode depicted in Fig. 2(a). The damping due to the dust particles is relatively small for very small $q$, becomes significant for larger values of $q$, and competitive with the ion-cyclotron damping for $q\simeq1$. Even for $q\simeq1$, Fig. 2(b) shows that the damping due to dust particles tends to dominate over the ion-cyclotron damping, even for the relatively small dust density which has been considered in the present case.

It is interesting to note that, in the absence of dust, the damping rate in the range of small $q$ is given analytically by Eq. (22), corresponding to the well-known Landau damping of Alfvén waves. For the parameters which we are utilizing, $u_i\simeq0.13$. The exponential factor appearing in Eq. (22) is nearly $1.0\times10^{-79}$ for $q=0.4$, and even smaller for smaller values of $q$. Landau damping of Alfvén waves is in practice completely negligible for the parameters which we are utilizing, while the damping due to the presence of dust par-

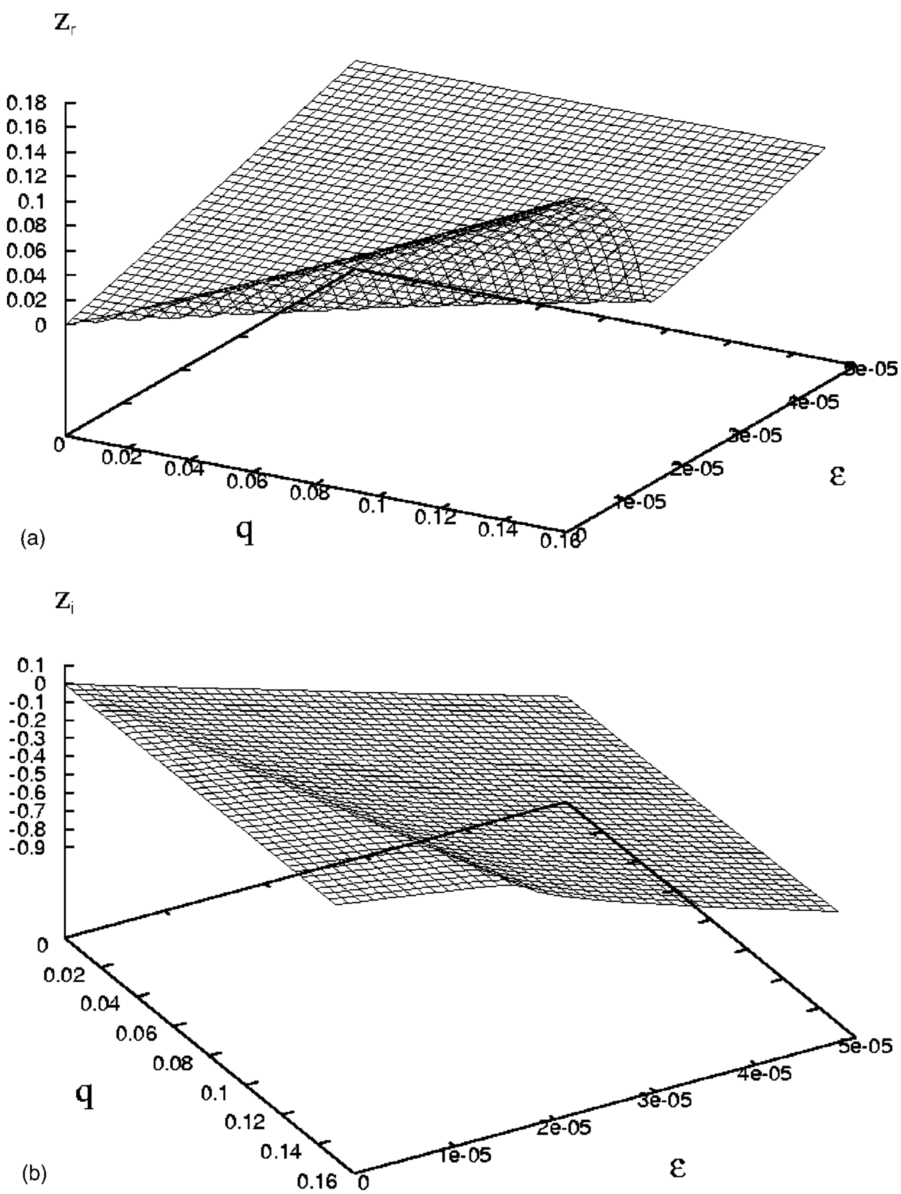


FIG. 3. (a) Real part of the normalized frequency ($z_r$) as a function of $q$ and $\epsilon$, for the upper root obtained for $s=1$ (the whistler branch), in the range of small values of $q$. (b) Imaginary part of the normalized frequency ($z_i$) as a function of $q$ and $\epsilon$, for the upper root obtained for $s=1$, in the range of small values of $q$. Other parameters as in Fig. 1.

ticles may become very significant even for small values of dust density, as seen in Fig. 2.

The region of small values of $q$ is explored in more details in the two panels of Fig. 3 and in the two panels of Fig. 4, considering a more extended range of $\epsilon$ than in the case of the previous figures. Figure 3(a) shows the behavior of the root labeled as $s=1$, $a$ in Fig. 1, which we have identified as the mode which for larger values of $q$ becomes the whistler branch. In the range of $q$ shown in the figure, the real part of the frequency grows linearly with $q$ for very small dust density, according to what is shown in Fig. 1. However, for $\epsilon \simeq 5.0 \times 10^{-5}$, the quantity $z_r$ appears almost insensitive to $q$, with value $z_r \simeq 0.12$. The corresponding damping rates appear in Fig. 3(b). It is seen that this mode, which is not damped for $\epsilon=0$, becomes heavily damped for larger $\epsilon$. The damping rate grows nearly linearly with the dust population for the whole range of $q$ appearing in the figure.

Figure 4 shows the behavior of the other root propagating in the positive direction, the root labeled as $s=-1$, $b$ in Fig. 1, corresponding to the mode which for larger values of $q$ has been identified with the branch of circularly polarized waves. Although Fig. 1 has shown that the real part of the frequency is almost insensitive to the density of dust particles, Fig. 4(a) shows that for $\epsilon \geqslant 1.0 \times 10^{-5}$ the absolute value $|z_r|$ tends to a value close to zero for the whole range of $q$ values depicted in the figure. Figure 4(b) shows that the damping rate attains maximum value for $\epsilon$ near $1.0 \times 10^{-5}$, in the range of $q$ depicted in the figure, and becomes much less significant for larger values of $\epsilon$.

A closer look into the real part of the solution appearing in Fig. 1 provides interesting information about the contribution of the dust particles to the dispersion relation. Figure 5 displays the values of the real part of the normalized frequency $z_r$ as a function of $\epsilon$ for three values of $q$ and both signs $s=+1$ and $s=-1$ for $\epsilon$ up to $5.0 \times 10^{-5}$, while Fig. 6 shows the corresponding values of the imaginary part. The values of $q$ depicted in panels (a)–(c) of the figures are, respectively, $q=0.05$, 0.1, and 0.15. According to what we have learned from Fig. 1, for $\epsilon=0$ the uppermost and lower-

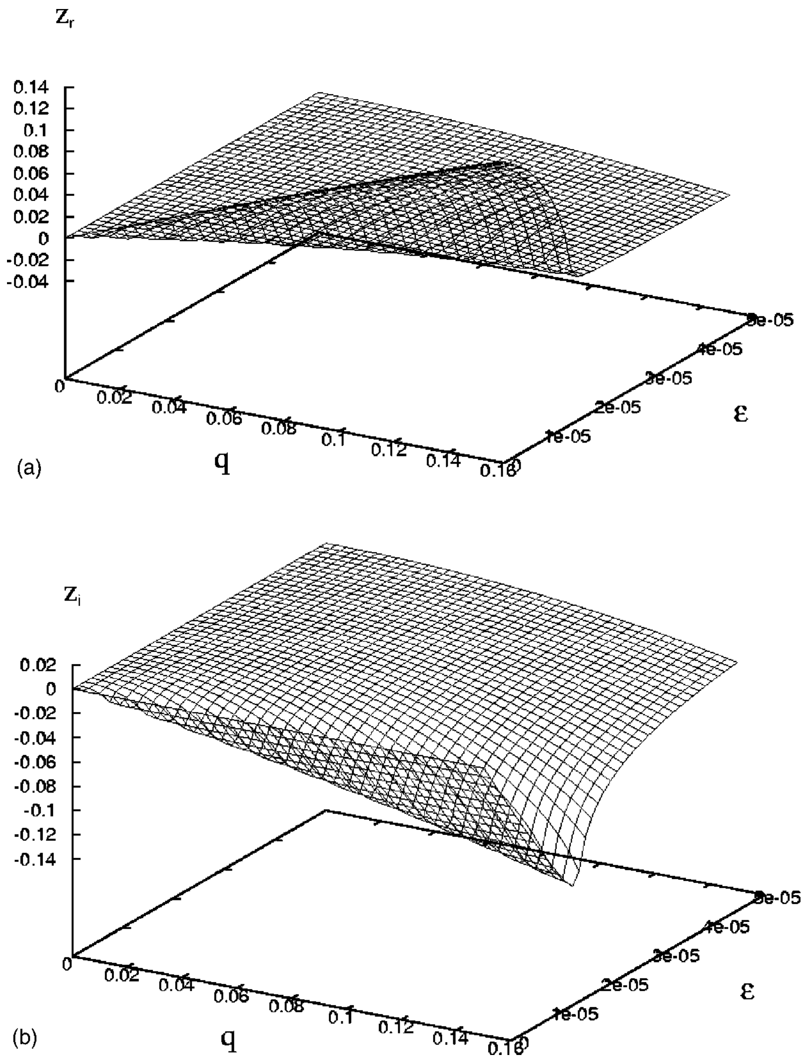


FIG. 4. (a) Real part of the normalized frequency ($z_r$) as a function of $q$ and $\epsilon$, for the upper root obtained for $s=-1$ (the branch of circularly polarized waves), in the range of small values of $q$. (b) Imaginary part of the normalized frequency ($z_i$) as a function of $q$ and $\epsilon$, for the upper root obtained for $s=-1$, in the range of small values of $q$. Other parameters as in Fig. 1.

most curves for $z_r$ shown in each of the panels of Fig. 5 correspond to the values of $z_r$ for the whistler branch obtained for $s=+1$ and $s=-1$, respectively. The other two curves in each panel pertain to the other mode, the circularly polarized branch.

It is seen from Fig. 5 that the two roots corresponding to the whistler branch display decreasing values of $|z_r|$ for increasing dust density, up to a certain point, and then show $|z_r|$ increasing linearly with $\epsilon$. The point of minimum value of $|z_r|$ is close to $\epsilon=0.7\times10^{-5}$ for $q=0.05$, $1.3\times10^{-5}$ for $q=0.1$, and $1.9\times10^{-5}$ for $q=0.15$. On the other hand, the two roots corresponding to the circularly polarized waves propagating in positive and negative directions, which are well separated for $\epsilon=0$, feature mode coupling for increasing dust density. The point of mode coupling approximately corresponds to the value of $\epsilon$ for which the minimum value of $z_r$ occurs for the whistler branch, increasing with the value of $q$, in the range of values of $q$ appearing in the figure. Alfvén waves propagating in a given direction in an inhomogeneous medium may arrive to a region where the mode-coupling conditions are satisfied and may therefore be transmitted or reflected. This coupling of waves propagating in positive and negative directions is a completely novel phenomenon which is caused by the presence of dust particles.

The analysis of the imaginary parts appearing in Fig. 6 shows that the damping of both modes, which is negligible for $\epsilon=0$, increases linearly with the dust density, up to nearly the same value of $\epsilon$ where mode-coupling occurs. At this point, the damping of the two modes are separated one from the other. The damping of the waves in the whistler branch continues to increase almost linearly with the dust density, while the damping of the waves in the branch of circularly polarized waves is gradually reduced for increasing values of $\epsilon$.

The effect of the plasma temperature can be appreciated in Figs. 7(a) and 7(b), which display, respectively, the values of the real and the imaginary parts of the normalized frequency, as a function of $\epsilon$, for $\epsilon$ up to $5.0\times10^{-5}$, for the representative value $q=0.1$ and both signs $s=+1$ and $s=-1$, for $T_i=5.0\times10^4$ K, with $T_e=T_i$. The comparison between Figs. 5(b) and 7(a), which displayed the case of $T_i=1.0\times10^4$ K, shows qualitatively similar results, with the point of mode coupling between forward and backward propagating circularly polarized waves being moved toward smaller values of $\epsilon$ with the increase of temperature. Similar trend can be seen for the imaginary part by comparing Figs. 6(b) and 7(b). It is noticeable that the separation between the damping rates of the whistler branch and of the branch of

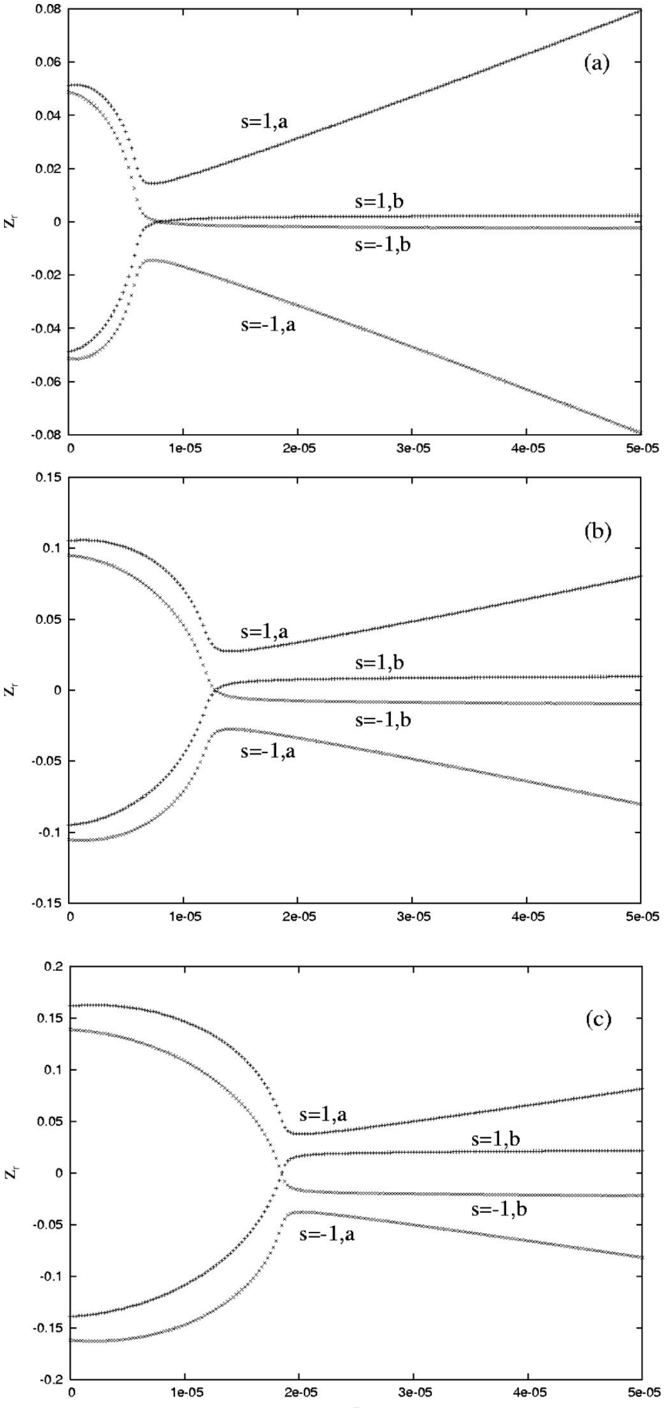


FIG. 5. Real part of the normalized frequencies ($z_r$) as a function of $\epsilon$, for $s=\pm 1$, and three values of $q$. (a) $q=0.05$; (b)=0.1; (c)=0.15. Other parameters as in Fig. 1.

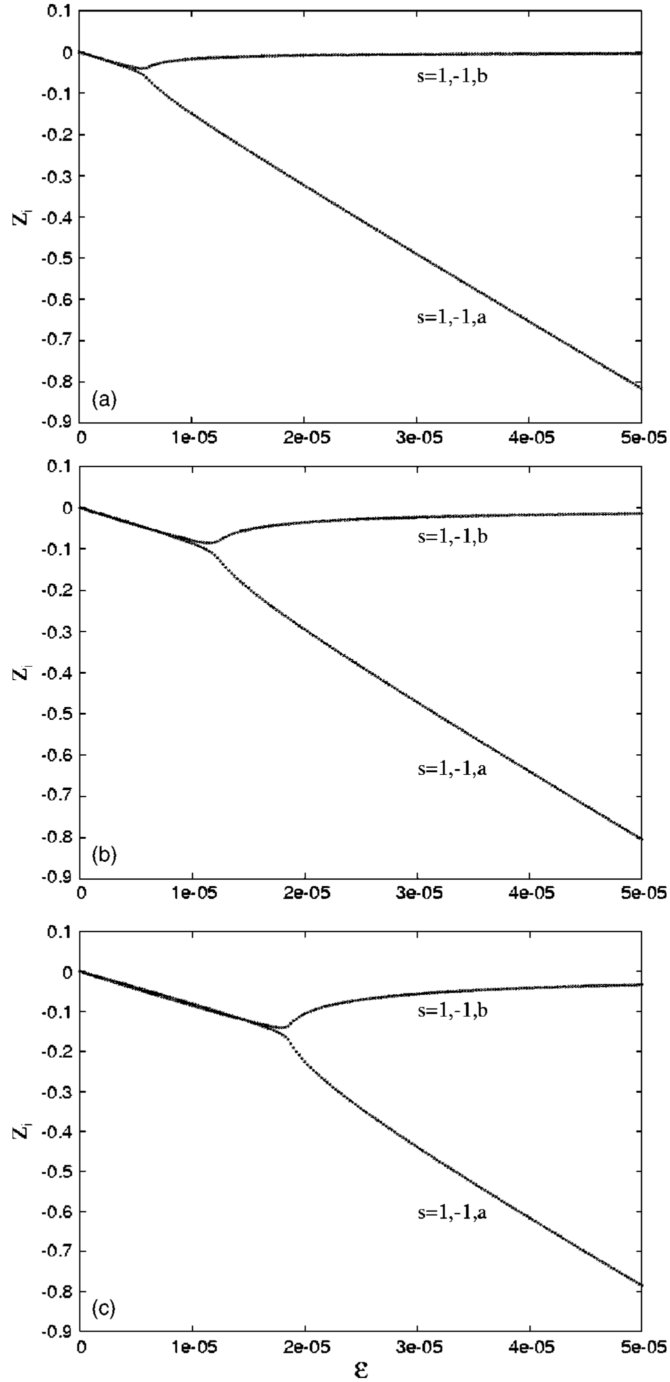


FIG. 6. Imaginary part of the normalized frequencies ($z_i$) as a function of $\epsilon$ for $s=\pm 1$, and three values of $q$. (a) $q=0.05$; (b)=0.1; (c)=0.15. Other parameters as in Fig. 1.

circularly polarized waves occurs for decreasing values of dust density, as the plasma temperature increases.

The effect of the difference of temperature between ions and electrons can be seen in Fig. 8, which shows the solutions of the dispersion relation for $\epsilon=5.0\times 10^{-6}$ and three values of the ratio $T_e/T_i$, $T_e/T_i=0.2$, 1.0, and 5.0. Figure 8(a) shows the relative independence of $z_r$ relatively to the ratio of temperature. Figure 8(b) shows that the damping rate of the Alfvén waves in the branch of whistler waves increases with the rise of $T_e/T_i$, for the whole range of $q$ values considered in the calculation. Figure 8(c) shows that the damping rate for the waves in the branch of circularly polarized waves is decreased in the range of small values of $q$, with the increase of $T_e/T_i$, while the damping rate for higher values of $q$ increases considerably with the increase of the temperature ratio.

Finally, we discuss an approximation which is frequently employed for the dispersion relation in dusty plasmas, namely, the neglect of the collision frequency $\tilde{\nu}_\beta$, while at the same time taking into account the difference in electron

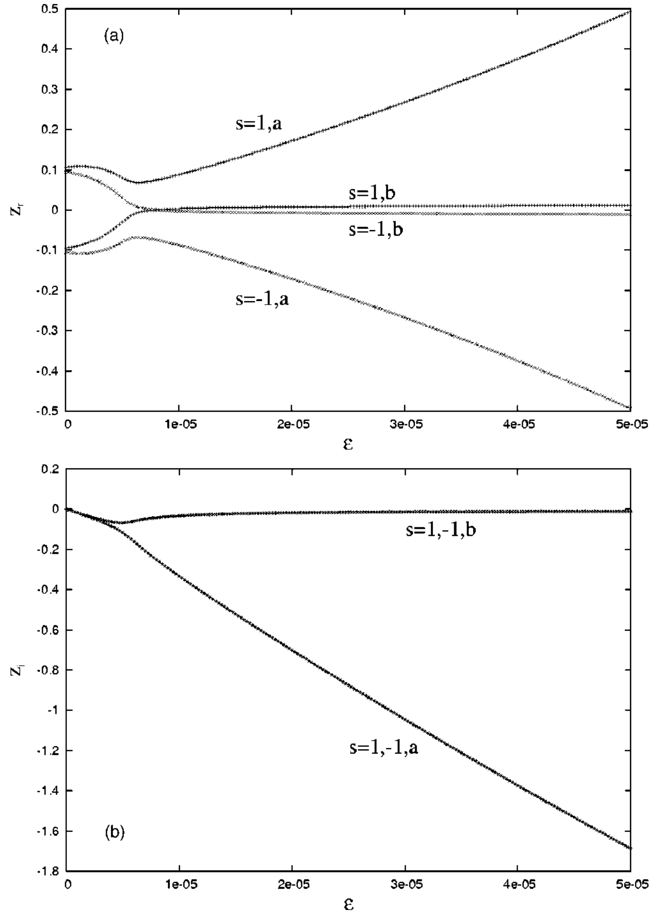


FIG. 7. (a) Real part of the normalized frequency ($z_r$) as a function of $\epsilon$, for $q=0.1$ and $s=\pm 1$, for ion temperature $T_i=5.0\times 10^4$ K, with $T_e=T_i$. (b) Imaginary part of the normalized frequency ($z_i$) as a function of $\epsilon$, for $q=0.1$ and $s=\pm 1$, for ion temperature $T_i=5.0\times 10^4$ K, with $T_e=T_i$. Other parameters as in Fig. 1.

and ion densities due to the presence of the dust particles.

In Fig. 9(a) we observe the real part of the two roots obtained from Eq. (19) for each of the signs $s=1$ and $s=-1$, but using as an approximation that $\tilde{\nu}_e=\tilde{\nu}_i=0$, for five values of $\epsilon$. The values of $\epsilon$ utilized are $\epsilon=0.0$, $1.25\times 10^{-5}$, $2.50\times 10^{-5}$, $3.75\times 10^{-5}$, and $5.0\times 10^{-5}$. Other parameters are the same as in Fig. 1. It is seen that the waves corresponding to the branch of whistler waves are more affected by the presence of the dust, but the effect is not very clear in the scale of Fig. 9. An amplification of the region of small values of $q$ can be seen in Fig. 10(a). It is seen that, as the dust density is increased, the values of $|z_r|$ increase for the waves in the whistler branch, and decrease for the waves in the branch of circularly polarized waves. However, the magnitude of the modifications due to the presence of the dust particles is much smaller than the magnitude of the modifications appearing when the inelastic collision frequency $\tilde{\nu}_\beta$ is taken into account, which can be appreciated, for instance, in Fig. 5.

In Fig. 9(b) we see the imaginary parts corresponding to the real parts appearing in Fig. 9(a). It is seen the vanishing value of $z_i$ for the root corresponding to the whistler branch, for the whole range of values of $\epsilon$ which has been considered. This is in contrast with the situation depicted in Fig.

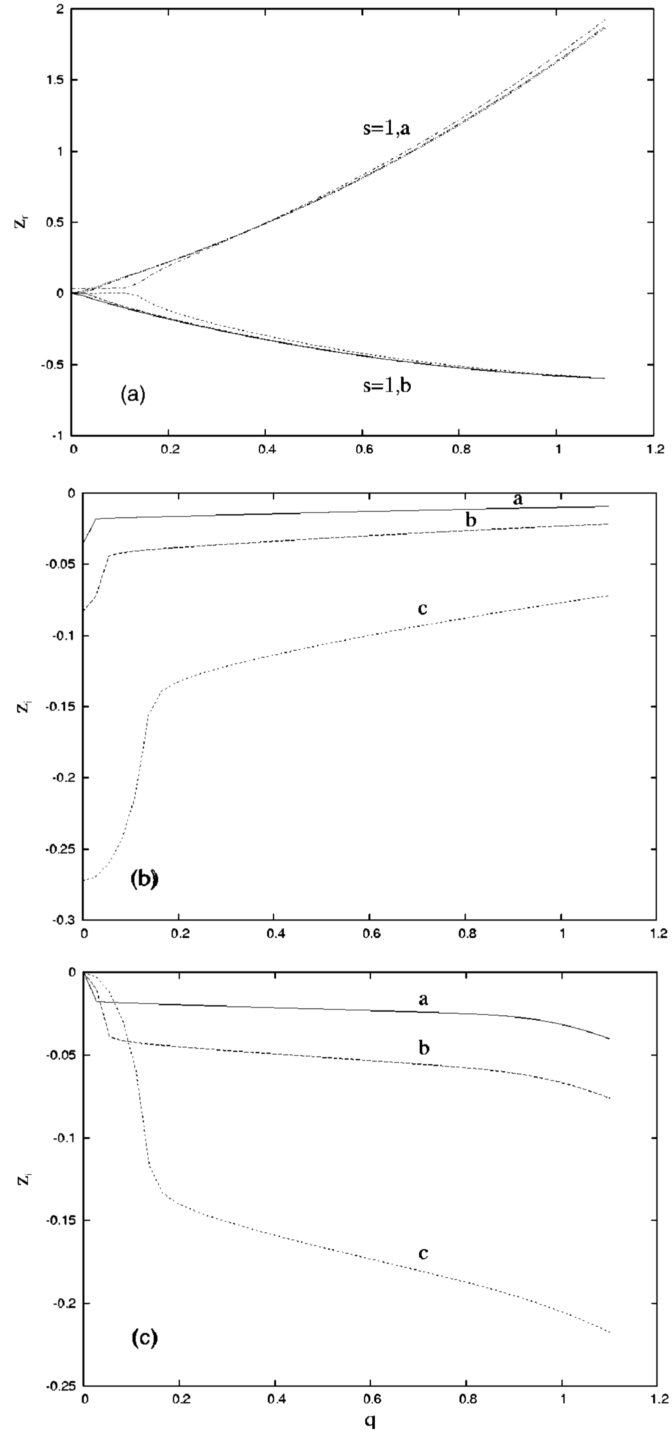


FIG. 8. (a) Real part of the normalized frequency ($z_r$), as a function of $q$, for $\epsilon=5.0\times 10^{-6}$, and three values of $T_e/T_i$, the two roots obtained using $s=1$. (b) Imaginary part $z_i$ of the roots labeled as $s=1$, $a$ in panel (a), as a function of $q$. (c) Imaginary part $z_i$ of the roots labeled as $s=1$, $b$ in panel (a), as a function of $q$. The curves are identified by "a" $T_e/T_i=0.2$, "b" $T_e/T_i=1.0$, and "c" $T_e/T_i=5.0$. Other parameters as in Fig. 1.

2(a), which shows that when the collision rates $\tilde{\nu}_e$ and $\tilde{\nu}_i$ are taken into account very significant damping rate for waves in the whistler branch occurs for nonvanishing dust density. Figure 9(b) also shows nonvanishing damping rates for $q\geqslant 0.8$, for the waves in the branch of circularly polarized waves. The damping rate for these waves is slightly modified relating to the usual Landau damping value. An amplification

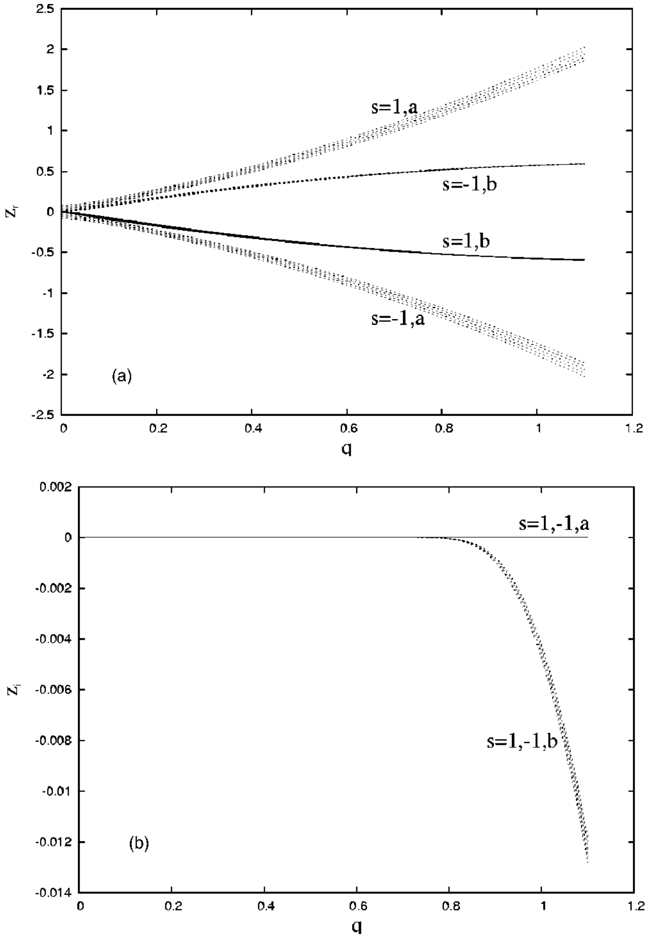


FIG. 9. (a) Real part of the normalized frequency $(z_r)$ for the two roots obtained using $s=1$ and for the two roots obtained using $s=-1$, as a function of $q$, for five values of $\epsilon$, and assuming $\tilde{\nu}_i=0$ and $\tilde{\nu}_e=0$. (b) Corresponding values of the imaginary part of the normalized frequency $(z_i)$, as a function of $q$, for five values of $\epsilon$, and assuming $\tilde{\nu}_i=0$ and $\tilde{\nu}_e=0$. The values of $\epsilon$ utilized are $\epsilon=0.0$, $1.25\times10^{-5}$, $2.50\times10^{-5}$, $3.75\times10^{-5}$, and $5.0\times10^{-5}$. Other parameters are the same as in Fig. 1.

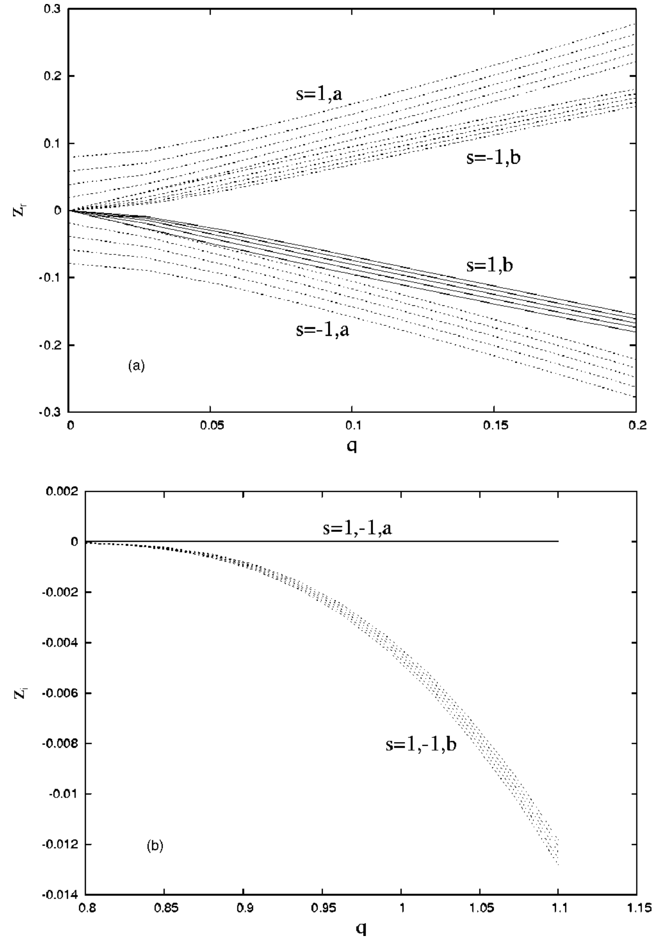


FIG. 10. (a) Real part of the normalized frequency $(z_r)$ for the two roots obtained using $s=1$ and for the two roots obtained using $s=-1$, as a function of $q$ in the range of small values of $q$, for five values of $\epsilon$, and assuming $\tilde{\nu}_i=0$ and $\tilde{\nu}_e=0$. (b) Corresponding values of the imaginary part of the normalized frequency $(z_i)$, as a function of $q$, in the range of values of $q$ where Landau damping may become important, for five values of $\epsilon$, and assuming $\tilde{\nu}_i=0$ and $\tilde{\nu}_e=0$. The values of $\epsilon$ utilized are $\epsilon=0.0$, $1.25\times10^{-5}$, $2.50\times10^{-5}$, $3.75\times10^{-5}$, and $5.0\times10^{-5}$. Other parameters are the same as in Fig. 1.

of the region of $q$ values for which significant damping occurs is seen in Fig. 10(b). The comparison between the damping rate shown in Figs. 9(b) and 10(b), obtained using the approximation $\tilde{\nu}_\beta=0$, and Fig. 2(b), obtained taking into account the collision frequency $\tilde{\nu}_\beta$, shows a completely different result. The approximation $\tilde{\nu}_\beta=0$ means that the only effect of the presence of dust particles is the loss of the balance between ion and electron densities, with the consequent modification of the Landau damping. On the other hand, when the charging of the dust particles is taken into account through the inelastic collisions, a different mechanism of absorption of wave energy is involved, which can result in much more efficient wave damping.

## VI. CONCLUSIONS

In the present paper we have used a kinetic description to analyze wave propagation in dusty plasmas, considering the case of propagation of waves exactly parallel to the external magnetic field, and considering Maxwellian distributions for electrons and ions in the equilibrium situation. The process of charging the dust grains is assumed to occur by the capture of electrons and ions during inelastic collisions between these particles and the dust grains. We have used this kinetic formulation to obtain the dispersion relation and damping rates for low-frequency electromagnetic waves. In order to simplify the calculations, we have assumed that the frequency of inelastic collisions between the charged dust particles with electrons and ions is approximated by an average value.

The results obtained show that, as in the case of dustless plasmas, the dispersion relation describes two different modes, identified for higher frequencies as the whistler waves and as the circularly polarized waves. For frequencies well below ion-cyclotron frequency, these two modes collapse together in the absence of dust, forming the well-known branch of the Alfvén waves. In the presence of dust particles with variable charge, however, these two modes become separated. The presence of the dust particles with variable charge also produces additional damping of the Alfvén waves, which may completely override conventional Landau damping for large wavelengths (small $q$ in our dimensionless variables). In fact, it has been seen that for small $q$ conventional Landau damping is completely negligible for the parameters considered. The results have shown that the damping of the two modes increase nearly linearly for small dust density. For continued increase of the dust density, waves in the circularly polarized branch have the damping rate further decreased, while waves in the whistler branch continue to have increased damping rates.

Another significant novel feature appearing from the present investigation is the occurrence of mode coupling due to the presence of dust particles, between waves in the branch of circularly polarized waves, propagating in opposite directions.

The results obtained indicate that Alfvén waves may be considerably damped due to the effect of the dust particles, which have therefore to be taken into account in the description of wave processes relevant to this environment. Moreover, it has been seen that, when the effect of variation on the charge of dust particles due to inelastic collisions is taken into account, the damping of the waves can be much more significant than predicted by the approximation which neglects the effect of inelastic collisions, keeping the charge imbalance between ions and electrons as the only effect of the presence of dust particles.

The results obtained also indicate clear need for further investigation. For instance, we intend to investigate in the near future the role of the approximation which has been utilized, namely, the assumption of an average collision rate ruling the charging process of the dust particles, instead of the more precise calculation using the velocity dependent collision frequency. We also intend to investigate in more detail the mode coupling process, which has been demonstrated to occur in the presence of dust. It would be also important to investigate the nonlinear stage of the interaction, in order to fully appreciate the effect of the heavy damping indicated by the present linear calculation.

## ACKNOWLEDGMENTS

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## APPENDIX: THE DIELECTRIC TENSOR

The dielectric tensor for a magnetized dusty plasma, homogeneous, fully ionized, with identical immobile dust particles and charge variable in time, could be written in the following way:[29,30]

$$\epsilon_{ij} = \epsilon_{ij}^C + \epsilon_{ij}^N. \tag{A1}$$

The term $\epsilon_{ij}^C$ is formally identical, except for the $i3$ components, to the dielectric tensor of a magnetized homogeneous conventional plasma of electrons and ions, with the resonant denominator modified by the addition of a purely imaginary term which contains the collision frequency of electrons and ions with the dust particles. For the $i3$ components of the dielectric tensor, in addition to the term obtained with the prescription above, there is a term which is proportional to the inelastic collision frequency of electrons and ions with the dust particles. The expression for $\epsilon_{ij}^C$ is[29,30]

$$\begin{aligned}\epsilon_{ij}^C = \delta_{ij} &+ \sum_\beta \frac{X_\beta}{n_{\beta 0}} \\ &\times \sum_{n=-\infty}^{+\infty} \int d^3p\, p_\perp \frac{\varphi_0(f_{\beta 0})}{D_{n\beta}} \left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{i3}+\delta_{j3}} R_{ij}^{n\beta} \\ &- \delta_{i3}\delta_{j3} \sum_\beta \frac{X_\beta}{n_{\beta 0}} \int d^3p\, L(f_{\beta 0}) \frac{p_\parallel}{p_\perp} \\ &+ \delta_{j3} \sum_\beta \frac{X_\beta}{n_{\beta 0}} \sum_{n=-\infty}^{+\infty} \int d^3p \left[ i\frac{\nu_{\beta d}^0(p)}{\omega} \frac{L(f_{\beta 0})}{D_{n\beta}} \left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{i3}} \right] R_{ij}^{n\beta},\end{aligned} \tag{A2}$$

where

$$D_{n\beta} \equiv 1 - \frac{k_\parallel p_\parallel}{m_\beta \omega} - \frac{n\Omega_\beta}{\omega} + i\frac{\nu_{\beta d}^0(p)}{\omega}, \tag{A3}$$

$$R_{11}^{n\beta} = \frac{n^2}{b_\beta^2} J_n^2(b_\beta), \quad R_{33}^{n\beta} = J_n^2(b_\beta), \tag{A4}$$

$$R_{12}^{n\beta} = -R_{21}^{n\beta} = i\frac{n}{b_\beta} J_n(b_\beta) J_n'(b_\beta), \tag{A5}$$

$$R_{13}^{n\beta} = R_{31}^{n\beta} = \frac{n}{b_\beta} J_n^2(b_\beta), \quad R_{22}^{n\beta} = J_n'^2(b_\beta), \tag{A6}$$

$$R_{23}^{n\beta} = -R_{32}^{n\beta} = -iJ_n(b_\beta) J_n'(b_\beta), \tag{A7}$$

$$\nu_{\beta d}^0(p) = \frac{\pi a^2 n_{d0}}{m_\beta} \frac{(p^2 + C_\beta)}{p} H(p^2 + C_\beta), \tag{A8}$$

$$\varphi_0(f_{\beta 0}) \equiv \frac{\partial f_{\beta 0}}{\partial p_\perp} - \frac{k_\parallel}{m_\beta \omega} L(f_{\beta 0}), \tag{A9}$$

$$L(f_{\beta 0}) \equiv p_\parallel \frac{\partial f_{\beta 0}}{\partial p_\perp} - p_\perp \frac{\partial f_{\beta 0}}{\partial p_\parallel}, \tag{A10}$$

$$X_\beta \equiv \frac{\omega_{p\beta}^2}{\omega^2}, \quad \omega_{p\beta}^2 = \frac{4\pi n_{\beta 0} q_\beta^2}{m_\beta}, \quad \Omega_\beta = \frac{q_\beta B_0}{m_\beta c}, \tag{A11}$$

$$b_\beta = \frac{k_\perp p_\perp}{m_\beta \Omega_\beta}, \quad C_\beta \equiv -\frac{2q_\beta m_\beta q_{d0}}{a}, \tag{A12}$$

where the subscript $\beta = e, i$ identifies electrons and ions, respectively, $q_{d0} = \varepsilon_d eZ_d$ is the equilibrium charge of the dust particle (positive, $\varepsilon_d = +1$, or negative, $\varepsilon_d = -1$) and $H$ denotes the Heaviside function.

The term $\epsilon_{ij}^N$ is entirely new and only exists in the presence of dust particles with variable charge. Its form is strongly dependent on the model used to describe the charging process of the dust particles. The expression for this term is[29,30]

$$\epsilon_{ij}^{N} = -\frac{4\pi i n_{d0}}{\omega} U_i S_j, \tag{A13}$$

with

$$U_i \equiv \frac{i}{\omega + i(\nu_{ch} + \nu_1)} \sum_\beta \frac{q_\beta}{m_\beta^2} \times \sum_{n=-\infty}^{+\infty} \int d^3p \frac{p_\perp \sigma'_\beta(p) f_{\beta 0} p}{D_{n\beta}} \left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{i3}} R_{i3}^{n\beta}, \tag{A14}$$

$$S_j \equiv \frac{1}{\omega n_{d0}} \sum_\beta q_\beta^2 \sum_{n=-\infty}^{+\infty} \int d^3p \frac{\nu_{\beta d}^0(p)}{\omega} \times \frac{\varphi_0(f_{\beta 0})}{D_{n\beta}} \left(\frac{p_\parallel}{p_\perp}\right)^{\delta_{j3}} R_{3j}^{n\beta} - \frac{\delta_{j3}}{\omega n_{d0}} \sum_\beta q_\beta^2 \int d^3p \frac{\nu_{\beta d}^0(p)}{\omega} \frac{L(f_{\beta 0})}{p_\perp} + i \frac{\delta_{j3}}{\omega n_{d0}} \sum_\beta q_\beta^2 \sum_{n=-\infty}^{+\infty} \int d^3p \frac{[\nu_{\beta d}^0(p)/\omega]^2}{D_{n\beta}} \frac{L(f_{\beta 0})}{p_\perp} R_{3j}^{n\beta}, \tag{A15}$$

where

$$\nu_{ch} = -\sum_\beta \frac{q_\beta}{m_\beta} \int d^3p \; \sigma'_\beta(p) p f_{\beta 0}, \tag{A16}$$

$$\nu_1 = \sum_\beta \frac{q_\beta}{m_\beta} \sum_{n=-\infty}^{+\infty} \int d^3p \frac{[i\nu_{\beta d}^0(p)/\omega]}{D_{n\beta}} \sigma'_\beta(p) f_{\beta 0} p R_{33}^{n\beta}. \tag{A17}$$

In the expressions (A14), (A16), and (A17), we have used the notation $\sigma'_\beta(p) \equiv (\partial\sigma_\beta/\partial q_d)|_{q_d=-Z_d e}$, and $\sigma_\beta$ is the charging cross section, given by[31]

$$\sigma_\beta = \pi a^2 \left(1 - \frac{2q_d q_\beta m_\beta}{ap^2}\right) H\left(1 - \frac{2q_d q_\beta m_\beta}{ap^2}\right). \tag{A18}$$

Effects of charge variation of dust particles occurs in the terms with $\nu_{\beta d}^0(p)/\omega$ and effects of presence of dust particles, introduced via quasineutrality relation ($n_{i0} \neq n_{e0}$), occurs in terms with $X_\beta \equiv \omega_{p\beta}^2/\omega^2$.


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