AN EMPIRICAL TEST OF THE THEORY OF CRYSTALLIZATION IN STELLAR INTERIORS

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ABSTRACT

Our theoretical calculations indicate that the massive pulsating DA white dwarf, BPM 37093, has a crystallized ion interior. Our estimates for the crystallized mass fraction depend on our assumption for the interior composition and range from 10% for pure carbon and low total mass to more than 90% for pure oxygen and high total mass; the mass of this white dwarf suggests on theoretical grounds that the core composition is unlikely to be lighter than oxygen. Our best estimate is that the crystallized mass fraction exceeds 90%.

BPM 37093 is the first known example of a pulsating white dwarf star with a potentially crystalline interior. We point out that future observations of the pulsation properties of this object are essential for carrying out asteroseismological measurements of the crystallized mass fraction of its interior. Uncertainties in the nature and extent of crystallization and its associated effects are the largest source of uncertainty in calculating the ages of the coolest white dwarf stars—an important chronometer for the Galactic disk. In addition, crystallization plays an important role in our understanding of neutron star crusts. Investigations of the pulsation properties of this star promise to provide us with the first empirical tests of the theory of crystallization in stellar interiors, now more than 35 yr old.

Subject headings: dense matter — equation of state — stars: evolution — stars: oscillations — stars: pulsations — white dwarfs

1. ASTROPHYSICAL CONTEXT

White dwarf stars provide independent chronometers for the age and star formation history of the Galactic disk (Winget et al. 1987; Iben & Laughlin 1989; Yuan 1989; Noh & Scalo 1990; Wood 1992; for recent reviews, see Wood 1995; Cowan et al. 1997). Recent ground-based and *Hubble Space Telescope* (*HST*) observations of white dwarfs in Galactic and globular clusters offer the prospect of cross-calibrating white dwarf chronometry with the more traditional technique of isochrone fitting (Claver 1995; Claver & Winget 1995); this is of particular importance because ages of globular clusters determined by isochrone fitting and related techniques are at odds with several recent determinations of H_0 (for a summary, see Jacoby 1994 and references therein; Bolte & Hogan 1995).

In addition, understanding the evolution and luminosity function of halo white dwarfs has become increasingly important for the interpretation of the data from the MACHO collaboration (Alcock et al. 1996) search for gravitational lensing resulting from objects in the halo. Their data reveal at least seven events with estimated average masses for the lenses of $\langle M_* \rangle \sim$ 0.5 $M_\odot$, which suggests that as much as 50% of the Galactic halo mass may be in the form of white dwarf stars (Bennett 1996). This has been investigated further by Adams & Laughlin (1996) and Chabrier, Segretain, & Méra (1996). As the latter authors point out, the inferred fraction of the mass of the halo in white dwarfs depends in an extremely sensitive way on how crystallization proceeds in the interiors of white dwarf stars.

White dwarf evolution is a cooling process: white dwarfs radiate at the expense of the residual thermal energy of their ions. As the thermal energy drops, the Coulomb interactions play an increasingly important role. The strength of these in-

teractions is determined by the total charge, Z, of the ions and the interionic spacing, and so depends only on the chemical composition and the density. When the ratio of the Coulomb energy to thermal energy is of order unity, matter enters the Coulomb liquid domain. When the ratio reaches ~173 (see, e.g., Farouki & Hamaguchi 1993), theory suggests that a first-order phase transition takes place and the fluid crystallizes; because the transition is first order, there is an associated release of the latent heat (Mestel & Ruderman 1967; Van Horn 1968), and this adds of order 1 Gyr to the age of the cool white dwarfs (Winget & Van Horn 1987). In addition, we do not know if crystallization proceeds as an alloy or if the C, O, and various trace elements undergo phase separation at crystallization and provide an additional energy source; the best current theoretical calculations suggest that some phase separation does take place (see Chabrier et al. 1993 for a review; Blasio & Lazzari 1996). This has a large effect on all of the applications described above because it increases the age estimates for the coolest white dwarfs by an additional 20% (Segretain et al. 1994; Hernanz et al. 1994).

Kirzhnits (1960), Abrikosov (1961), and Salpeter (1961) first predicted the occurrence of crystallization in matter under conditions appropriate to the interiors of cool white dwarfs and, as it would later turn out, the outer layers of neutron stars (see, e.g., Blasio & Lazzari 1996 and references therein). Since then, more than 35 yr have passed, and this theory has been shown to have profound consequences for our understanding of the history and composition of the stellar population of the Galaxy—yet there remained no direct observational confirmation of the occurrence of crystallization. In this Letter we demonstrate that the DAV white dwarf BPM 37093 should have substantial crystallization according to current theoretical models. We then point out how the techniques of asteroseismology can be used to measure the crystallized mass fraction of BPM 37093 and provide a crucial test of the theory of crystallization in white dwarf interiors. Remarkably, this circumstance was anticipated almost two decades ago in the pioneering work of Hansen & Van Horn (1979).

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2. OBSERVED PROPERTIES OF BPM 37093

Pulsations in the DA white dwarf BPM 37093 were first reported by Kanaan et al. (1992). Observations by Giovannini (1996) confirmed that this is a low-amplitude pulsating DA white dwarf (DAV star) with a dominant period of ~600 s. Kanaan (1996) reports additional data obtained at the Cerro Tololo Inter-American Observatory (CTIO) in 1995 and 1996 and discusses it together with the earlier data.

Kanaan's work demonstrates several key properties of this star, which we summarize here. First, he establishes that the star has at least four independent frequencies, hereafter modes, present. All are at low amplitude and exhibit complex behavior. These results are puzzling in the context of the other ZZ Ceti stars. First, the star is near the red edge of the observed DAV instability strip (Giovannini 1996). The general trends found in DAV stars have been summarized by Winget & Fontaine (1982) and more recently by Clemens (1994, 1995), Giovannini (1996), Kanaan (1996), and especially Bergeron et al. (1995 and references therein). Stars near the red edge tend to have complex power spectra dominated by long-period modes (of order 600 s or longer) and the largest amplitudes. BPM 37093 lies near the red edge and has the characteristic long periods but has anomalously low amplitudes. Thus, BPM 37093 is unusual among the other ZZ Ceti stars solely on the basis of its photometric properties.

Recent determinations of the temperature and gravity point to another unusual characteristic of BPM 37093. The best published values come from the landmark analysis of the known ZZ Ceti stars by Bergeron et al. (1995), who were careful to fit the UV and optical spectra and to obtain consistent UV and optical temperatures. They give $T_{eff} = 11,730$ K and $\log g = 8.81$. With the evolutionary models of Wood (1990, 1992), this implies a mass of $M/M_{\odot} = 1.09$. This makes it far and away the most massive known DAV; the next most massive DAV is G207-9 with $M/M_{\odot} = 0.83$. The errors are not stated explicitly in Bergeron et al. (1995), but if we use the values from the self-consistent results of Bragaglia, Renzini, & Bergeron (1995; these two analyses used the same 10 optical spectra and essentially the same model atmospheres), we obtain for the mass and temperature of BPM 37093, respectively, $M/M_{\odot} = 1.09 \pm 0.065$ and $T_{eff} = 11,730 \pm 740$ K. Giovannini (1996), using a 1.3 hr spectrum obtained at CTIO and Koester's model atmospheres (see, e.g., Koester, Schulz, & Weidemann 1979), measured $M/M_{\odot} = 1.03 \pm 0.04$ and $T_{eff} = 12,700 \pm 300$ K.

3. THEORETICAL CONSEQUENCES OF THE OBSERVED PROPERTIES

The total mass from the results of Bergeron et al. (1995) implies that its interior has no significant carbon and borders on the minimum theoretical mass for Ne/O core composition (Iben 1991). Again, this serves to set BPM 37093 apart from the other DAV stars, which we expect to have predominantly C/O cores. The large gravity has a further significance, which is the focus of this Letter. This becomes clear on an inspection of Figure 1. Here we give the location of BPM 37093 in the $M/M_{\odot}$–$\log T_{eff}$ plane according to Bergeron et al. (1995) with the error bars from Bragaglia et al. (1995) as described above. In addition, we superpose the locus of points representing constant interior crystallized mass fraction in theoretical models for pure C and pure O interior compositions from the models of Wood (1992)—recall that these are the models used by Bergeron et al. to determine the masses, so the analysis is self-consistent.
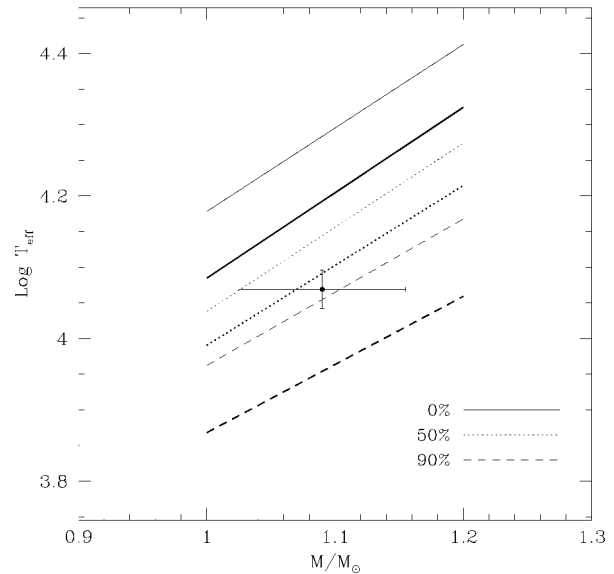

FIG. 1.—Theoretical crystallization curves based on models by Wood (1990) plotted with the best-fit point for BPM 37093 from Bergeron et al. (1995) and error bars from Bragaglia et al. (1995). The darker set of lines is for carbon core models and the lighter set is for oxygen core models. The different styles of lines denote different percentages of crystallization.

Figure 1 demonstrates clearly that BPM 37093 is well within the domain where we expect that a substantial mass fraction of the star is crystallized, even in the unlikely event that the interior is as light as pure C. The best values imply a crystallized mass fraction of nearly 90% for a pure O interior and, by extrapolation, a greater fraction for heavier core compositions. This makes BPM 37093 unique among all known pulsating compact objects, in that it is the only one expected on theoretical grounds to have a substantially crystalline interior. This in itself is of great interest only if there is some way of measuring the location of the interior liquid/crystal boundary on the basis of observations of the photosphere. Our preliminary investigations indicate that asteroseismology gives us a way to do this.

4. ON THE POSSIBILITY OF ASTEROSEISMOLOGICAL DETERMINATION OF THE CRYSTALLIZED MASS FRACTION IN BPM 37093

Because the pulsations of the DAV, or ZZ Ceti, stars are global in nature, with sufficient observational data it is possible to measure internal structure effects well below the photosphere and to make independent measures of the total stellar mass (see, e.g., Winget et al. 1991, 1994; Kepler & Bradley 1995 and references therein).

The first possibility that suggests itself for detecting and measuring the effects of crystallization is measuring the rate of period change. This possibility was considered by Bradley (1996), who independently noticed that BPM 37093 might be crystallized. Here the idea is that secular evolution of the star can be detected by measuring the time rate of change for a pulsation period (see Winget, Hansen, & Van Horn 1983; Kepler et al. 1991, 1995). We would expect that the evolutionary timescale would be changed by the presence of crystallization, either because the release of latent heat of crystallization extends the cooling time or, if crystallization has proceeded far enough, because Debye cooling of the crystalline interior causes accelerated evolution (see, e.g., Lamb & Van Horn 1975;

Winget & Van Horn 1987); in addition, as we will see below, the dominant effect on the rates of period change from crystallization may be the movement of the crystallization boundary causing the periods to increase. Unfortunately, measuring the period change with current techniques will take on the order of 10–20 yr for a DAV star (Kepler & Bradley 1995; Kepler et al. 1995), after one has identified a stable pulsation mode to measure. Any hope of using this technique to detect crystallization is dealt a further blow by existing observations (Kanaan 1996; Kanaan et al. 1992) that demonstrate that BPM 37093 is a complex pulsator. Unfortunately, measurement of the rate of period change in a peak that is not stable in phase or amplitude on timescales of years is essentially impossible.

Fortunately, recent work by Kleinman and collaborators (Kleinman 1995a, 1995b) and Kleinman et al. (1997) shows that complex DAV stars are normal mode pulsators and that by obtaining data over several observing seasons, one can see many peaks come and go, but each represents either a normal mode or a linear combination of normal modes. In this way, we can hope to identify the mean period spacing in the star (for a discussion see Kepler & Bradley 1995 and references therein).

Our preliminary analysis (described in more detail in Montgomery et al. 1997) of models of crystallizing white dwarf stars indicates that there is a significant effect on the nonradial g-mode frequencies, consistent with the early results of Hansen & Van Horn (1979). This effect manifests itself in a readily quantifiable way in terms of the mean period spacing. For our preliminary analysis, we have adopted the approximation of setting the vertical displacement of the g-modes to zero at the solid/liquid interface, since the vertical displacement must be continuous across this boundary and is assumed to be small in the crystallized region (this approach has been applied to the fluid/solid interface in neutron stars; see Bildsten & Cutler 1995). We then moved this boundary condition outward through the stellar model in order to approximate increasing amounts of crystallization. The results shown in Figure 2 are for a model with $M/M_\odot = 1.1$ and $T_{eff} = 12,023$ K. The filled circles represent the normalized mean period spacing as a function of crystallized mass fraction as determined from the differencing of numerically computed periods in the range of 500–1000 s. For example, for a 50% crystallized model ($M_{xtal}/M_\star = 0.50$), the period spacing is a factor of 1.15 of the uncrystallized period spacing, which is a 15% increase. For $M_{xtal}/M_\star = 0.90$, this increase is approximately 25%.

The solid line in Figure 2 is derived from an analytical expression for the mean period spacing (Unno et al. 1989, eq. [16.41]); it is given by the following equation:

$$\frac{\langle\Delta P\rangle}{\langle\Delta P_0\rangle} = \frac{\int_{r_a}^{r_b} N \, dr/r}{\int_{r_x}^{r_b} N \, dr/r}, \tag{1}$$

where N is the Brunt-Väisälä frequency, r is the radial coordinate, r_a and r_b are the inner and outer turning points of propagation of the mode in the uncrystallized case, respectively, and $r_x(M_{xtal}/M_\star)$ is the inner turning point in the crystallized case. It is clear that this analytical formula gives nearly the same result as that obtained directly from period differencing, with the added advantage of a more transparent interpretation. We can see from equation (1) that as the star crystallizes and r_x moves farther outward, the denominator decreases (since the integrand is real and positive between r_a and r_b), which leads
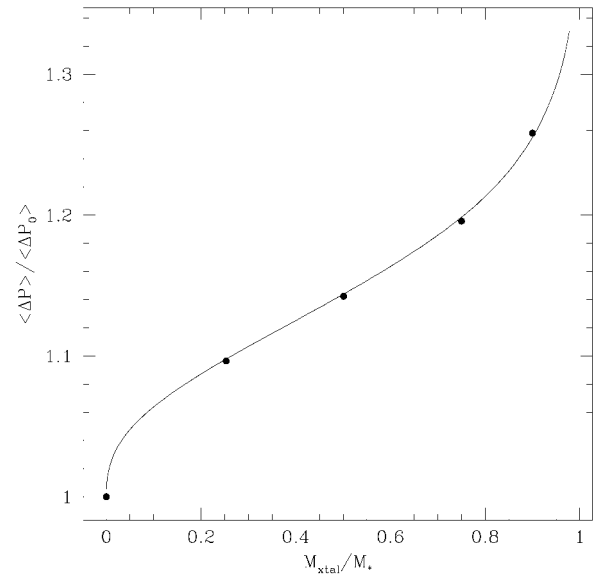

FIG. 2.—The normalized mean period spacing, $\langle\Delta P\rangle/\langle\Delta P_0\rangle$, is plotted against the effective crystallized mass fraction of the model, $M_{xtal}/M_\star$. $\langle\Delta P_0\rangle$ is the mean period spacing of the uncrystallized model in the period range of 500–1000 s, and $\langle\Delta P\rangle$ is the mean period spacing as a function of $M_{xtal}/M_\star$. The filled circles represent the results from period differencing of the actual period structure of the model with the boundary condition applied at the top of the crystallized core, and the solid line gives the result based on the analytical expression shown in eq. (1).

to an increase in $\langle\Delta P\rangle/\langle\Delta P_0\rangle$. This trend is clearly illustrated in Figure 2.

From an observational standpoint, if we have four or more modes of the same degree, l, the prospects of measuring the period spacing to an accuracy of 1 s or better are excellent (Bradley & Winget 1994; Winget et al. 1994). For the present model, this accuracy would translate into an error of ~3% in $\langle\Delta P\rangle$, much smaller than the size of the effect we have estimated. Of course, $\langle\Delta P\rangle$ is in reality a function of many model parameters, such as the thickness of the surface hydrogen and helium layers, the total stellar mass, and the effective temperature (Bradley 1996). From the massive models in Bradley (1996), we estimate that a change in the hydrogen layer mass from $M_H = 10^{-5}M_\star$ to $M_H = 10^{-6}M_\star$ results in a 10% increase in the period spacing, which is of the same order as the effect due to crystallization. This means we will need additional independent constraints on at least some of these parameters in order to make statements regarding the degree of crystallization in our models of these stars. Fortunately, such constraints exist; for example, if we observe enough modes in BPM 37093 to see a trapping cycle, this can provide an independent constraint on the hydrogen layer mass (Winget et al. 1991).

The preliminary results of Montgomery et al. (1997) underscore the importance of extensive observation of BPM 37093 in order to measure as many periods as possible. With more modes, it should be possible to apply the techniques of Kleinman (1995a, 1995b) to measure a mean period spacing without making individual identification of the degree, l, of the pulsation modes necessary. On the other hand, currently approved *HST* observations of the relative UV/optical amplitude in BPM 37093 promise to make individual l identifications possible and to improve the mass determination. Taken together, these observations promise a very sensitive measurement of the crystallized mass fraction of the DAV star BPM

37093, and thereby a critical test of the theory of white dwarf crystallization.

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