Stochastic dynamics in the interaction of magnetized electrons and electromagnetic waves with ordinary-mode polarization

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In the present work we perform analytical and numerical investigations on the conditions under which there occurs stochastic exchange of energy between unidirectionally magnetized electrons and perpendicular electromagnetic waves that propagate with ordinary-mode polarization. It is observed that for low amplitudes of the fluctuating field, the electronic dynamics is essentially regular for small initial energies. On the other hand, for larger energies, the dynamics may be chaotic. It is also shown that stochastic energy excursions may be larger than resonant ones.

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With the advent of high-power free-electron lasers [1], gyrotrons [2], and the modern concepts of stochastic particle acceleration [3], the analysis of relativistic nonlinear interactions between magnetized particles and finite-amplitude electromagnetic waves has become an area of major interest since it is of fundamental importance to identify the optimal conditions for wave-particle energy exchange. Actually, it has already been shown that, among other situations, such an exchange can take place in the case of parallel (with respect to the external magnetic field) wave propagation [4], oblique propagation [3], and perpendicular propagation of extraordinary modes interacting with electrons of zero longitudinal momentum [5].

In this regard, there has been some recent interest in the cyclotron interaction of magnetized electrons and large-amplitude ordinarily polarized modes propagating perpendicularly to the ambient magnetic field [6,7]. An approximation used in this context is to take the electronic orbits to be affected only when isolated resonance conditions involving the cyclotron and wave frequencies are satisfied. These conditions are written in the form

$$m\omega_c \cong \omega , \tag{1}$$

where $m=\pm 1, \pm 2, \ldots$, with ω_c and ω as the cyclotron and wave frequencies, respectively. It is known that in cases of weak coupling between field and particles, the isolated-resonance approximation is appropriate; the resonances do not touch each other because their widths are very small and the perturbed orbits occupy approximately the same region in the phase space as the unperturbed ones. On the other hand, when the coupling grows larger, the resonances may touch, establishing some sort of web over which particles can be driven very far away from their initial location [8–11].

As already mentioned, the works quoted above [6,7] are directed toward the study of the action of isolated resonances on the orbits. We shall see that under the specific conditions assumed of small values of momenta and field amplitudes, this is the relevant situation. Here, the simultaneous action of several resonances shall be taken into account. This seems to be the suitable approach to the study of the moderately relativistic highly suprathermal electrons that are involved in the electron-cyclotron interactions [12]. Moreover, we intend to show that once the overlap of various resonances for a particular set of initial parameters occurs, it persists for larger values of the perpendicular momentum, leading to a stochastic dynamical transition between initial and final electronic orbital states with different characteristics.

Let us consider the relativistic Hamiltonian of our system, recalling that it is constructed in order to describe the motion of an electron submitted to the combined action of a constant and homogeneous magnetic field pointing along the z axis and a perturbing wave field propagating along the y axis with the corresponding electric field fluctuating along the z axis. Such a Hamiltonian reads

$$\mathcal{H}=mc^2\left\{1+\frac{1}{m^2c^2}\left[\left(P_x+\frac{eB_0}{c}y\right)^2+P_y^2+\left(P_z-\frac{e}{c}A_z\right)^2\right]\right\}^{1/2} , \tag{2}$$

where $\mathbf{P}$ is the canonical momentum, B_0 is the ambient magnetic field, and A_z is the vector potential for the considered fluctuating electromagnetic mode. As said before, this wave field represents an ordinary like mode propagating along a perpendicular direction (we choose here the y axis), being therefore given by the form

$$A_z=a\cos(ky-\omega t) , \tag{3}$$

with a as the constant amplitude of the wave and k as its wave vector.

At this point it is advantageous to introduce canonical quantities that are explicitly conserved in the absence of the wave field. The respective variable transformations are to be performed with help of the generating function $\mathcal{F}$ defined by

$$\mathcal{F}(\mathbf{P},\mathbf{r}')=\frac{c}{eB_0}(P_xP_y-P_y^2/2tgy')-x'P_x-z'P_z , \tag{4}$$

where primed and unprimed variables denote new and old coordinates, respectively.

If we define $H \equiv \mathcal{H}'/mc^2$, $I \equiv eB_0 P_y'/m^2c^3$, replacing $P_z'/mc \rightarrow P_z$ and $eA_z/mc^2 \rightarrow A_z$, it becomes possible to recast the Hamiltonian in the form

$$H=(1+2I+P_z^2-2P_z a\cos\psi + a^2\cos^2\psi)^{1/2}\ , \qquad (5)$$

where the function H produces a canonical set of equations if time and space are normalized accordingly to $t\rightarrow(eB_0/mc)t$ and $\mathbf{r}\rightarrow(eB_0/mc^2)\mathbf{r}$ and where we use the additional definitions, $\psi \equiv k\sqrt{2I}\,\sin\alpha - \omega t$ and $y' \equiv \alpha$, from which one can readily see that z is a cyclic coordinate and the longitudinal momentum P_z is a conserved quantity.

As the normalized amplitude a of the electromagnetic wave is in general much smaller than 1, it becomes possible to expand the Hamiltonian as a power series on that quantity. Doing this, one obtains in the lowest significant order

$$H \approx \frac{1}{\omega_c} - \omega_c P_z a \cos\psi$$
$$\equiv h_0(I,P_z) + h_1(I,P_z,\psi)\ , \qquad (6)$$

where we have introduced the relativistic cyclotron frequency as

$$\omega_c \equiv h_0^{-1} \equiv (1+2I+P_z^2)^{-1/2}\ ,$$

taking $|P_z| \geq |a|$ (a suitable approximation for systems with not-too-low longitudinal currents) to discard second-order terms. When $a\rightarrow 0$, the momentum I also becomes a constant of motion; in that case the Hamiltonian is simply h_0 and the dynamics is to be evaluated from the equation

$$\dot{\alpha} = \partial_I h_0 = \omega_c\ , \qquad (7)$$

which says that on the perpendicular (to the steady magnetic field) plane the electrons describe a periodic motion whose frequency decreases with I and/or P_z.

Now, by making use of Bessel expansions

$$\cos(k\cos\theta) = \sum_{m=-\infty}^{+\infty} J_m(k)\cos(m\theta)\ , \qquad (8)$$

one can use the concept of resonance overlap [8–11] to estimate whether or not the dynamics is approximately integrable. In order to do so, we first calculate the location of each resonance defined by Eq. (1) and then we investigate for which values of the relevant parameters, adjacent resonances do overlap.

Using standard procedures [8–11], one can write this overlap condition as

$$r \equiv 2\frac{\delta I_p}{\Delta I_p} = 2\omega[2P_z a J_p(k\sqrt{2I_p})]^{1/2} > 1\ , \qquad (9)$$

where we define δI_p ($I \equiv I_p + \delta I_p$) as the width of the p resonance introduced in Eq. (1) and ΔI_p as the distance between the p and $p+1$ resonances. To derive Eq. (9), one expands the Hamiltonian h_0 up to second-order terms in δI_p to include the relativistic detuning [6,7] but considers the Bessel functions with $I \rightarrow I_p$. This is an inaccurate approximation when I_p is so small that $I_p \approx \delta I_p$, but we proceed along this way, because it provides reasonable order-of-magnitude estimates.

Next we perform a numerical analysis where we investigate the nonlinear dynamical behavior of our system assuming that initially it is close to a situation for which an "initial harmonic number" can be defined:

$$p_0 \equiv \frac{\omega}{\omega_c(I_0)}\ , \qquad (10)$$

with the subscript zero indicating the value at $t=0$. We investigate small and moderately large values of I_0

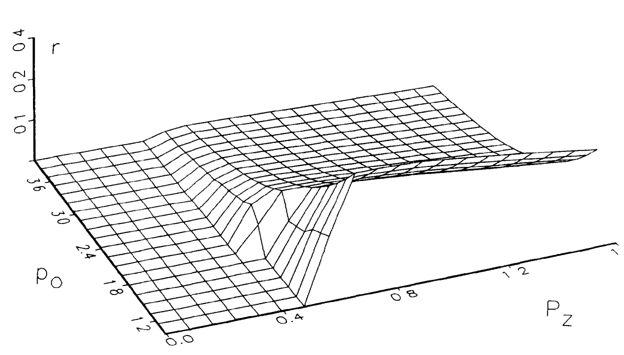

FIG. 1. The ratio $r = r(p_0, P_z)$ for $I=0.005$.

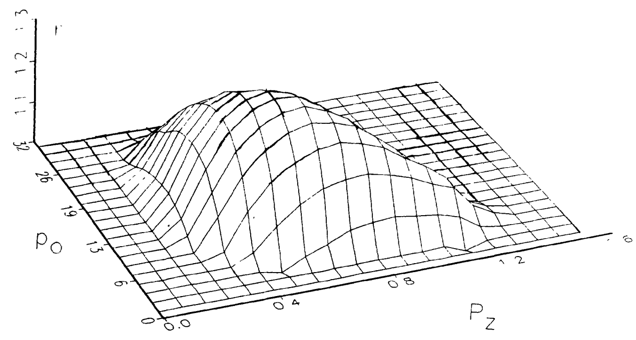

FIG. 2. The section $r \geq 1.0$ of the ratio for $I=1.0$.

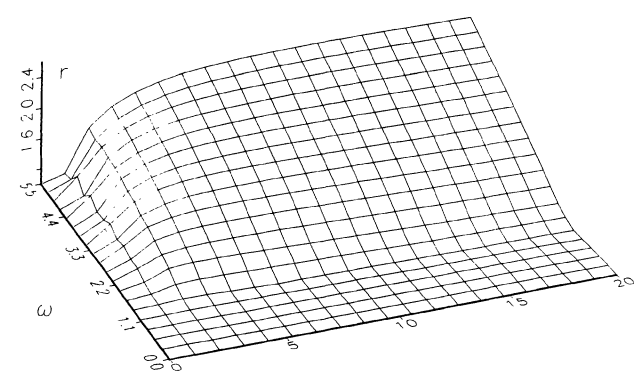

FIG. 3. The section $r \geq 1.0$ of r, now calculated as a function of I and ω; $P_z = 1.0$.

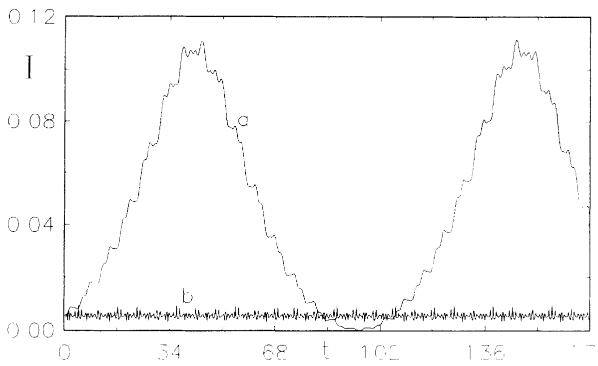

FIG. 4. I vs the normalized time for $I_0=0.005$, $P_z=0.1$, and $\alpha_0=\pi/2$, with (a) $p_0=1$ and (b) 7.

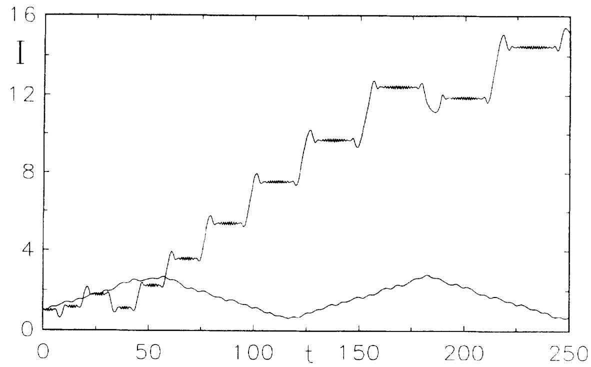

FIG. 5. I vs the normalized time for $I_0=1.0$, $P_z=1.0$, and $\alpha_0=\pi/2$, with (a) $p_0=1$ and (b) 7.

($I_0=0.005$, 1.0), with $a=-0.2$. The case of small values of I_0 and P_z with a typical wave amplitude similar to the one we are using is analyzed in Refs. [6] and [7]. We shall compare both situations to show that new physical aspects are present when one considers larger values of the initial momenta.

To begin with, in Fig. 1 we plot the behavior of the ratio r as a function of both the harmonic number p_0 and the longitudinal momentum P_z, with the wave frequency set as a harmonic of the cyclotron one, $\omega=p_0\omega_c(I_0,P_z)$, with $I_0=0.005$. It is seen that for that particular value of I_0, r is always smaller than 1. In Fig. 2 we plot the section $r\geq 1$ of $r=r(p_0,P_z)$ for $I=1.0$. In contrast to the previous case, r is larger than 1 within a limited region of the p_0-P_z plane; in both cases the width of the resonance islands reduces with p_0. In Fig. 3 we study the behavior of r as a function of I and ω for $P_z=1.0$, considering, from now on, $\omega=k$ ($c=1$). This figure reveals a peculiar feature; it shows that once overlap does take place for a particular set of parameters and initial conditions, it persists for larger values of I that may be attained as the system evolves in time. This may result in relatively large excursions of the angular momentum in the chaotic regime.

The general conclusion is that for the typical (and representative) wave-field intensity employed in this paper, overlap may be present only for relatively large values of I_0 and P_z. This implies that in the case $I\ll 1$, the largest energetic excursions will be probably obtained for the (integrable) first harmonic resonance, a result compatible with other works [6,7]. For larger values of I, on the other hand, it may happen that stochastic excursions are larger than resonant ones if p_0 and P_z pertains to the stochasticity domain appearing in Fig. 2. Next, we integrate the canonical equations for I and α in order to check the validity of the above assertions.

In Fig. 4 we plot I versus t for $I_0=0.005$, $P_z=0.1$, $\alpha_0=\pi/2$, and $p_0=1$ and 7; in Fig. 5 the same is done for $I=1.0$ and $P_z=1.0$. From Fig. 4, it is possible to observe that for both values of p_0, there occur bounded periodic oscillations of I with time, with the smaller amplitude for the largest value of p_0, a result that confirms the analytical predictions. From Fig. 5, on the other hand, we see that for the particular value $p_0=7$ that pertains to the domain of chaoticity displayed in Fig. 2, the excursion of I is larger than the one corresponding to the first resonance, which again confirms the analytical conclusions.

To summarize, in this work the nonlinear interaction between magnetized electrons and electromagnetic waves with ordinary-mode polarization has been studied. It was shown that for small values of I_0 and P_z, the dynamics is essentially regular, with the largest excursion of the angular momentum I corresponding to the first harmonic resonance, a result totally coherent with the analysis done in previous papers [6,7].

On the other hand, for larger values of I_0, it was shown that chaoticity may be present for a limited range of the initial harmonic number p_0 and momentum P_z. Interestingly, once chaoticity is established for a particular initial value of I, it persists for larger values of this quantity. In this regard it was shown that the largest excursions of I correspond not to the regular first harmonic resonance as in the former case, but to values of p_0 and P_z well inside the stochasticity domain.

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