Nonlocality of the isobar propagation and the effective Δ-nucleus spin-orbit interaction

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The imaginary central and spin-orbit components of the Δ-nucleus optical potential are investigated in a π-exchange model. It is found that the effective spin-orbit interaction of the isobar reflects large nonlocalities from true pion absorption and from the quasielastic Δ decay. The parameters for the absorption and the spin-orbit strength are in qualitative agreement with recent phenomenological results.

[NUCLEAR SCATTERING Δ isobar propagation; relation of nonlocality and spin-orbit interaction.]

The formulation of the isobar doorway model[1–7] has provided new insight into the dynamics of nuclear reactions at intermediate energies. In particular it turned out that in view of the selective sensitivity of the Δ-particle N-hole ($\Delta\bar{N}$) states on the nuclear medium, higher order corrections—beyond that of pion multiple rescattering—are crucial for a quantitative understanding. Unfortunately, a microscopic calculation of such medium corrections—among which true π-absorption and reflection terms are the most important ones[3]—requires rather drastic assumptions about the elementary input. Alternatively Hirata *et al.* and recently Horikawa *et al.* parametrized the higher order corrections in terms of a local one-body operator with a central[2] and an effective Δ-nucleus spin-orbit interaction[7]

$$V_\Delta(r;\omega)=\left\{[V_c(\omega)+iW_c(\omega)]-[V_{ls}(\omega)+iW_{ls}(\omega)]\,\vec{l}\,\vec{\Sigma}\frac{r_0^2}{r}\frac{d}{dr}\right\}\rho(r)/\rho_0 \qquad (1)$$

as a function of the scattering energy ω and the nuclear density $\rho(r)$ (r_0 is a scale parameter, taken to be 1 fm). For an energy-independent spin-orbit term $W_{ls}(\omega)$ a best fit of elastic π-scattering data on ^{12}C was obtained with (compare Fig. 3)

$$W_c(\omega)\simeq -40\text{ MeV} \qquad (2)$$

together with

$$W_{ls}(\omega)\simeq -W_c(\omega)/10\ . \qquad (3)$$

Though the ansatz in Eq. (1) is fairly successful in actual calculations, its shortcomings are obvious: Incorporating different medium corrections in such a simple parametrization necessarily prevents their detailed and systematic investigation. In addition, an interpretation of the various effective coupling constants, obtained from a fit of Eq. (1) to experimental data, is not unique, as the resulting parameters have to reflect the shortcomings of the parametrization itself [for example, they artificially have to mock-up nonlocal effects in the central part of the isobar-nucleus optical potential, absent in the ansatz in Eq. (1)].

In a microscopic approach to the parametrization from Eq. (1) we concentrate in the following on the absorptive part of $V_\Delta(r;\omega)$: for this piece a diagrammatic expansion is promising due to the small number of inelastic channels (in addition, the real part of the Δ-nucleus potential receives large contributions from σ and ω exchange and requires a much more detailed model than sketched below).

In the same spirit we evaluate the two leading diagrams from Fig. 1 for π exchange only, mocking-up the influence of heavy mesons, such as the ρ meson, by a relatively small π cutoff with $\Lambda_\pi \sim 800$ MeV.[8] With static πNN and $\pi N\Delta$ vertex functions we then obtain from old-fashioned perturbation theory for the

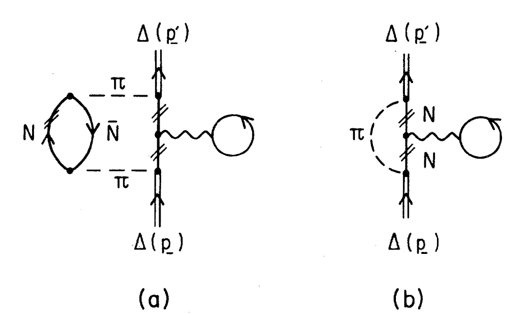

FIG. 1. Leading diagrammatic contributions to the imaginary isobar self-energy (to first order in the nuclear density) through the coupling to the 2p-1h continuum due to true π absorption (π, 2N) (a) and to the quasielastic channel (b). Above the wiggly lines represent the shell model potential of the nucleon; the double lines indicate Pauli blocking for the nucleon.

2π-exchange diagram in Fig. 1(a)

$$V_{\pi\pi}(\vec{p},\vec{p}\,';\omega)=\left(\frac{f_\pi}{m_\pi}\right)^2\left(\frac{f_\pi^*}{m_\pi}\right)^2\frac{1}{(2\pi)^3}\int\frac{d\vec{k}N(\vec{k};\vec{p},\vec{p}\,')}{\omega_\pi(\vec{k}+\vec{p})\omega_\pi(\vec{k}+\vec{p}\,')}\frac{F_\pi{}^2(\vec{k}+\vec{p}\,')F_\pi{}^2(\vec{k}+\vec{p})F_\Delta(k_0)}{2T_N(k)-\omega+i\epsilon}$$
$$\times\frac{1}{2}\left\{\frac{1}{\omega_\pi(\vec{k}+\vec{p}\,')+T_N(k)}+\frac{1}{\omega_\pi(\vec{k}+\vec{p}\,')+T_N(k)-\omega+i\epsilon}\right\}$$
$$\times\frac{1}{2}\left\{\frac{1}{\omega_\pi(\vec{k}+\vec{p})+T_N(k)}+\frac{1}{\omega_\pi(\vec{k}+\vec{p})+T_N(k)-\omega+i\epsilon}\right\} \quad (4)$$

with the numerator

$$N(\vec{k};\vec{p},\vec{p}\,')=\vec{S}_1^\dagger(\vec{k}+\vec{p}\,')\,\vec{\sigma}_2(\vec{k}+\vec{p}\,')\vec{S}_1(\vec{k}+\vec{p})\,\vec{\sigma}_2(\vec{k}+\vec{p})\vec{T}_1^\dagger\vec{\tau}_2\vec{T}_1\vec{\tau}_2\ . \quad (5)$$

Above f_π and $f_\pi^*=2f_\pi$ (Ref. 9) denote the πNN and $\pi N\Delta$ coupling constants; $\omega_\pi(q)=(q^2+m_\pi{}^2)^{1/2}$ is the pion energy and $T_N(k)$ the kinetic energy of a nucleon in the intermediate state (corrections on the level of single particle energies are dropped). The form factors $F_{\pi(\Delta)}(q)$ (Ref. 9) in Eq. (4) include off shell corrections both for the virtual pions (we use the same cutoff mass Λ_π at the the πNN and the $\pi N\Delta$ vertices) and for the Δ-isobar[10,11] (k_0 is the on shell pion momentum in the πN c.m. system for the scattering energy ω with $k_0=k_R\sim 230$ MeV/c at resonance).

The kinematics of the two-nucleon process allows a rather natural separation of the nonlocal potential $V_{\pi\pi}(\vec{p},\vec{p}\,';\omega)$ of Eq. (4) into a local piece and the leading nonlocal correction: At resonance the momentum of the nucleons emitted satisfies

$$k_N\simeq\sqrt{M\omega}\simeq 500\ \text{MeV}/c >> p,p' \quad (6)$$

for typical momenta p and p' of the isobar. By dropping then all the $\vec{p}$ and $\vec{p}\,'$ dependence relative to $\vec{k}_N$ except for the invariant $\vec{S}_1^\dagger\vec{p}\,'\vec{S}_1\vec{p}$ the only dependence of $V_{\pi\pi}(\vec{p},\vec{p},\omega)$ on the isobar momentum is contained in the spin-orbit term of the numerator, which then reads, after summing over the intermediate NN state, as

$$N(\vec{k};\vec{p},\vec{p}\,')=\frac{k^2}{3}\left[k^2+\frac{i}{2}\underline{\Sigma}(\vec{p}\,'\times\vec{p})\right]\ , \quad (7)$$

where the transition matrix $\vec{\Sigma}$ is defined by $\langle\frac{3}{2}\,\|\,\vec{\Sigma}\,\|\,\frac{3}{2}\rangle=2\sqrt{15}$.

In going over to coordinate space we introduce the spin-orbit operator for the isobar by

$$i\vec{\Sigma}(\vec{p}\,'\times\vec{p})\rightarrow\vec{l}\,\vec{\Sigma}\frac{1}{r}\frac{d}{dr}\ . \quad (8)$$

As the remaining part of the Box diagram is independent of $\vec{p}$ and $\vec{p}\,'$ its Fourier transform yields schematically

$$V_{\pi\pi}(\vec{r}-\vec{r}_N;\omega)\sim\delta(\vec{r}-\vec{r}_N)\ . \quad (9)$$

By folding in the nuclear density we recover the form of Eq. (1) for the Δ-nucleus potential with the imaginary central and spin-orbit part given, respectively, as

$$W_c(\omega)=-\frac{1}{48\pi}\left(\frac{f_\pi}{m_\pi}\right)^2\left(\frac{f_\pi^*}{m_\pi}\right)^2\frac{Mk_N{}^5}{\omega_\pi{}^2(k_N)}F_\pi{}^4(k_N)F_\Delta(k_0)\rho_0\left(\frac{1}{\omega_\pi(k_N)+E_N(k_N)-E_\Delta}+\frac{1}{\omega_\pi(k_N)+E_N(k_N)-E_N}\right)^2 \quad (10)$$

and

$$W_{ls}^1(\omega)=-W_c(\omega)/2k_N{}^2\ . \quad (11)$$

(Above ρ_0 denotes the nuclear density at the origin.)

The two leading corrections to Fig. 1(a) come from the influence of the nuclear shell model potential and from the Pauli blocking.[2,3,12] While the first contribution increases $W_{ls}(\omega)$ by

$$\Delta W_{ls}^1(\omega)\cong\left(\frac{1}{6}\frac{MW_c(\omega)}{k_N{}^2}\right)V_{ls}^N \quad (12)$$

[$V_{ls}^N\simeq 18$ MeV (Ref. 13) is the coefficient for the spin-orbit interaction of the nucleon], Pauli blocking reduces the total spin-orbit strength in ^{12}C by a factor of

$$P(k_N)\cong 1-\frac{\sqrt{2}}{18}\frac{7+4a^2(k_N-\sqrt{2}/a)^2}{ak_N}\times\exp\left[-a^2\left(k_N-\frac{\sqrt{2}}{a}\right)^2\right] \quad (13)$$

for a Δ isobar with an average momentum $(\langle p^2\rangle)^{1/2}\simeq(\sqrt{2}/a)$ [$a=1.69$ fm (Ref. 7)]. The effect of both corrections is very small due to their mutual cancellation.

Similarly, we obtain for the spin-orbit contribution from Fig. 1(b) [the notation is the same as for Fig. 1(a)]

$$W_{ls}^2(r;\omega) = V_{ls}^N \frac{r_0^2}{r}\frac{d}{dr}\frac{\rho(r)}{\rho_0}\left[\frac{f_\pi^*}{m_\pi}\right]^2 (\vec{T}^\dagger\vec{T}) F_\Delta(k_0)\frac{1}{(2\pi)^3}\int \frac{d\vec{k}F_\pi^2(k)}{2\omega_\pi(k)}\frac{\vec{S}^\dagger\vec{k}\,\vec{\sigma}\,\vec{1}\vec{S}\vec{k}}{[\omega_\pi(k)+E_N(k)-E_\Delta+i\epsilon]^2} . \quad (14)$$

Evaluating the integral with standard techniques we obtain with

$$\vec{S}^\dagger\vec{k}\,\vec{\sigma}\,\vec{1}\vec{S}\vec{k} = \frac{k^2}{9}\vec{\Sigma}\,\vec{1} \quad (15)$$

the spin-orbit coefficient

$$W_{ls}^2(\omega) = \frac{\Gamma_\Delta(\omega)}{2}\left[\frac{\omega}{k_0^2} - \frac{1}{3}\frac{1}{M+\omega}\right] V_{ls}^N P(k_0) , \quad (16)$$

where $\Gamma_\Delta(\omega)$ is just the width of the Δ isobar at the scattering energy ω,[14] while $P(k_0)$ again accounts for Pauli blocking of the intermediate nucleon.

Our main findings for ^{12}C are presented in Figs. 2 and 3. In Fig. 2 the energy dependence of $W_{ls}^1(\omega)$ and $W_{ls}^2(\omega)$ is shown for representative values of Λ_π and Λ_Δ. Characteristically, the two coefficients show an opposite trend with increasing ω as expected from their gross structure

$$W_{ls}^1(\omega) \propto 1/\omega; \quad W_{ls}^2(\omega) \propto \omega^{3/2} . \quad (17)$$

For quantitative details we consider for $\Lambda_\pi = 800$ MeV (Ref. 8) two extreme choices with $\Lambda_\Delta = \infty$ (no cutoff) and $\Lambda_\Delta = 200$ MeV (Ref. 11) for the off shell continuation of the Δ isobar. The influence of Λ_Δ is significant; presently, its uncertainty just reflects current problems in defining the Δ isobar microscopically, especially off the resonance (for different philosophies compare, for example, Refs. 15–17).

In Fig. 3 we compare our results with the findings by Horikawa *et al.* For $\Lambda_\pi \sim 800$ MeV without an additional Δ cutoff we qualitatively reproduce the $W_c(\omega)$ from Ref. 7. As the same model fits the total π-absorption cross section $\pi d \rightarrow NN$ (Ref. 8) we conclude from our qualitative agreement that the phenomenological quantity $W_c(\omega)$ in Ref. 7 receives its dominant contribution from the true π-absorption process (π,NN). For $\Lambda_\Delta = 200$ MeV the agreement is much worse as the resulting energy dependence of $W_c(\omega)$ is too steep. The same situation persists for

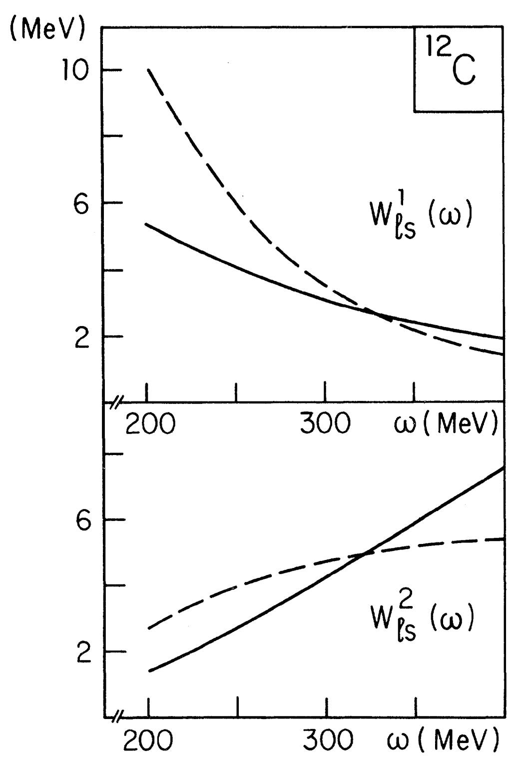

FIG. 2. Energy dependence of the coefficients $W_{ls}^1(\omega)$ and $W_{ls}^2(\omega)$ (without Pauli corrections) for $\Lambda_\pi = 800$ MeV and two different cutoff masses $\Lambda_\Delta = \infty$ (full lines) and $\Lambda_\Delta = 200$ MeV (dashed lines), respectively.

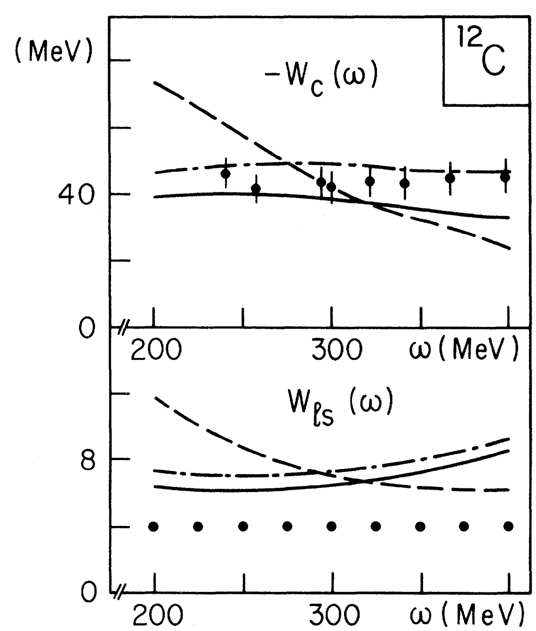

FIG. 3. Energy dependence of true π absorption $W_c(\omega)$ [Eq. (11)] and the summed strength $W_{ls}(\omega) = W_{ls}^1(\omega) + W_{ls}^2(\omega)$ of the spin-orbit interaction of the Δ isobar. Shown are the results for $\Lambda_\pi = 800$ and 900 MeV without Δ cutoff (full and dashed-dotted lines) and for $\Lambda_\pi = 800$ MeV together with $\Lambda_\Delta = 200$ MeV (dashed line), as well as the corresponding results from Horikawa *et al.* (Ref. 7) (full dots).

the total spin-orbit strength $W_{ls}(\omega) = W_{ls}{}^1(\omega) + W_{ls}{}^2(\omega)$: only without a Δ cutoff we find $W_{ls}(\omega)$ approximately constant as in the fit from Ref. 7. For the discrepancy in the absolute magnitude by a factor 1.5–2 there are two obvious interpretations: on the one side we presumably overestimate $W_{ls}(\omega)$, as our model incorporates neither ρ-exchange nor short range correlations [both mechanisms should cut down $W_{ls}(\omega)$]; on the other side, $W_{ls}(\omega)$ from Ref. 7 might indeed underestimate the complex spin-orbit interaction of a Δ isobar, as π scattering in the isobar-hole model is not yet understood on such a quantitative level.

Summarizing, we can account for some aspects of the parametrization of Ref. 7 in our simple microscopic model. We find that for a local parametrization of the central part of the Δ-nucleus optical potential highly nonlocal medium corrections—dominated by true pion absorption and reflection—have to be mocked-up by a large spin-orbit interaction of the isobar. This explains why isobar doorway calculations, which keep the nonlocalities in the higher order corrections, are similarly successful without a strong spin-orbit term.[3,18] Furthermore, the result indicates that only within a more detailed microscopic framework a comparison between the phenomenological spin-orbit interaction of the isobar from Ref. 7 and with the quark model,[19] for example, is meaningful.

It is clear that on a quantitative level the diagrammatic approach has its own problems. The sensitivity of the result on the cutoff masses is an unpleasant feature (though the same parameters already enter into a calculation of the ΔN interaction in first order); more serious are the difficulties in developing on the same basis a quantitative picture for the real parts of the Δ-nucleus potential, as it is not fully clear how well a diagrammatic expansion converges. For a conclusive answer further investigations have to be awaited.

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